BOOK - I

pages : 128

3/10/14

CONTROL SYSTEMS

1. Automatic Control system — B.C. KUO (Exercise Qns must).
2. Control system Engineering — NAGRATH & GOPAL
3. " " " — NISE
4. Control system : Design & Principle — M.GOPAL
5. Modern Control system — OGATA
6 " " " — B.S. MANKE

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CONTROL SYSTEMS

weightage → IES → 100 marks
Gate → 10 marks.

IES SYLLABUS

→ Transfer Function
- Basics
- N/ws, DE, signal
- Block diagrams, signal flow graphs.

} GATE + IES

- Mechanical system (only for IES)
  - Translational sys
  - Rotational sys
  - → FVT Analogy
  - → FIT Analogy

→ Time Domain Analysis
- Transient Analysis
- Steady state Analysis.

} GATE + IES

→ Stability
- Time domain technique. → RH/RL
- Frequency domain technique ⇒ BP, NP

} GATE + IES

M&N circles, Mag and phase plots, Nicole's chart. } IES

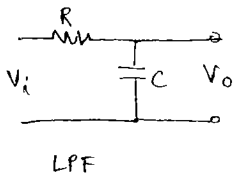
→ COMPENSATORS AND CONTROLLERS → GATE + IES

→ state space Analysis → GATE

⇒ Transfer function : Mathematical equivalent model of a system.

⇒ Order of T.F : Number of storage elements or no of time constants.

for eg.



$$\frac{V_o(s)}{V_i(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{sCR+1}$$

ie, 1 time constant $\tau_1 = RC$, Hence order = 1

⇒ If poles > zero → strictly proper T.F → Low pass filter (order of denominator greater than num)

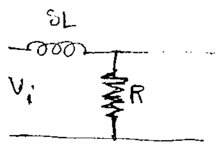
poles = zero → proper T.F

poles < zero → Improper T.F → High pass filter (order of num greater than den)

→ Low pass filter is used to remove H.F noise.

→ Control systems are designed for systems working at Low frequency ~~filte~~ to get desired results.

another LPF



$$\frac{V_o(s)}{V_i(s)} = \frac{R}{SL+R} = \frac{1}{S\frac{L}{R}+1}$$

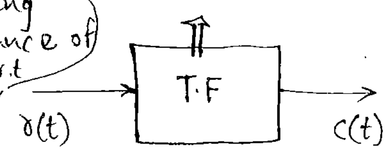
P > Z , Hence LPF.

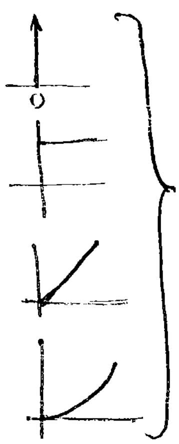
ie, 1 time constant, $\tau_1 = \frac{L}{R}$, Hence order 1

2nd chapter

Evaluating performance of syst w.r.t time

1^{st} order & 2^{nd} order.



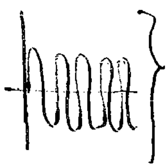


TIME DOMAIN ANALYSIS

and c(t) is evaluated w.r.t time.

& find in t_d, t_r, t_p, t_s, % M_p, e_{ss}.

⇒ Any higher order (3^{rd}, 4^{th} ...) are only a combination of 1^{st} & 2^{nd} order systems only.



FREQUENCY DOMAIN ANALYSIS

Control system specification.

1. Speed → t_r, t_s
2. Accuracy → e_{ss}
3. stability → Gain margin & phase margin. optimum value (5–10 dB).
4. Sensitivity → Temp, Noise

3rd chapter.

⇒ stability is meant only for finding the stability of closed loop systems.

→ For both open loop system and closed loop system always ~~can~~ have a open loop transfer function.

open loop ~~TF~~ system → Open loop T.F $G(s)$

closed loop system (FB will be there) → Open loop T.F $G(s)H(s)$

$G(s)$ → open loop T.F of open loop.
$H(s)$ → Feed back T.F

→ If $H(s) = 1$, then OLTF = $G(s)$ then it is a unity feedback closed loop system.

→ For a ~~cl~~ open loop T.F, we can directly determine the stability criteria using the T.F itself by determining the position of poles and zeros.

→ But in closed loop system, it is different, we cannot ~~d~~ determine stability directly from open loop T.F. Here we need to find the closed loop T.F $\left(CLTF = \frac{G(s)}{G(s)+1}\right)$. And from that finding stability we uses certain techniques. ~~OB~~ OLTF cannot be used because feedback dislocates the poles and zeros.

* For unity feedback, to the denominator of OLTF, ~~and~~ ~~add 1~~ add numerator, we get CLTF.

→ M and N circles, Mag & phase plots, Nicolas chart are to find the frequency response → Gain margin
→ Phase margin
→ ~~Regin~~ Bandwidth

→ For stability the priority order for various techniques are.

1st → Nyquist plot.
2nd → Root Locus.
3rd → Bode Plot.
4th → RH criteria.

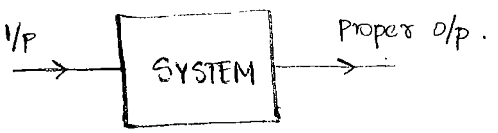
→ Time Domain technique is better than Frequency domain technique, becoz TD technique can analyse both steady state and transient ~~are~~ properties. But FD technique have only steady state.

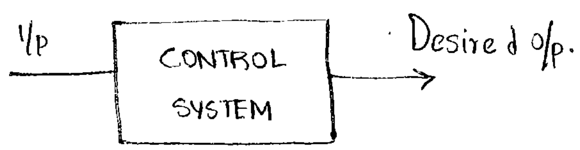
⇒ Compensators and Controllers are used to get the desired specifications and results.

It is a simple electrical n/w which ~~require~~ provide desired poles and zeros.

⇒ state space Analysis is used for Dynamic Systems. (L/NL / Time Variant / Time Invariant)

→ TF can be applied for Linear Time invariant System.





SYSTEM: Group of physical components used together for getting a ~~specific or~~ proper o/p. (may or maynot be desired output.)

CONTROL SYSTEM: Group of physical components used together for getting a Desired o/p.

" System is a group of physical components arranged in such a way that it should give the proper o/p, to the given input. The proper output may or may not be the desired output."

" control system is a group of physical components arranged in such a way that it gives the desired o/p by means of control or regulation either direct or indirect method to the given i/p."

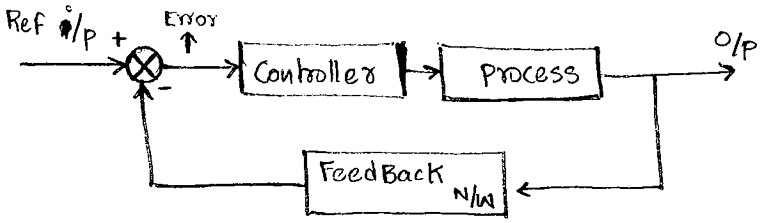
eg: A fan without Blade is not a system → No proper o/p → No air flow.
A fan without Regulator is a system → proper o/p → air flow.
A fan with Regulator is a control system → Desired o/p.

→ control systems are classified into two ways based on controlling ~~access~~ action.

1. open loop control system
2. closed loop control system

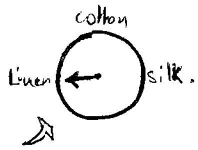
<u>OPEN LOOP CONTROL SYSTEM</u> (MANUAL)

<u>CLOSEDLOOP CONTROL SYSTEM</u> (AUTOMATIC)



> Referance Input is nothing but desired o/p (what we required).

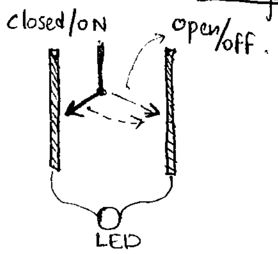
> ~~In a~~ consider the case of IRON BOX.



→ In automatic iron box, referance i/p is given by the <u>controlling knob</u>.

→ In open loop, Ref i/p cannot be set automatically. Only controlled manually.

→ In closed loop, controller is a <u>relay switch</u>.



→ Error may be +ve or −ve. Based on the sign of error, the relay will be on or off.

→ Feedback ckt is a <u>sensor</u> which senses o/p temp.

Error = Reference i/p − o/p temp.

Let Reference is 100°c. If current o/p is 99°c

Error is +ve, Relay closed, Temp↑ → 101°c

then Error is −ve, Relay open Temp↓ → 99°c

→ Above process goes ~~to~~ on continuing.

→ Controller in a ~~closed~~ open loop is a <u>manual operating switch</u>.

→ <u>Process is same</u> in both open loop and closed loop. Heating in both.

→ In open loop, controller action is completely independent of o/p.

eg: Traffic light signal action is open loop. Either traffic is there or not, light durat^n is the same.

OPEN LOOP CONTROL

A system in which the controller action independent of output, then it is called open loop control system. (for eg: Fan, ~~AC~~, Tubelights, traffic lights and so on). Any ~~Bo~~ system which is not having provision to select the reference input and not having a sensor.

CLOSED LOOP CONTROL SYSTEM

A system in which the controller action depends on the o/p, is called the closed loop control system. (for eg. A.C, Fridge, Human beings, Automatic Iron box, and so on). Any system ~~has~~ which is having a provision to select the reference input and having a sensor.

FEEDBACK NETWORK

It is a property of the closed loop system which brings the output to input and compared ~~to~~ with reference input. So that appropriate control action formed to make the error equal to zero.

- Error = 0 $\longrightarrow$ System is stable and it gives desired o/p.
- Feed back n/w is made using R, L, C ckt.

  For −ve feed back $\longrightarrow$ R

  +ve feed back $\longrightarrow$ L, C
- The maximum gain of feedback n/w ratio is 1. The best feedback is unity negative feedback.
- The steady state errors valid only for unity feedback systems.
- The −ve feedback is the best because it improves the relative stability. (Loop gain > 0).
- The feedback n/w may consists the transducer which converts the energy from one form to another.

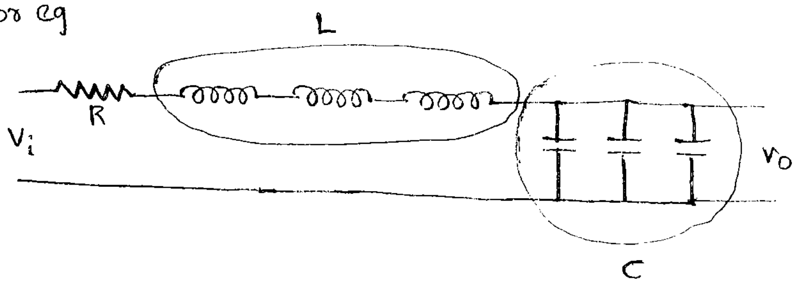
TRANSFER FUNCTION

It is nothing but the mathematical ~~model o~~ equivalent model of the system.

The order of transfer function gives the no of storage elements or

Note: whenever same kind of elements are connected either series or parallel, it should be treated as a single component.

For eg



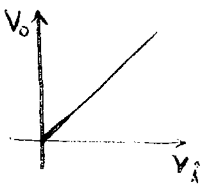
is a second order system.

Definition 1 for T.F : The transfer function of <u>Linear Time Invarient</u> system is defined as the ratio of Laplace transform ~~to the~~ of output to the Laplace transform of input, with all initial condition equal to zero.

$$\boxed{T \cdot F = \left.\frac{L[o/p]}{L[i/p]}\right|_{I_i = 0}}$$

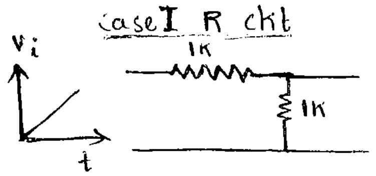
Linear : Means the transfer characteristics must be linear.

ie, V_o v/s V_1 must be of the form

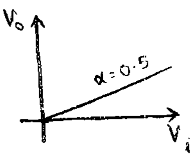


ie, using R, L and C, only we get linear transfer characteristics.

case I R ckt

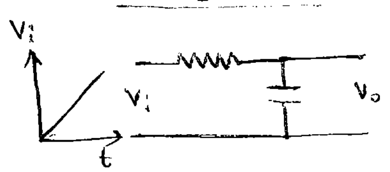


$V_o = V_i\left(\frac{1}{2}\right) = 0.5V_i \Rightarrow$

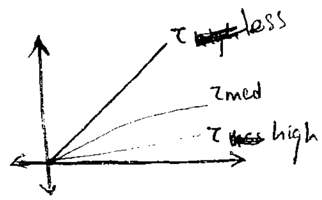


Hence Linear.

case II RC ckt.



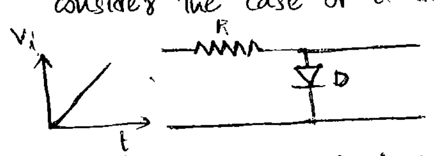
$\tau = RC$

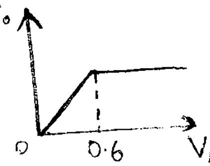


Linear.

→ All the ckt elements having ON and OFF operation will act as non linear ckts.

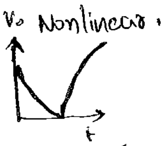
consider the case of a diode





Hence Non Linear.

V_o Nonlinear.



→ Always take the initial ~~ckt~~ condition to be zero. consider the capacitor with an initial non zero energy. Then first it removes charge before charging Hence non linear.

" LTI system is nothing but R,L,C ckt because the R,L,C ckt gives the Linear transfer characteristics and the RLC component values not changes w.r.t time."

In the transfer function, analysis the initial conditions must be zero to get the linear transfer characteristics."

Definition 2 for T·F : The transfer function of a LTI system is defined as Laplace transform, Impulse response with initial conditions zero.

Impulse → SYSTEM → Impulse Response.

Impulse Response is used because, its Laplace transform is 1, Hence it gives system response itself.

ie, no input term in impulse response.

$$\boxed{TF = \left.\frac{L[\text{Impulse Response}]}{\underbrace{L[\text{Impulse}]}_{=1}}\right|_{I_i=0} = L[\text{Impulse Response}]_{I_i=0}}$$

→ All other inputs like step, ramp, parabolic, etc, then the response will follow input and hence called Forced Response.

→ "Impulse response consist only system parameters. (K and τ)"

→ The Impulse Response is also called System Response, Natural Response or Free Forced Response

BASICS

- The standard form of the system is described in the form of open loop transfer function.

- If the system is open loop, then it is described as G(s). It is also called open loop transfer function of a system.

- If the system is closed loop, then it is described as G(s) H(s). And it is known as open loop transfer function of a non-unity feedback system.

- If H(s) = 1, then G(s) is called open loop transfer function of a unity

The standard form of the system is $\boxed{\dfrac{C(s)}{R(s)} = G(s)} = \dfrac{K(1+s\tau_1)(1+s\tau_2)\ldots}{s^n(1+s\tau_a)(1+s\tau_b)\ldots}$ CONSTANT FORM

where K, τ are called system parameters.

- The closed loop system described in the form of open loop transfer function of unity feedback system.

  s^n represents the Type n system., ~~and order.~~

- Type gives the number of poles ~~in the s plane~~ at the origin.
- order gives the total number of poles ~~in at the~~ in the s plane.

Q Find the system gain, ~~and~~ type and order of the given system

$$\frac{C(s)}{R(s)} = \frac{10(s+5)^2}{s^3(s+2)(s+10)}$$

$\longrightarrow$ Poles zero form

convert to $\Downarrow$

Standard form (Time constant form).

$$\Downarrow$$

$$= \frac{10 \times 5^2(1+s0.2)^2}{s^3 \times 2 \times 10(1+0.5s)(1+0.1s)}$$

$$= \frac{12.5(1+s0.2)^2}{s^3(1+0.5s)(1+0.1s)}$$

$$\boxed{K = \frac{\text{Numr}^r\ \text{Constant}}{\text{Denor}^r\ \text{Constant}}}$$

System gain $K = \underline{12.5}$

Type $= \underline{3}$

Order $= \underline{5}$

In order, type is also included.

Q Find the type and order of a given unity feedback system of closed loop transfer function $\dfrac{C(s)}{R(s)} = \dfrac{2s+5}{s^3+5s^2+4s+5}$

soln.

~~order $\rightarrow$ 3~~

Given $\dfrac{G}{1+G}$

~~$s(s^2+5)($~~

~~Type $\Longrightarrow$~~

So convert to equivalent ~~to~~ open loop ~~ckt~~ transfer fn $G(s)$

For that $G(s) = \dfrac{\text{Numerator of CLTF}}{\text{Denomr}^r \text{ of CLTF} - \text{Numr}^r \text{ of CLTF}}$

$$= \frac{2s+5}{s^3+5s^2+4s+5-(2s+5)}$$

$$= \frac{2s+5}{s^3+5s^2+2s} = \frac{2s+5}{s(s^2+5s+2)}$$

Order → 3
Type → 1

Note:

- To get the type and order of a closed loop system required open loop transfer function of unity feedback system. ie, $G(s)$ [$H(s)$ must be 1]

→ otherwise go for $G(s)$ from $\frac{G(s)}{1+G(s)}$ (from closed loop T.F)

Note:

To get the openloop transfer function from closed loop, substract the numerator term from the denominator, ~~then it~~ when the feedback is unity.

CHARACTERISTIC EQUATION

→ "System behaviour can be ~~rate~~ realized from the denominator term only."

→ "characteristic equation is the denominator equated to zero."

→ System response consist only denominator term.

for eg: $\frac{s-5}{(s+2)(s+10)} = \frac{K_1}{s+2} + \frac{K_2}{s+10} = K_1 e^{-2t} + K_2 e^{-10t}$ Here no significance for the numerator.

→ For a closed loop system, the characteristic equation is given by

$$\boxed{1+G(s)H(s) = 0}$$

→ The roots of characteristic equation is called Poles.

At poles, T.F becomes infinity.

For 1st order T.F $\cancel{\frac{K}{s+a}}$ $\frac{K}{(1+s\tau_a)}$.

characteristic equ. $1+s\tau_a = 0$ $s = -\frac{1}{\tau_a}$ At $s = -\frac{1}{\tau_a}$ T.F $= \infty$

→ "Poles are the −ve of inverse of time constant at which T.F is ∞"

→ "zeros are the

~~the pole~~

The pole effect the system stability and system response, but not the zeros.

TIME CONSTANT

The time constant gives the system behaviour. If the time constant is very large, then it is called slow response system because it takes the large time to reach the steady state. Practically any system takes the five time constants (5τ) to reach the steady state.

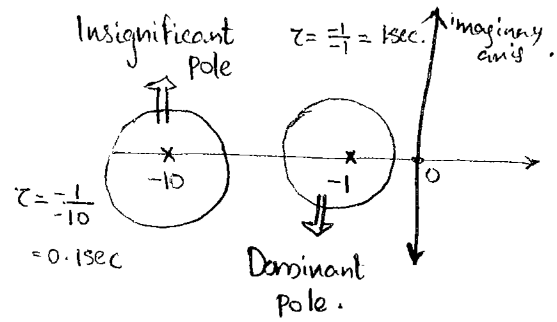
$$\boxed{\text{Time Constant} = \frac{-1}{\text{Real part of the Dominant Pole}}}$$

Dominant pole $\longrightarrow$ The pole which is near to the imaginary axis.

Q Find the equivalent 1^{st} order system to the given second order system

Find the system time constant

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}$$



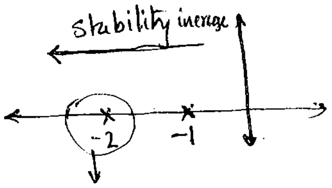
Condition :

Insignificant pole (ISP) $\tau \leq 5$ times of the Dominant Pole (D.P) τ.

$$\boxed{\text{I.S.P.}(\tau) \leq \frac{\text{D.P}(\tau)}{5}}$$

Then Insignificant poles can be neglected.

ie



not $\leq \frac{\text{D.P}(\tau)}{5}$, hence pole at -2 cannot be neglected.

$\rightarrow$ Actually stability increases away from the imaginary axis. ie, Dominant pole is actually is the worst pole. because it is the worst pole that effects stability.

Insignificant pole : ➤ The pole which lies in the left most side.

➤ The insignificant pole time constant must be less than or equal to 5 times of the dominant pole time constant.

➤ The insignificant poles are neglected because even though the insignificant pole is neglected, there is no much change in the system response.

Note : The best pole is the Insignificant pole because it gives the very quick response and more relatively stable.
Because of the dominant pole, the system response become slow. and relative stability decreases.

for $\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}$

System response

$R(s) = 1$

$$c(s) = \frac{1}{(s+1)(s+10)} = \frac{K_1}{s+1} + \frac{K_2}{s+10}$$

$K_1(s+10) + K_2(s+1) = 1$

$K_1 (9) = \frac{1}{9}$

$s = -10$

$K_2 = -\frac{1}{9}$

$$= \frac{1}{9(s+1)} - \frac{1}{9(s+10)}$$

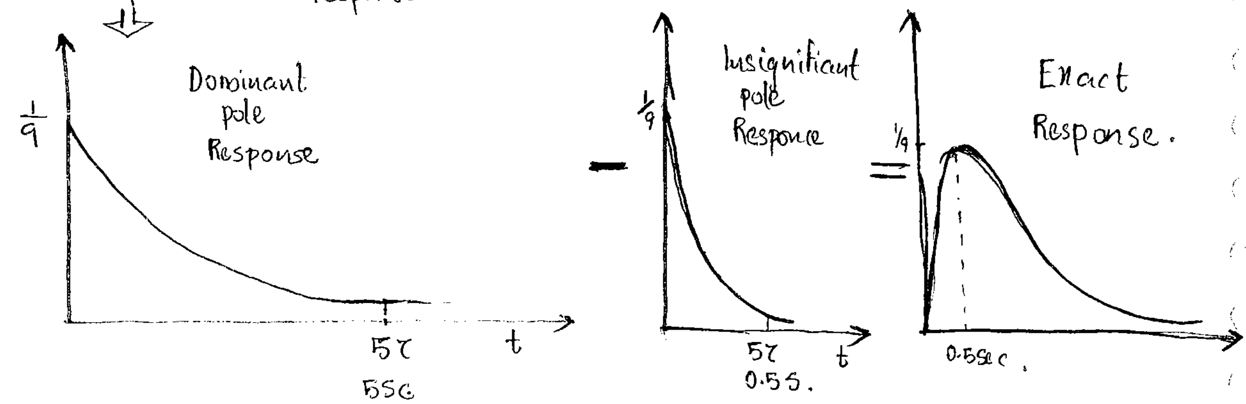
Inverse Laplace Transform,

$$\therefore c(t) = \left(\frac{1}{9}e^{-t} - \frac{1}{9}e^{-10t}\right)$$

Compare with $e^{-t/\tau}$

$\tau = 1sec$ Dominant pole. Response

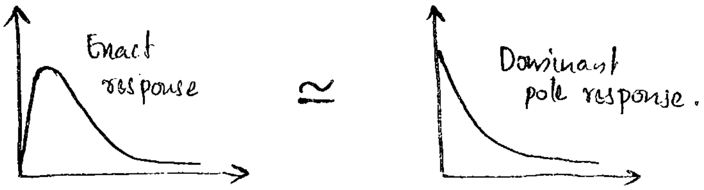
Insignificant Pole. Response $\tau = 0.1 sec$.



Note :

To get the time constant from the response compare the response with $e^{-t/\tau}$

Exact response can be approximated with Dominant Pole Response.



→ The insignificant pole should be neglected only in the time constant form.

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}$$

↘ we can't neglect it directly, because the gain term may change.

Hence convert to time constant form,

$$\cancel{\frac{C(s)}{R(s)} = \frac{0.1}{(s+1)(s+1}}$$

$$\frac{C(s)}{R(s)} = \frac{0.1}{(1+s)(1+0.1s)}$$

↘ Now Neglect.

ie, Equivalent T.F is

$$\frac{C(s)}{R(s)} = \frac{0.1}{(1+s)} \quad , \text{ ie, } \tau = 1\text{sec}.$$

then only ILT gives $0.1e^{-t}$

Q Find the equivalent transfer function to the following system.

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)(s+100)}$$

Both the poles are insignificant.

$$\frac{10^{-1}\ 10^{-2}}{(1+s)(1+0.1s)(1+0.01s)}$$

$$\text{Equivalent T.F} = \frac{0.001}{(s+1)}$$

Q Identify the poles location to the given unit step response.

$$c(t) = 1 - e^{-2t} + 3te^{-3t} + 4^{-4t}\sin 2t.$$

~~C(s) =~~ No need of Laplace transform, directly find poles,

The ~~constant~~ exponential part is the location of pole.

t directly given, implies repeatition of poles.

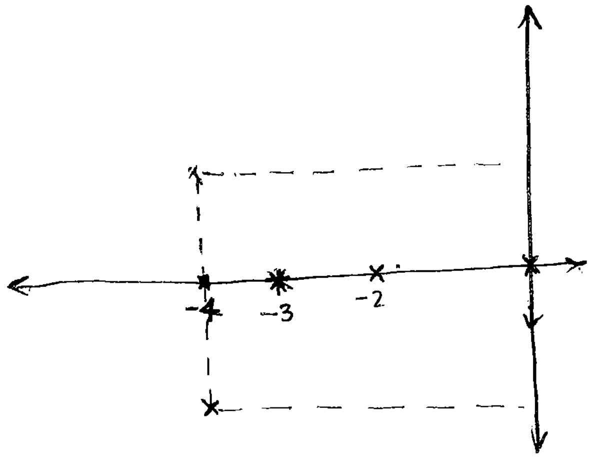
Take by term

(i) $1 \rightarrow$ pole at zero.

(ii) $-e^{-2t} \rightarrow$ pole at -2

(iii) $3te^{-3t} \rightarrow$ 2 poles at -3

(iv) $4^{-4t}\sin 2t \rightarrow \dfrac{2}{(s+4)^2+2^2}$ $s = -4 \pm 2j$



~~zero~~

RULES

1 → exponent power represent the location of poles.
2 → Sine or cos represent the imaginary part of poles.
3 → 't' term represents the repeatition of poles.

| System Response |
| System Stability |
| System Time Constant |

⇒ POLES

For stability put $t = \infty$ in system response

then (i) finite result : ~~stability~~ stable
(ii) '∞' value : Unstable.

System response only consider poles ~~to get together~~ response terms. No zero response terms exists in the system response. Zeros gives the location where ~~it~~ the response starts.

"while finding system response, system stability and system time constant, consider only poles, but not the zeros because the system response consist the no zero response term, are present in system response.

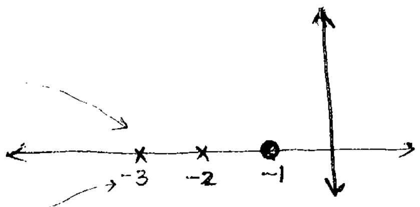
For eg: $\frac{C(s)}{R(s)} = \frac{(s+1)}{(s+2)(s+3)}$

$R(s) = 1$

$$C(s) = \frac{-1}{s+2} + \frac{2}{s+3}$$

Inverse Laplace Transform,

$$c(t) = -1e^{-2t} + 2e^{-3t}$$



> "To find the stability, substitute $t \to \infty$ in the system response if it gives finite value then it is stable. If it gives the infinite value, then it is unstable."

> "The timeconstant is mainly required to draw the response."

TRANSFER FUNCTION TO THE ELECTRICAL NETWORKS

Transfer the ckt in the form of fig 1

and then the transfer fn is given by

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{\text{Impedance Accross Output}}{\text{Total circuit Impedance.}}}$$

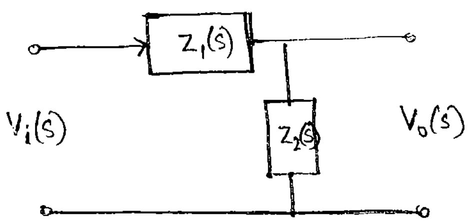
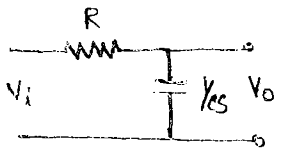


fig 1

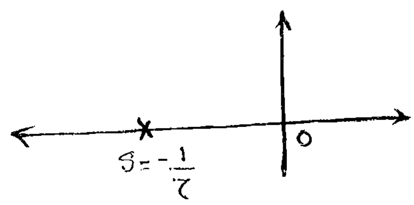
Q. Find the transfer function for the given electrical n/w and locate the poles in the s plane. Find the system Response.



$$T.F \quad \frac{V_o(s)}{V_i(s)} = \frac{1/cs}{R + 1/cs}$$

$$T.F = \frac{1}{sCR+1}$$

$\tau = RC$, then $T.F = \frac{1}{s\tau + 1}$

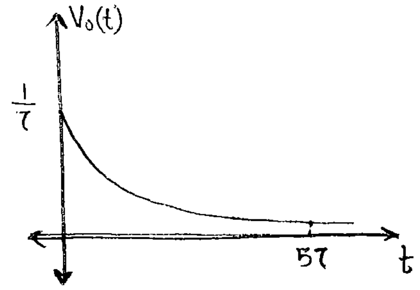


For system response, $V_i(s) = 1$

$$V_o(s) = \frac{1}{\tau(s+\frac{1}{\tau})}$$

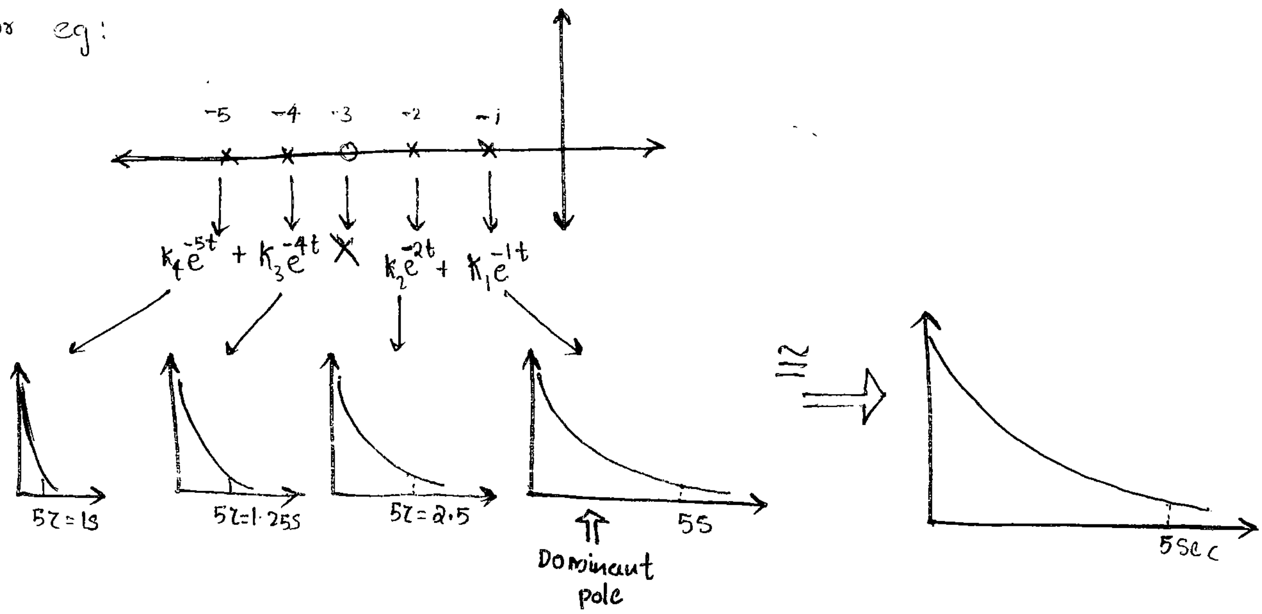
Inverse Laplace transform,

$$\boxed{V_o(t) = \frac{1}{\tau}\left(e^{-t/\tau}\right)}$$



⇒ From poles location to system responses.

for eg:



" If many poles lies on the -ve real axis at different location, then the system response is exponential decay, irrespective ~~posi~~ of position of zeros."

STABILITY

The ~~moment~~ movement of pole in the the s plane is nothing but varying the system component value.

Absolutely Stable System means, the system is stable for all the values of system components or system parameters, like K from 0 to ∞

Conditional stable system The system is stable for certain range of system parameters or components like K from 0 to 100.

Addition of pole or zero to transfer function means adding an R,L,C component to the system. The R,L,C components added to the system ...

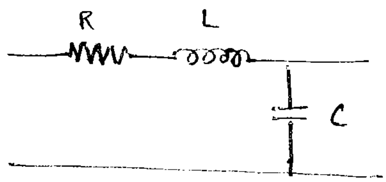
(i) Series Connection : Here R,L,C components ~~series connection~~ added in the forward path.

(ii) parallel connection : Here R,L,C, components added in the feedback path.

Q (a) Find the transfer function by considering ~~Ro~~ $R = 0\Omega$, $L = 1H$ and $C = 1F$

(b) Locate the poles in the s plane.

(c) Find the system responce



(a) $$\frac{C(s)}{R(s)} = \frac{1/sC}{R + sL + 1/sC}$$

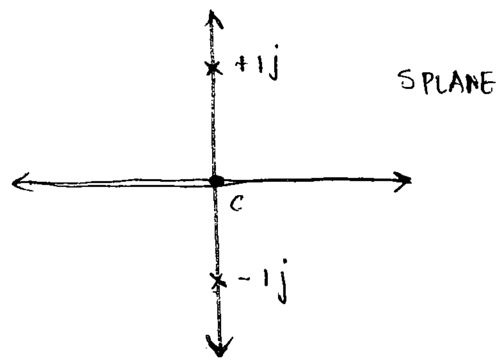
$$= \frac{1}{RCs + s^2LC + 1}$$

$$\frac{C(s)}{R(s)} = \frac{1}{s^2LC + RCs + 1}$$

$R = 0$, $L = 1H$, $C = 1F$

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 1}$$

(b) $$\frac{C(s)}{R(s)} = \cancel{\frac{1}{(s+1)}} \quad \cancel{\sin t}$$



Finding ~~tan~~. ILT

$$c(t) = \sin t$$

$$\tau = \frac{-1}{\text{Real part of pole}} = \frac{-1}{0} = +\infty$$

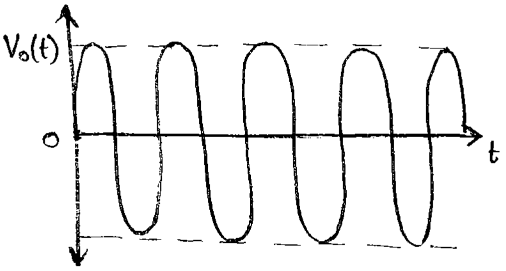
frequency of operation is ~~real part o~~ magnitude of imaginary component.

Here Non repeated poles on $J\omega$ axis → Hence marginally stable.

System response.

$V_i(S) = 1$

$V_o(t) = \sin(t)$



constant Amplitude.
Un Damped Oscillation. → constant frequency
Sustained Oscillation.

- whenever the poles on the Imaginary axis which are non repeated, then the system response is constant amplitude and the frequency of oscillations which are called undamped Oscillations. Any system which produce undamped Oscillations is called <u>Undamped system</u> and the system become marginal stable.

Q Repeat the above problem by considering R = 1Ω, L = 1H, C = 1F.

$$\frac{C(S)}{R(S)} = \frac{1}{S^2 + S + 1}$$

$$\frac{\frac{1}{4}}{S^2 + 2S + \frac{1}{4}}$$

$$= \frac{1}{\left(S+\frac{1}{2}\right)^2 + 1 - \frac{1}{4}}$$

$$\frac{C(S)}{R(S)} = \frac{9}{\left(S+\frac{1}{2}\right)^2 + \frac{3}{4}}$$

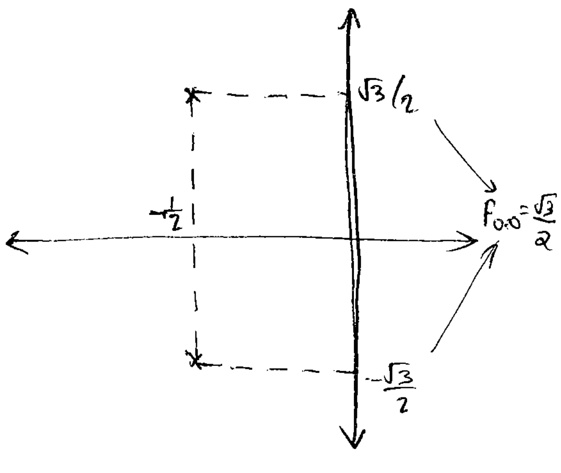
$$\frac{-1 \pm j\sqrt{1-4}}{2}$$

$$-1 \pm j$$

$$\text{Poles} = -\frac{1}{2} + j0.866 = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$= -\frac{1}{2} - j0.866 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

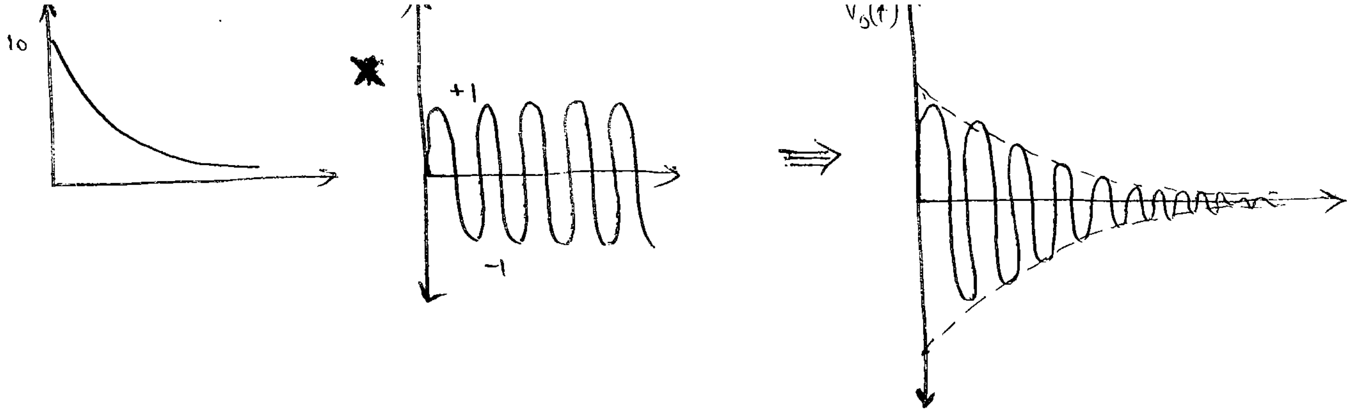
Find the system response



$$V_o(t) = e^{-\frac{1}{2}t} \sin\frac{\sqrt{3}}{2} t \left(\frac{2}{\sqrt{3}}\right)$$

$$L[\sin at] = \frac{a}{S^2 + a^2}$$

$$\tau = \frac{-1}{(-\frac{1}{2})} = 2\ \text{sec}$$



→ In such cases, consider the real part and imaginary part of the poles, then draw response for real part and imaginary part separately, take the product of two as above.

~~→ Here the pole is~~ .

➢ "whenever the poles lies in the left of s plane, which are complex conjugate, the system response is exponential decay, frequency of oscillation, which are called <u>damped Oscillations</u> and and the system is stable".

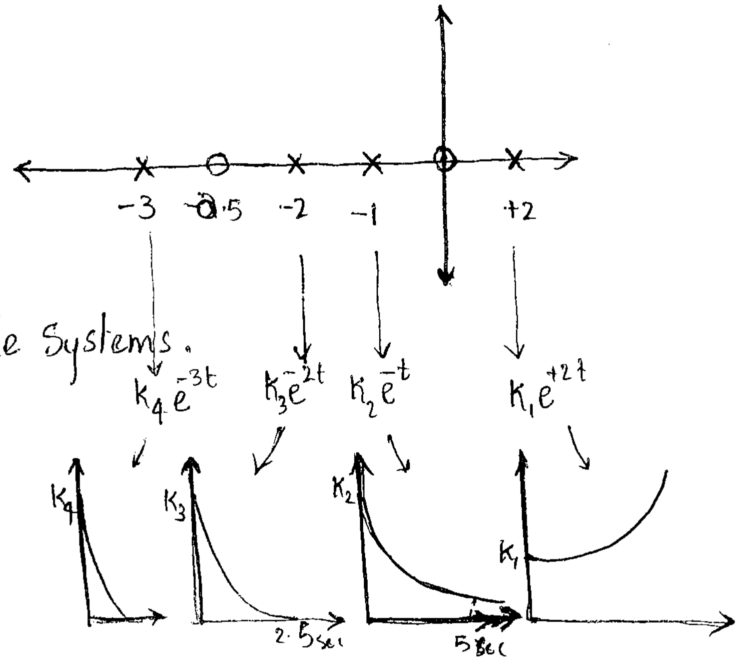
➢ Any systems which produce exponential decay and frequency of oscillation is called <u>Underdamped Oscillations</u>. (maintaining the damping).

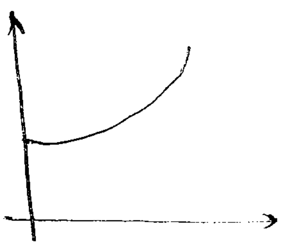
Q. Find the system time constant and response for the give pole location in the s plane.

The given system is <u>Unstable</u>.

Hence the time constant is not defined.

Time Constant defined for only Stable Systems.

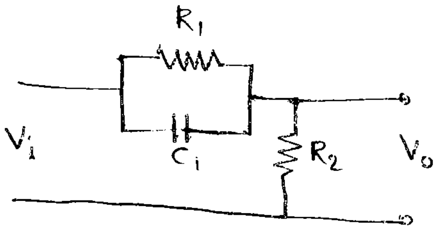




If one or more poles lies in the right of s plane, then the system response is exponential rise to infinity and system is unstable irrespectable of position of poles and zeros in the left hand side.

Q. Find the T.F to the lead network in the time constant form.

Lead N/w



R_1

$\frac{1}{SC}$

$$\frac{C(s)}{R(s)} = \frac{R_2}{R_2 + \frac{R_1/SC}{R_1 + 1/SC}}$$

$$= \cancel{\frac{R_2}{R_2 SC + R_1}} = \frac{R_2}{R_2 + \frac{R_1}{R_1 SC + 1}}$$

$$\frac{C(s)}{R(s)} = \frac{R_2 (R_1 sC + 1)}{R_2 (R_1 SC + 1) + R_1}$$

$$\frac{C(s)}{R(s)} = \frac{R_1 R_2 SC + R_2}{R_1 + R_2 + R_1 R_2 SC}$$

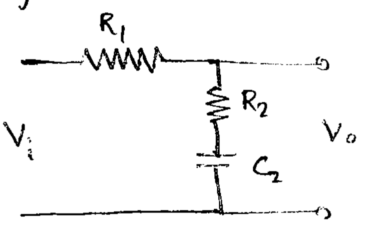
$$= \frac{R_2 \left(\frac{R_1 \cancel{R_2} C}{\cancel{R_2}} s + 1\right)}{R_1 + R_2 \left(\frac{S R_1 R_2 C}{R_1 + R_2} + 1\right)} = \left(\frac{R_2}{R_1 + R_2}\right) \frac{(R_1 C s + 1)}{\left(\frac{S R_1 R_2 C}{R_1 + R_2} + 1\right)}$$

$K = \frac{R_2}{R_1 + R_2} = \alpha \rightarrow$ Lead constant. $(\alpha < 1)$

$\tau =$ Lead time constant $= R_1 C_1$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1 + \tau s)}{(1 + \alpha \tau s)}}$$

Lag N/W



$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + 1/sC_2}{R_1 + R_2 + 1/sC_2}$$

$$= \frac{R_2 sC_2 + 1}{R_1 sC_2 + R_2 sC_2 + 1}$$

$$= \frac{(R_2 sC_2 + 1)}{s(R_1C_2 + R_2C_2) + 1}$$

$$= \frac{(R_2 sC_2 + 1)}{(sC_2(R_1 + R_2) + 1)}$$

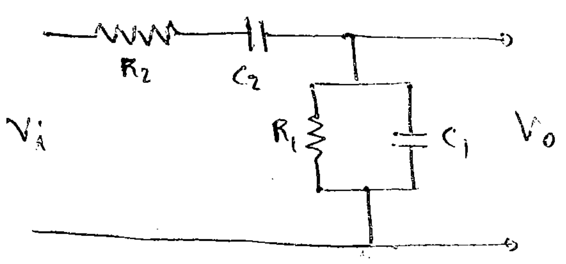
Divide & multiply by R_2

$$= \frac{(\overbrace{R_2C_2}^{\tau = R_2C_2} s + 1)}{\left(s R_2C_2 \underbrace{\frac{(R_1+R_2)}{R_2}}_{\beta} + 1\right)}$$

Lag Time constant $\boxed{(\tau) = R_2 C_2}$

Lag constant $\boxed{\beta = \frac{R_1 + R_2}{R_2}}$

H.W



$$\frac{V_o(s)}{V_i(s)} = \frac{\dfrac{\frac{R_1}{sC_1}}{R_1 + 1/sC_1}}{\dfrac{\frac{R_1}{sC_1}}{R_1+1/sC_1} + R_2 + 1/sC_2} = \frac{\dfrac{R_1}{R_1 sC_1 + 1}}{\dfrac{R_1}{R_1 sC_1 + 1} + R_2 + \dfrac{1}{sC_2}}$$

$$= \frac{\dfrac{R_1}{\cancel{(R_1 sC_1 + 1)}}}{\dfrac{R_1 + \left(R_2 + \frac{1}{sC_2}\right)(R_1 sC_1 + 1)}{\cancel{(R_1 sC_1 + 1)}}} = \frac{R_1}{R_1 + R_1 R_2 sC_1 + R_2 + \dfrac{R_1 C_1}{C_2} + \dfrac{1}{sC_2}}$$

$$= \frac{R_1}{R_1 s + R_1 R_2 s^2 C_1 + R_2 s + \dfrac{R_1 C_1}{C_2} s + \dfrac{1}{C_2}}$$

$$= \frac{1}{s^2(R_1 R_2 C_1) + s\left(R_1 + R_2 + \dfrac{C_1}{C_2}\right) + \dfrac{1}{C_2}}$$

$\tau_1 = R_1 C_1$

$\tau_2 = R_2 C_2$

$$= \frac{C_2}{s^2(R_1 R_2 C_1 C_2) + s\left((R_1+R_2)C_2 + C_1\right) + 1}$$

$\alpha = \dfrac{R_2}{R_1 + R_2}$ $\qquad \dfrac{R_2}{R_1+R_2} \times R_1 C_1$

$\beta = \dfrac{R_1 + R_2}{R_2}$ $\qquad \dfrac{R_1 + R_2}{R_1 R_2 C_1}$

$$\cancel{= \frac{C_2}{\quad}}$$

$\beta \tau_2 = \dfrac{R_1 + R_2}{\cancel{R_2}} \times \cancel{R_2} C_2$ $\qquad \dfrac{1}{R_2 C_2} + \dfrac{1}{R_1 C_1}$ $\qquad = \dfrac{1}{\alpha \tau_1}$

$$= \frac{C_2 \quad \frac{1}{\tau_1 \tau_2}}{s^2(\tau_1 \tau_2) + s\left((R_1+R_2)C_2 + C_1\right) + 1}$$

$\beta \tau_2 = \cancel{R_1 + }\ R_1 C_2 + R_2 C_2$

$$\cancel{= \frac{C_2 \frac{1}{\tau_1 \tau_2}}{s^2 + s\dfrac{((R_1+R_2)C_2 + C_1)}{\tau_1 \tau_2} + \dfrac{1}{\tau_1 \tau_2}}}$$

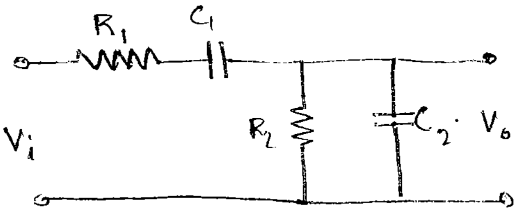
$$= \frac{\cancel{C_2} \times \dfrac{1}{R_1 C_1 R_2 \cancel{C_2}}}{s^2 + s\left(\dfrac{R_1 C_2 + R_2 C_2 + C_1}{R_1 R_2 C_1 C_2}\right) + \dfrac{1}{R_1 R_2 C_1 C_2}}$$

$$= \frac{\dfrac{1}{R_1 R_2 C_1}}{s^2 + s\left(\dfrac{1}{R_2 C_2} + \dfrac{1}{R_1 C_1} + \dfrac{1}{R_1 R_2 C_2}\right) + \dfrac{1}{R_1 R_2 C_1}} = \frac{\dfrac{1}{R_1 R_2 C_1}}{s^2 + s\left(\cancel{\alpha \tau_1}\ \dfrac{1}{\alpha \tau_1}\right. +}$$

$\dfrac{1}{R_1 R_2}$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{s\left(\frac{1}{\alpha} - 1\right)\tau_2}{s^2(\tau_1 \tau_2) + s\left(\tau_1 + \tau_2 + (\beta - 1)\tau_2\right) + 1}}$$

Find the transfer function to the electric n/w given in the figure and plot the poles & zeros in the s plane by considering $R_1 = R_2 = 1\Omega$, $C_1 = C_2 = 1F$



$$\frac{V_i(s)}{V_0(s)} = \frac{\dfrac{\frac{R_2}{sC_2}}{R_2 + 1/sC_2}}{\dfrac{\frac{R_2}{sC_2}}{R_2 + 1/sC_2} + R_1 + \frac{1}{sC_1}} = \frac{\dfrac{R_2}{R_2 sC_2 + 1}}{\dfrac{R_2}{R_2 sC_2 + 1} + \dfrac{R_1 sC_1 + 1}{sC_1}}$$

$$= \frac{\dfrac{1}{s+1}}{\dfrac{1}{s+1} + \dfrac{s+1}{s}}$$

$$= \frac{\dfrac{R_2}{\cancel{(R_2 sC_2 + 1)}}}{\dfrac{R_2 C_1 s + (R_1 sC_1 + 1)(R_2 sC_2 + 1)}{sC_1 \cancel{(R_2 sC_2 + 1)}}}$$

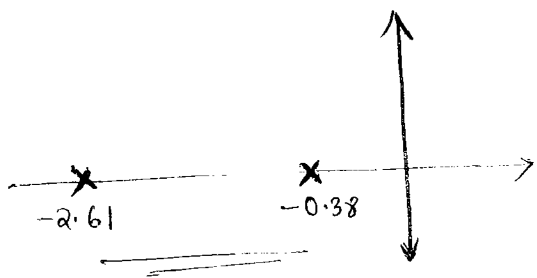
$$= \frac{\dfrac{1}{\cancel{s+1}}}{\dfrac{s + (s+1)^2}{s\cancel{(s+1)}}} = \frac{s}{s^2 + 3s + 1}$$

$$\frac{C(s)}{R(s)} = \frac{R_2 C_1 s}{R_2 C_1 s + s^2 R_1 R_2 C_1 C_2 + s(R_1 C_1 + R_2 C_2) + 1}$$

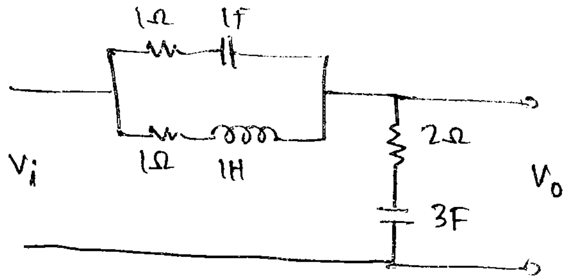
$R_1 = R_2 = 1\Omega$ $C_1 = C_2 = 1F$

$$\frac{C(s)}{R(s)} = \frac{s}{s^2 + 3s + 1}$$

$$= \frac{s}{(s + 0.38)(s + 2.61)}$$



Q



$$\frac{\cancel{\left(1+\frac{1}{s}\right)(1+s)}}{\cancel{1+1/s+1+s}} = 1$$

$$\cancel{\frac{s+1\ (s+1)}{s\left(2+s+1/s\right)}}$$

$$\cancel{\frac{(s+1)^2}{2s+s^2+1}}$$

$$\frac{(s+1)^2}{\cancel{s^2+2s+1}} \qquad 1+\frac{1}{s}\cancel{+s+}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{2+\frac{1}{3s}}{2+\frac{1}{3s}+\frac{(s+1)^2}{s^2+2s+1}}$$

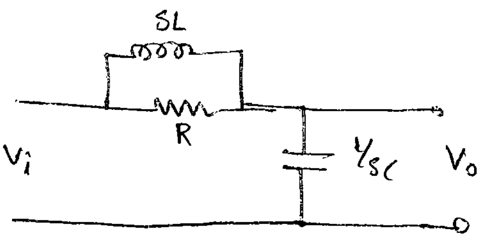
$$= \frac{2+\frac{1}{3s}}{2+\frac{1}{3s}+1}$$

$$= \frac{6s+1}{6s+1+3s}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{6s+1}{9s+1}$$

~~R, C acts as~~

Q Find the transfer function in terms of τ_1 and τ_2.



$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{\frac{RSL}{R+SL}+\frac{1}{sC}}$$

$$= \frac{\frac{1}{sC}}{\frac{s^2RLC+R+SL}{(R+SL)sC}} = \frac{(R+sL)}{s^2 + \frac{sL}{RC} + \frac{1}{LC}}$$

$$B = \frac{1}{s^2 + \frac{s}{RL} + \frac{1}{LC}}$$

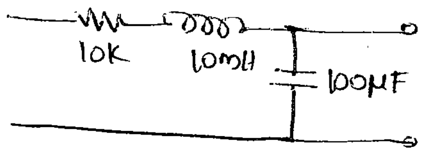
$$= \frac{1}{\left(s+\frac{1}{2RL}\right)^2 + \frac{1}{LC} - \frac{1}{R^2L^2}} \qquad \frac{1}{R^2L^2}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\left(s+\frac{1}{2RL}+j\sqrt{\frac{1}{LC}-\frac{1}{R^2L^2}}\right)\left(s+\frac{1}{2RL}-j\sqrt{\frac{1}{LC}-\frac{1}{R^2L^2}}\right)}$$

$\tau_1 = RC \quad \tau_2 = L/R \qquad \tau_1\tau_2 = LC$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{s\tau_2+1}{s^2\tau_1\tau_2 + s\tau_2 + 1}}$$

Q



$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{s\times 100\mu}}{10^4 + s10^{-2} + \frac{1}{s\times 100\mu}}$$

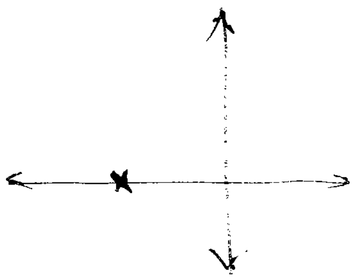
$$B \quad \frac{\frac{1}{sC}}{R+sL+\frac{1}{sC}}$$

$$\frac{1}{RCs+s^2LC+1}$$

$$\frac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + \frac{10^4}{10^{-2}}s + \frac{1}{10^{-2}\times 10^{-4}}}$$

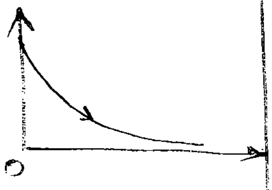
$$= \frac{10^6}{s^2 + 10^6 s + 10^6}$$



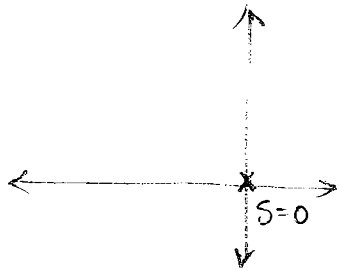
T.F = $\frac{1}{S+a}$

System Response = $1 . e^{-at}$

~~Draw the~~ Response

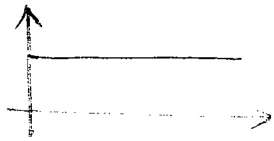


Alternate way for stability critirian. whenever response follows the input. Here input is impulse. Impulse tends to zero. Response also tends to zero. Hence Stable.

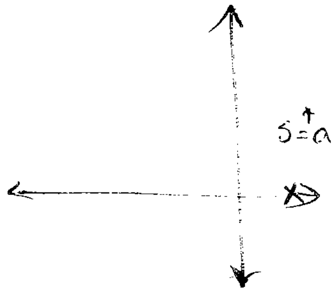


T.F = $\frac{1}{S}$

System Response = $u(t)$

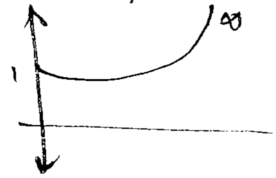


Here non-repeated pole on $j\omega$ axis, then it is marginally stable.

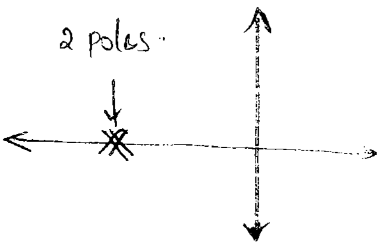


T.F = $\frac{1}{S-a}$

System response = $1 \cdot e^{at}$

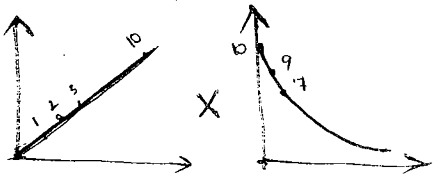


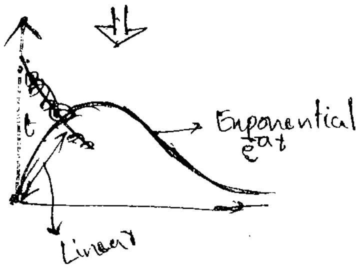
Response away from input (impulse) Hence Unstable.



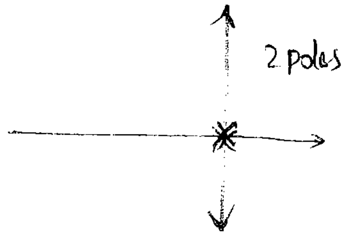
T.F = $\frac{1}{(S+a)^2}$

System Response = te^{-at}



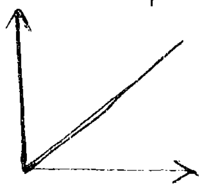


→ Lower values of t → Linear dominant

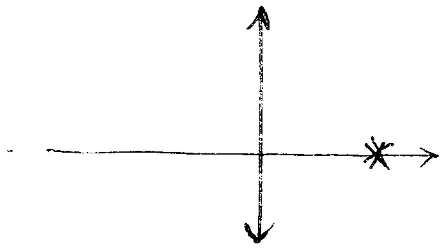


T.F = $\frac{1}{S^2}$

System response = $t\, u(t)$

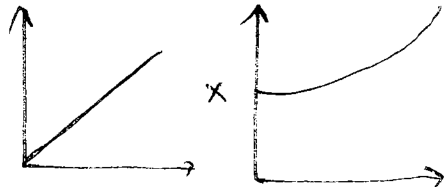


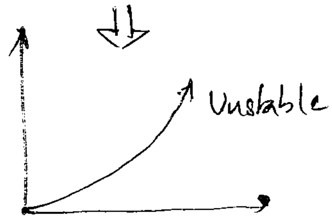
→ Unbounded and Unstable.



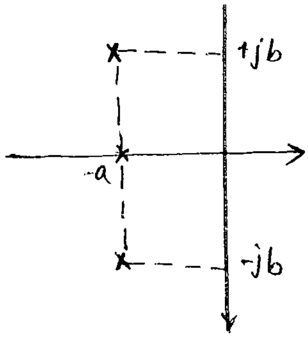
T.F = $\frac{1}{(S-a)^2}$

System response = te^{at}



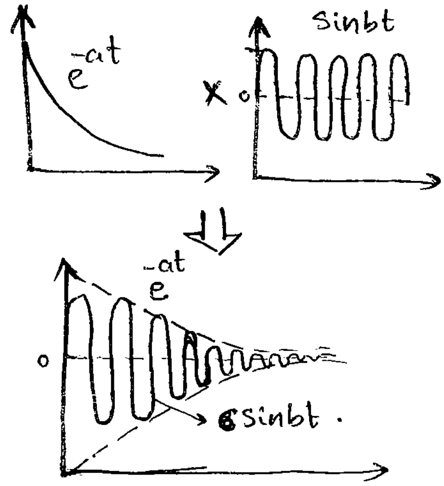


→ Unbounded and Unstable.

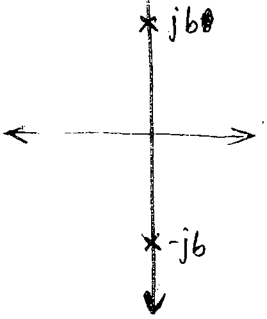


$$T.F = \frac{1}{(S+a)^2 + b^2}$$

System Response $= \frac{1}{b} e^{-at} \sin bt$

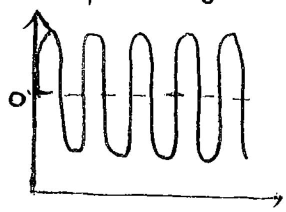


Damped oscillation
System stable.

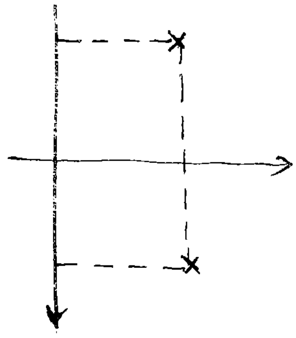


$$T.F = \frac{1}{S^2 + b^2}$$

System Response $= \frac{1}{b} \sin bt$

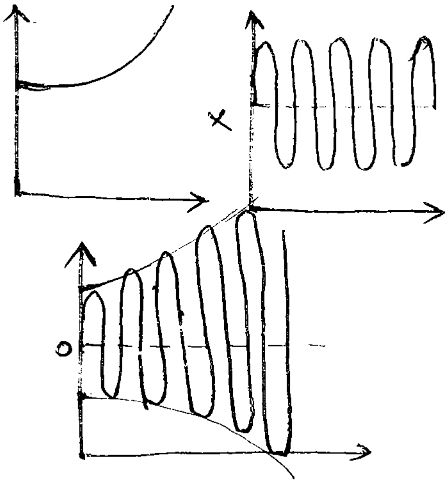


→ Non repeated poles on $j\omega$ axis. Hence marginally stable.



$$T.F^2 = \frac{1}{(S-a)^2 + b^2}$$

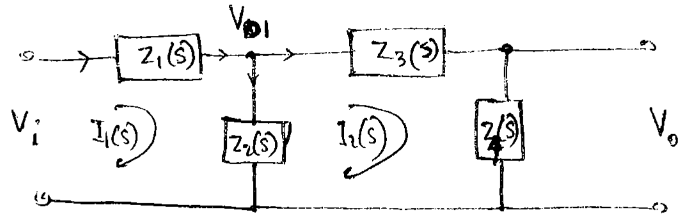
System Response $= \frac{1}{b} e^{at} \sin bt$



→ Not bounded.
→ Unstable system.

T.F / S-PLANE	SYSTEM RESPONSE
purely real	Exponential terms.
Imaginary	sin/cos terms.
Real + Imaginary	Product of exponential and sin/cos terms.
Repeated Real	Product of t and exponential
Repeated Imaginary	Product of t and sin/cos terms
Repeated (Real + Imaginary)	Product of t, exponential and sin/cos terms.

Q



KCL

$$\frac{V_i - V_{01}}{Z_1(s)} - \frac{V_{01}}{Z_2(s)} - \frac{V_{01} - V_o}{Z_3(s)} = 0$$

$$V_i\left[\frac{1}{Z_1(s)}\right] - V_{01}\left[\frac{1}{Z_1(s)} + \frac{1}{Z_2(s)} + \frac{1}{Z_3(s)}\right] + \frac{V_o}{Z_3(s)} = 0$$

$$V_{01} = \frac{\frac{V_i}{Z_1(s)} + \frac{V_o}{Z_3(s)}}{\left[\frac{1}{Z_1(s)} + \frac{1}{Z_2(s)} + \frac{1}{Z_3(s)}\right]}$$

$$\frac{V_{01} - V_o}{Z_3(s)} + \frac{V_o}{Z_4(s)} = 0$$

$$\frac{V_{01}}{Z_3} - V_o\left[\frac{1}{Z_3(s)} - \frac{1}{Z_4(s)}\right] = 0$$

$$\frac{\frac{V_i}{Z_1(s)} + \frac{V_o}{Z_3(s)}}{\left[\frac{1}{Z_1(s)} + \frac{1}{Z_2(s)} + \frac{1}{Z_3(s)}\right] Z_3(s)} - V_o\left[\frac{1}{Z_3(s)} - \frac{1}{Z_4(s)}\right] = 0$$

KVL 1

$$V_i(s) = I_1(s)\left[Z_1(s) + Z_2(s)\right] - I_2(s)\left[Z_2(s)\right] \longrightarrow ①$$

KVL 2

$$0 = -I_1(s)\, Z_2(s) + I_2(s)\left[Z_2(s) + Z_3(s) + Z_4(s)\right]$$

$$V_o(s) = I_2(s)\, Z_4(s)$$

$$① \Rightarrow \quad I_1(s) = \frac{V_i(s) + I_2(s) Z_2(s)}{Z_1(s) + Z_2(s)}$$

$$0 = -\frac{\left(V_i(s) + I_2(s) Z_2(s)\right)}{Z_1(s) + Z_2(s)} \left[Z_2(s) \right] + I_2(s) \left[Z_2(s) + Z_3(s) + Z_4(s) \right] \rightarrow (2)$$

$$\Rightarrow \frac{V_i(s)}{Z_1(s) + Z_2(s)} = I_2(s) \left[\frac{-Z_2^2(s)}{Z_1(s) + Z_2(s)} + Z_2(s) + Z_3(s) + Z_4(s) \right]$$

$$V_i(s) Z_2(s) = I_2(s) \left[-Z_2^2(s) + \left[Z_2(s) + Z_3(s) + Z_4(s) \right] \left[Z_1(s) + Z_2(s) \right] \right]$$

$$I_2(s) = \frac{V_i(2) Z_2(s)}{\left[\left[Z_2(s) + Z_3(s) + Z_4(s) \right] \left[Z_1(s) + Z_2(s) \right] - Z_2^2(s) \right]}$$

$$V_0(s) = I_2(s) Z_4(s)$$

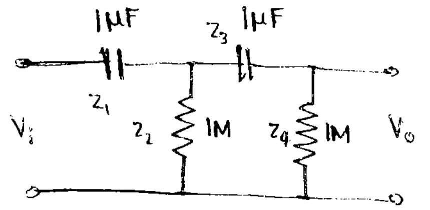
$$\boxed{\frac{V_0(s)}{V_i(s)} = \frac{V_i(2) Z_2(s) Z_4(s)}{\left[\left[Z_2(s) + Z_3(s) + Z_4(s) \right] \left[Z_1(s) + Z_2(s) \right] - Z_2^2(s) \right]}}$$

<u>short cut</u>

$$\frac{V_0(s)}{V_i(s)} = \frac{\text{Product of shunt Impedances}}{\text{First Impedance (sum of others)} + 2^{nd} \text{ Impedance (sum of remaining)}}$$

$$\boxed{\frac{V_0(s)}{V_i(s)} = \frac{Z_2 Z_4}{Z_1(Z_2 + Z_3 + Z_4) + Z_2(Z_3 + Z_4)}}$$

Q



$$\frac{V_o(s)}{V_i(s)} = \frac{10^6 \times 10^6}{\frac{1}{s\times10^{-6}}\left(10^6+10^6+\frac{1}{s\times10^{-6}}\right) + 10^6\left(\frac{10^6}{s}+10^6\right)}$$

$$= \frac{\cancel{10^{12}}}{\frac{\cancel{10^6}}{s}\left(\cancel{10^6}+\cancel{10^6}+\frac{\cancel{10^6}}{s}\right) + \cancel{10^{12}}\left(\frac{1}{s}+1\right)}$$

$$= \frac{1}{\frac{1}{s}\left(2+\frac{1}{s}\right) + \left(\frac{1}{s}+1\right)}$$

$$= \frac{1}{\frac{2}{s}+\frac{1}{s^2}+\frac{1}{s}+1}$$

$$= \frac{s^2}{2s+1+s+s^2}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{s^2}{s^2+3s+1}}$$

Q The unit step response to the above ~~system~~ n/w is

$$V_o(s) = \frac{1}{\cancel{s}} \times \frac{s^{\cancel{2}}}{s^2+3s+1}$$

$$= \frac{s}{(s+0.38)(s+2.61)}$$

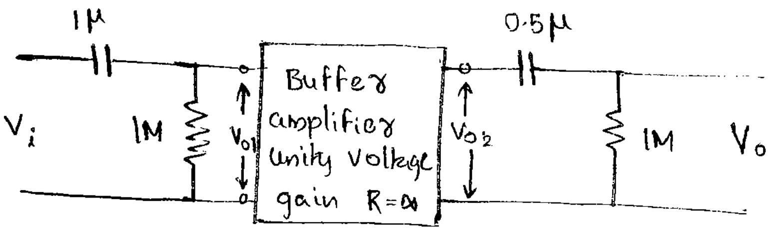
$\cancel{\frac{s}{s+0.38}}$

$k_1(s+2.61) + k_2(s+0.38) = 0$

$$= \frac{-0.17}{s+0.38} + \frac{1.17}{s+2.61}$$

$$= -0.17\,e^{-0.38t} + 1.17\,e^{2.61t}$$

Q Determine the transfer function and calculate the output voltage for unit step input, at t=0.



$$\frac{V_o}{V_i} = \frac{V_o}{V_{o2}} \times \frac{V_{o2}}{V_{o1}} \times \frac{V_{o1}}{V_i}$$

$$\frac{V_{o1}(s)}{V_i(s)} = \frac{\cancel{10^6}}{\cancel{10^6} + \cancel{10^6}\frac{1}{s}} = \frac{1}{\frac{1}{s}+1}$$

$$\frac{V_{o2}(s)}{V_{o1}(s)} = 1$$

$$\frac{V_o(s)}{V_{o2}(s)} = \frac{\cancel{10^6}}{\cancel{10^6} + \frac{2\times\cancel{10^6}}{s}} = \cancel{\frac{1}{1+s}} \quad \cancel{1+2s}$$

$$\frac{V_o}{V_i} = \cancel{\frac{1}{(s+1)(2s+1)}} \quad \frac{1}{\left(\frac{1}{s}+1\right)\left(1+\frac{2}{s}\right)}$$

$$= \frac{1}{\frac{1+s}{s} \times \frac{s+2}{s}}$$

$$= \frac{s^2}{(1+s)(2+s)}$$

Unit step response $= U(s)\,\dfrac{s^2}{(s+1)(s+2)}$

$$= \frac{1}{s} \times \frac{s^2}{(s+1)(s+2)}$$

$$= \frac{s}{(s+1)(s+2)}$$

$k_1(s+2) + k_2(s+1) = 0$

$$V_o = \frac{-1}{s+1} + \frac{2}{s+2}$$

Inverse laplace transform

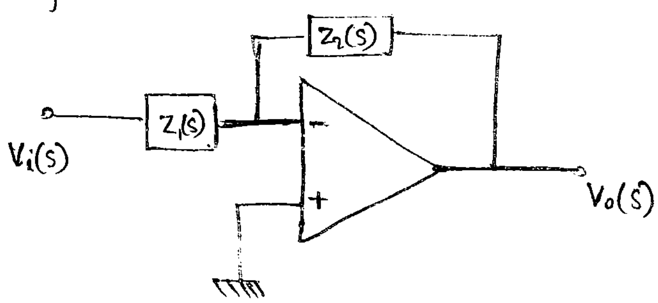
$$v_o(t) = -e^{-t} + 2e^{-2t}$$

at $t=0$

$$V_o = -1+2 = +1V$$

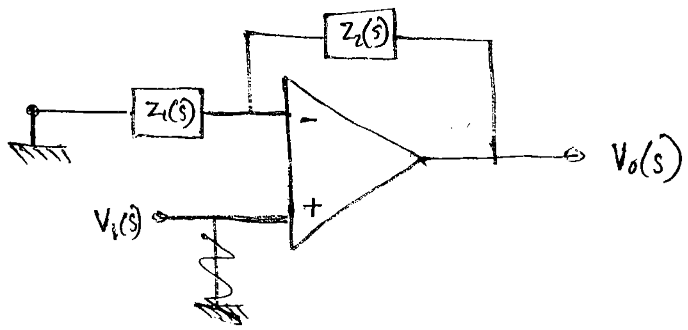
TRANSFER FUNCTION TO OPERATIONAL AMPLIFIERS

(i) Inverting Amplifier.

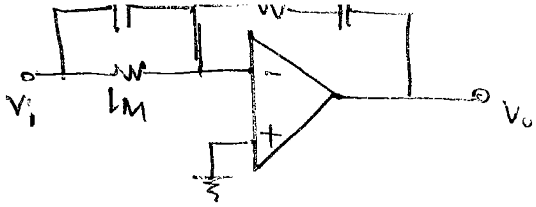


$$\boxed{\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}}$$

(ii) Non Inverting Amplifier.



$$\boxed{\frac{V_o(s)}{V_i(s)} = 1 + \frac{Z_2(s)}{Z_1(s)}}$$



$$Z_1(s) = \frac{10^6 \times \frac{1}{s10^{-6}}}{10^6 + \frac{10^6}{s}} = \frac{\frac{10^{12}}{s}}{10^6\left(1+\frac{1}{s}\right)} = 10^6 \times \frac{\frac{1}{s}}{\frac{s+1}{s}} = \frac{10^6}{s+1}$$

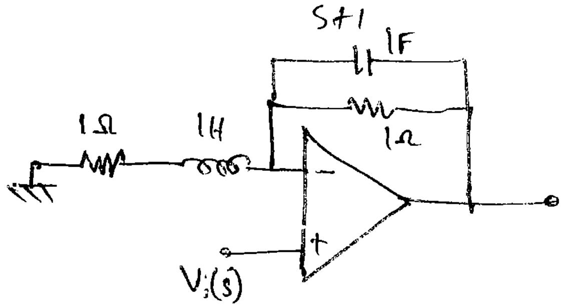
$$Z_2(s) = 10^6 + \frac{0.5\,10^6}{\cancel{0.5}s}$$

$$= 10^6\left(1+\frac{1}{2s}\right)$$

$$= \left(\frac{2s+1}{2s}\right)10^6$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = \cancel{\frac{\frac{10^6}{s+1}}{\left(\frac{2s+1}{2s}\right)10^6}} = \cancel{\frac{-2s}{(s+1)(2s+1)}}$$

$$= \frac{\left(\frac{2s+1}{2s}\right)\cancel{10^6}}{\frac{\cancel{10^6}}{s+1}} = \frac{-(2s+1)(s+1)}{2s} = \frac{-\left[2s^2+3s+1\right]}{2s}$$



~~$V_o(s)$~~ $$Z_1(s) = \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{1}{s+1}$$

$$Z_2(s) = 1+s$$

$$\frac{V_o(s)}{V_i(s)} = 1 + (1+s)^2 = s^2+2s+2$$

TRANSFER FUNCTION TO DIFFERENTIAL EQUATIONS

Q, Find the transfer function to the following system where x is input and y is o/p.

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 10y = x(t-\tau)$$ (Transportation delay / lag system)

$$s^2 y + 5sy + 10y = x e^{-s\tau}$$

$$Y(s)(s^2+5s+10) = X(s)\, e^{-s\tau}$$

$$\boxed{T.F = \frac{Y(s)}{X(s)} = \frac{\text{Input Related Terms}}{\text{Output Related Terms}}}$$

$$\frac{Y(s)}{X(s)} = \frac{e^{-s\tau}}{s^2+5s+10}$$

Q, Repeat the above problem,

$$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8 = 2\frac{dx}{dt} + x$$

$$s^3 Y(s) + 2s^2 Y(s) + 4sY(s) + 8 = 2s\,X(s) + X(s)$$

$$Y(s)\left[s^3+2s^2+4s\right] + 8/s = [2s+1]\,X(s)$$

$$Y(s) = \frac{2s+1}{s^3+2s^2+4s}$$

Any term which we can't locate it as input or output related term, we can neglect it to zero.

So term corresponds to 8 is zero.

Q. Write the D.E to the given transfer function.

$$\frac{Y(s)}{X(s)} = \frac{2s+5}{s^2+6s+7}$$

$$Y(s)\left[s^2+6s+7\right] = (2s+5)\,X(s)$$

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 7y = 2\frac{dx}{dt} + 5x$$

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 7y + \text{const} = 2\frac{dx}{dt} + 5x$$

TRANSFER FUNCTION TO THE SIGNAL RESPONSE

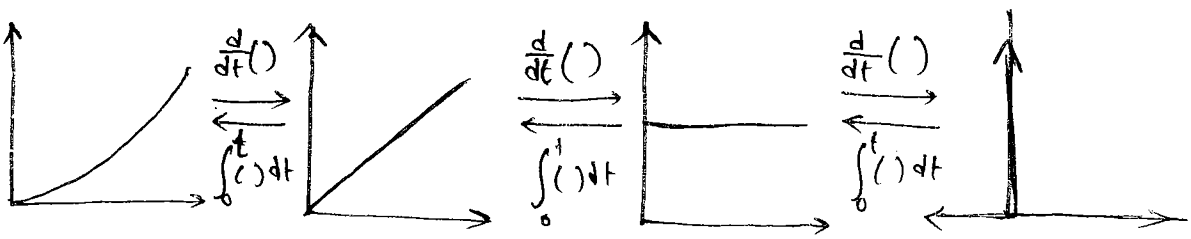
Q. $T.F = \dfrac{L[o/p]}{L[i/p]} \rightarrow (1)$, $T.F = L[\text{impulse response}] \rightarrow (2)$

Here always use first eqn for T.F.

For eg: give a unit ramp response. $(t - 10e^{-2t} + 4e^{-3t} + \dots)$

$$T.F = L[\text{impulse response}] = L\left[\frac{d}{dt}\left[\frac{d}{dt}\left[\text{ramp response}\right]\right]\right]$$

This is a tedious process. Not go for that



"To get a trans for from the response, use the transfer function $= \dfrac{L[o/p]}{L[i/p]}$"

Q, The unit step response of the system is

$$y(t) = \frac{5}{2} - \frac{5}{2}e^{-2t} + 5t$$

Its transfer function is

$$Y(s) = \frac{5}{2s} - \frac{5}{2(s+1)} + \frac{5}{s^2} \qquad X(s) = \frac{1}{s}$$

$$T.F = \frac{\frac{5}{2s} - \frac{5}{2(s+1)} + \frac{5}{s^2}}{1/s}$$

$$= s\left[\frac{5}{2} - \frac{5s}{2(s+1)} + \frac{5}{s}\right]$$

$$= \cancel{\left[\frac{5}{2}\right.} \quad \frac{5}{2}\left[1 - \frac{s}{s+1} + \frac{2}{s}\right]$$

$$= \frac{5}{2}\left[\frac{s(s+1) - s^2 + (2s+2)}{s(s+1)}\right]$$

$$= \frac{5}{2}\left[\cancel{s^2} + s - \cancel{s^2} + 2s + 2\right]$$

$$= \frac{5}{2}\left[\frac{3s+2}{s(s+1)}\right]$$

$$= \underline{\underline{\frac{10s+10}{s(s+1)}}}$$

Q, The Impulse response is $c(t) = (-4e^{-t} + 6e^{-2t})$

The equivalent step response is

$$-4\int e^{-t} + 6\int e^{-2t}$$

$$= \left[4e^{-t} - 3e^{-2t}\right]_0^t \qquad = \underline{\underline{4e^{-t} - 3e^{-2t} - 1}}$$

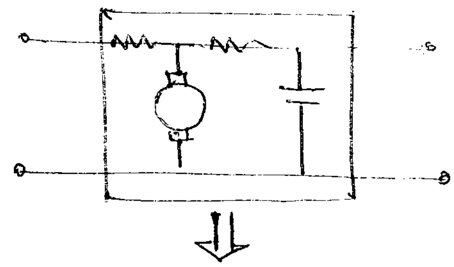
$$c(t) = \ldots e^{-t} \ldots e^{-2t}$$

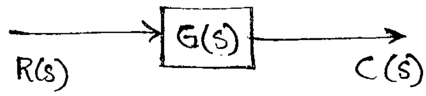
BLOCK DIAGRAMS

→ Purpose is to find the overall transfer function of the system.

"Block Diagram is a short and pictorial representation of a system between input and output."

→ System can be represented in two form,

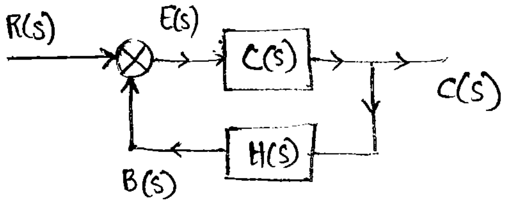




Open loop form.

Open loop Transfer function $\frac{C(S)}{R(S)} = G(S)$

Closed loop Form.



$E(s) \rightarrow$ Error signal.

$B(S) \rightarrow$ Feedback signal.

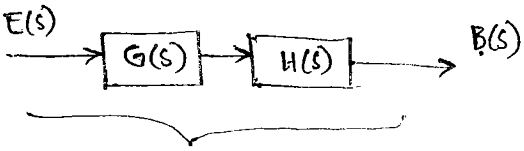
$G(S) \rightarrow$ Forward path Gain $= \frac{C(S)}{E(S)}$

$H(S) \rightarrow$ Feedback path Gain $= \frac{B(S)}{C(S)}$

(i) Open loop transfer function (OLTF)

closed loop system always represented as $G(S)H(S)$.

For a non-unity feedback system, can be represented as open loop transfer function of non unity feedback system.



OLTF of non unity feedback system.

~~The facto~~ If $H(S) = 1$

OLTF $= G(S)$, then it is OLTF of unity feedback system.

The factor $G(S)H(S)$ represents the actual ~~system~~ closed loop system which is also called loop gain (open).

(ii) Closed Loop Transfer Function. (CLTF)

$$C(S) = G(S)E(S)$$

where $E(S) = R(S) \pm B(S)$

$$E(S) = R(S) \pm C(S)H(S)$$

~~E(S) + B(S)~~
$E(S) = R(S) + B(S) \rightarrow$ +ve Feedback.
$E(S) = R(S) - B(S) \rightarrow$ -ve Feedback.

$$C(S) = G(S)\left[R(S) \pm C(S)H(S)\right]$$

$$C(S) = R(S)G(S) \pm G(S)C(S)H(S)$$

$$C(S)\left[1 \mp G(S)H(S)\right] = G(S)R(S)$$

$$\boxed{CLTF = \frac{C(S)}{R(S)} = \frac{G(S)}{1 \mp G(S)R(S)}}$$

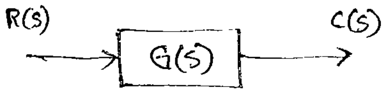
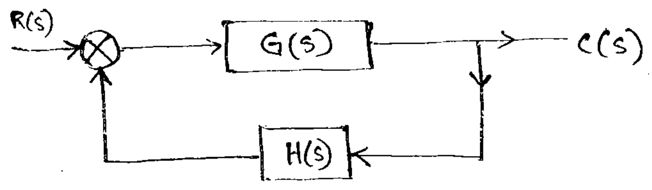
(a) $CLTF = \dfrac{G(S)}{1+G(S)R(S)}$ ⇒ Negative Feedback.

(b) $CLTF = \dfrac{G(S)}{1-G(S)R(S)}$ ⇒ Positive Feedback.

→ In a practical system, the phase shift b/w input and feedback signal is 0°, or ±360°, for positive Feedback.

→ whereas for negative feedback, the phase shift b/w input and feedback signal is ±180°, or out of phase.

COMPARISON B/W OPEN LOOP & CLOSED LOOP SYSTEMS

OPEN LOOP SYSTEM	CLOSED LOOP SYSTEM
	
$OLTF = \dfrac{C(S)}{R(S)} = G(S)$	$CLTF = \dfrac{G(S)}{1+G(S)H(S)}$

→ In general the open loop system is more stable, because there is no factor to effect the open loop stability.

3
⇒ **ACCURACY**

The open loop system accuracy depends on input and process.

4
⇒ **SENSITIVITY** w.r.t Noise

The open loop system is more sensitive for the disturbance, noise and environmental conditions, because whatever the changes occurs in the $G(s)$, that directly affects the output (eg: 10% change in $G(s)$ gives 10% change in $c(s)$).

~~Denominator term doesn't change (not sensitive) called DESENSITIVE FACTOR (1+GH)~~

5
⇒ **BANDWIDTH**

Comparitevely low Bandwidth.

6
⇒ **RELIABILITY**

Reliability depends number of discrete components used in the system. The open loop system is more reliable because it has less number of components.

7
⇒ In open loop control system, It is not necessary to measure the output. Errors are not generated. Sensors are not essential.

Design is easy.

→ The main disadvantage of feedback is it decreases gain by the factor of $1+G(s)H(s)$

2
→ closed loop is less stable.

consider a loop gain $G(s)H(s)$

$$G(s)H(s) = -1$$

$$G(s)H(s) = 0$$

$$G(s)H(s) > 0$$

"The closed loop stability depends on the loop gain"

→ If Loop Gain $= G(s)H(s) = -1$, CLTF $= \infty$, then the closed loop system stability is effected.

→ If Loop Gain $= G(s)H(s) = 0$, CLTF $=$ OLTF. then the closed loop system stability equal to open loop system stability.

→ If Loop Gain ≥ 0 $G(s)H(s) > 0$, then the closed loop system become more stable than open loop system.

3
⇒ **ACCURACY**

The closed loop system accuracy depends on feedback n/w ratio. If the feedback n/w use the stable value, then the closed loop system becomes highly accurate than open loop s/m.

4
⇒ **SENSITIVITY**

The closed loop control system is less sensitive for the disturbance, noise and environmental condition because the chang in output is very less (Less than 1%).

eg:
10% change in $G(s)$

$$c(s) = \frac{G(s)^{\,10\%}}{1+G(s)H(s)} \rightarrow \text{No change} \rightarrow \text{Desensitivity fac}$$

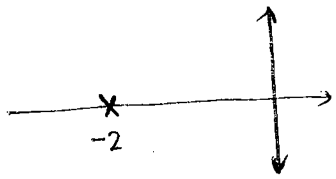
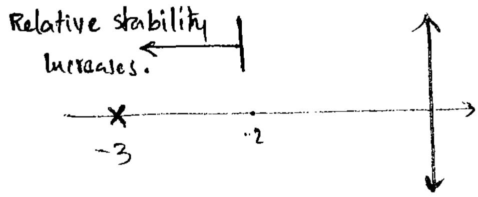
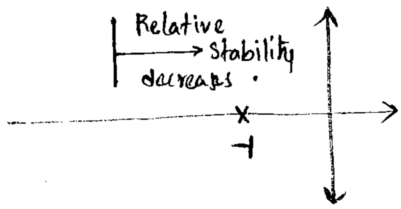
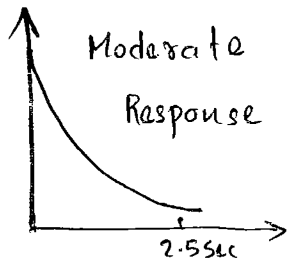
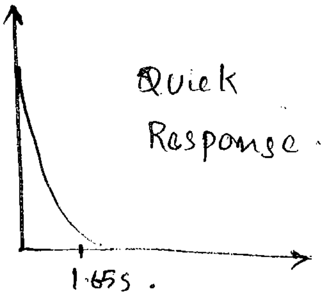
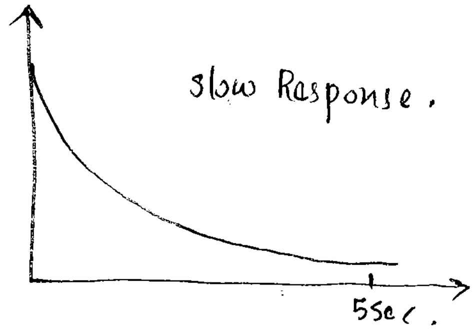
⇒ **BANDWIDTH**

For any practical system, the Gain Bandwidth product must be constant. With feedback the gain is reduced by the factor of $1+G(s)H(s)$. ie, Bandwidth is increased by a factor of $1+G(s)H(s)$. Bandwidth represent the speed of response. Large B.W gives a very quick response.

Bandwidth $\boxed{B.W \propto \frac{1}{t_r}}$ $\boxed{BW = \frac{0.35}{t_r}}$

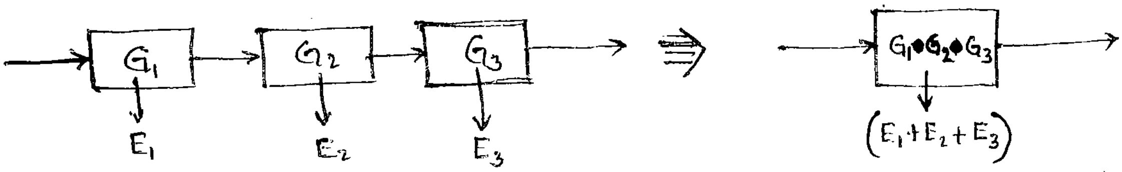
⇒ Less Reliable due to more # of components.

⇒ Output must be measured. sensors are essential, Errors are generated. Design is complex.

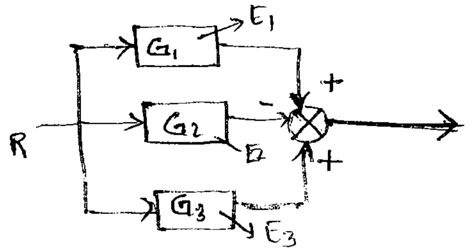
OPEN LOOP WITHOUT FEEDBACK	CLOSED LOOP WITH -ve FEEDBACK	CLOSED LOOP WITH +ve FEEDBACK
Let $\frac{C(s)}{R(s)} = G(s) = \frac{1}{s+2}$	~~Let~~ $G(s) = \frac{1}{s+2}$ $\frac{C(s)}{R(s)} = \frac{1}{s+3}$	$G(s) = \frac{1}{s+2}$ $\frac{C(s)}{R(s)} = \frac{1}{s+1}$
		
$\tau = 0.5$ sec $BW = \frac{1}{\tau} = 2Hz$.	$\tau = 0.33$ sec $BW = 3Hz$.	$\tau = 1$ sec. $BW = 1Hz$.
		

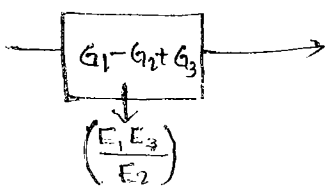
BLOCK REDUCTION TECHNIQUES

(i) BLOCKS ARE IN SERIES

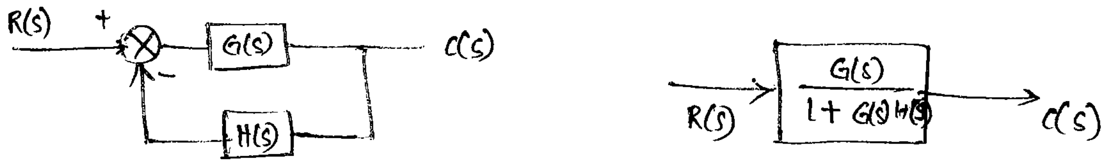


(ii) BLOCKS IN PARALLEL

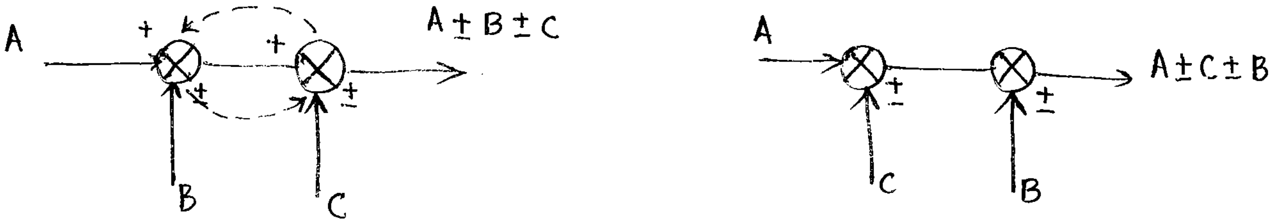




(iii) LOOP

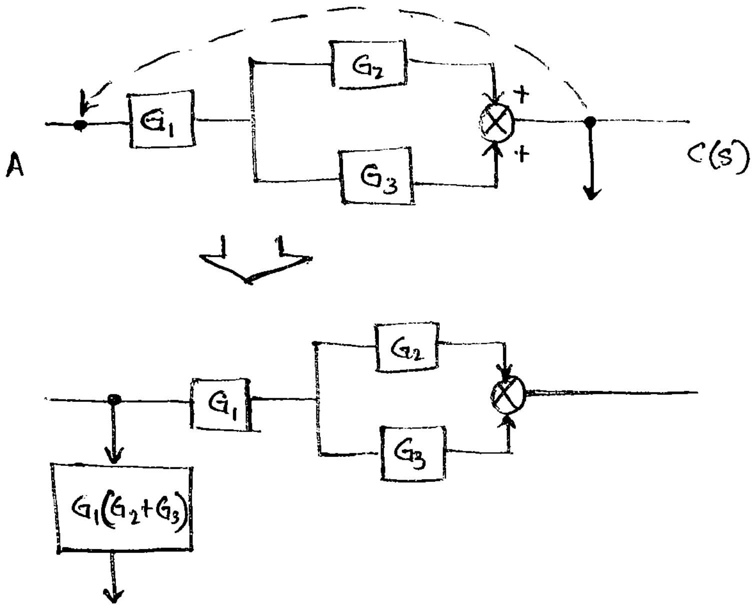


(iv) INTERCHANGING THE SUMMING POINT

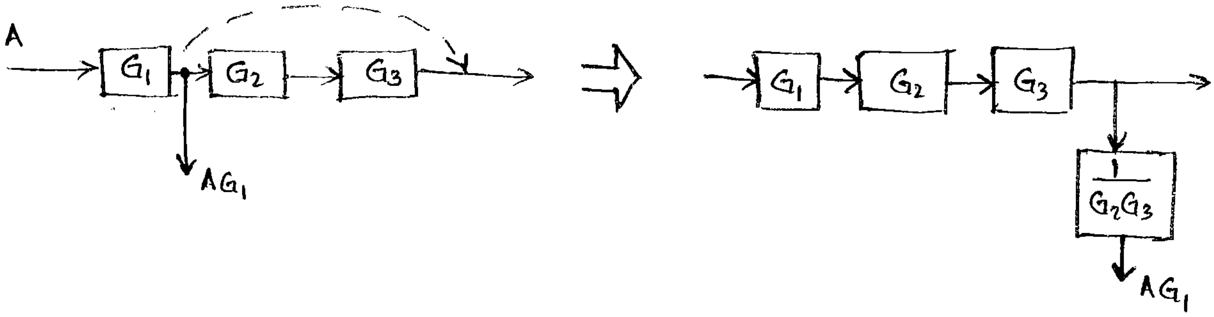


(v) ADJUSTING BLOCK GAIN AND TAKE UP POINT

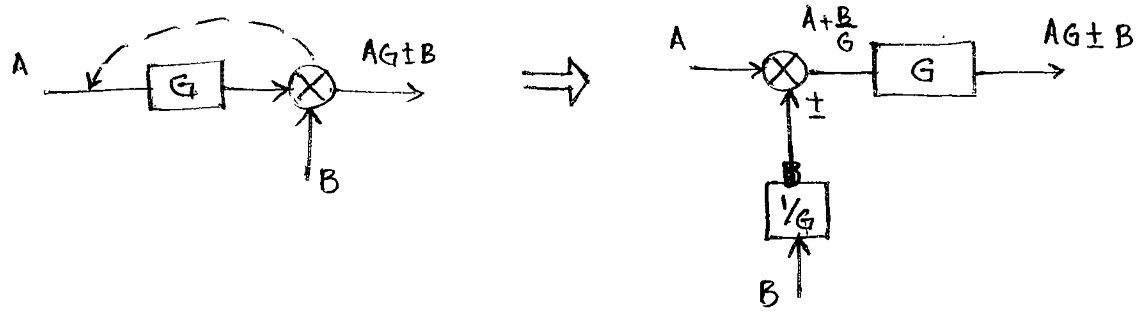
1,

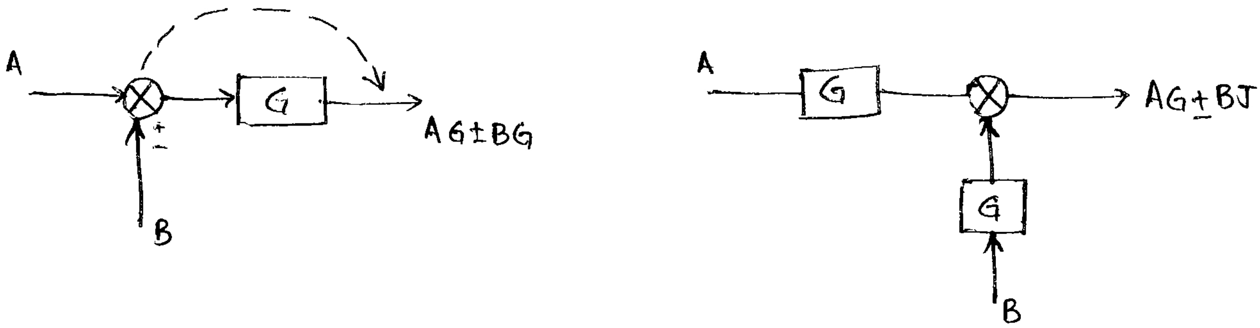


2,

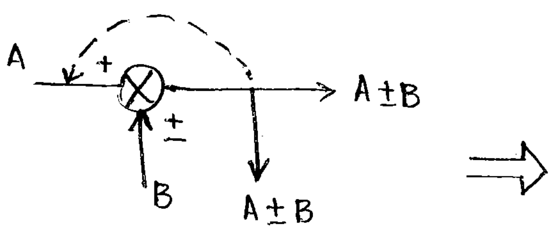


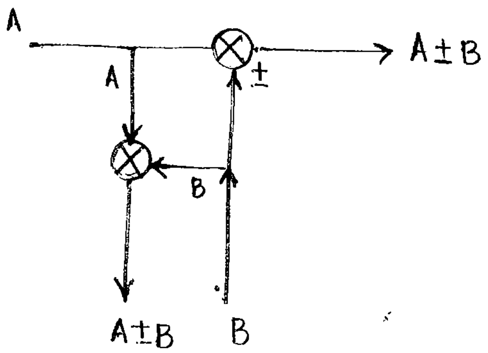
ADJUSTING BLOCK GAIN AND SUMMING POINT



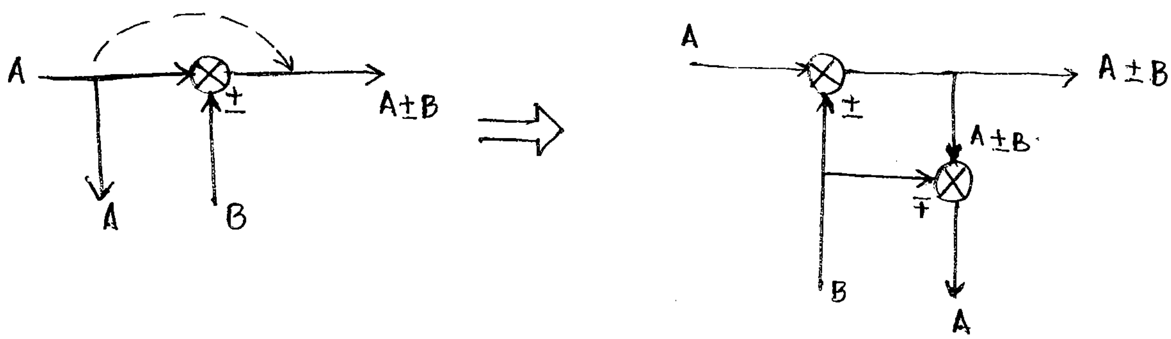


ADJUSTING TAKEUP POINT AND SUMMING POINT





Here while interchanging, an additional summing point will come.



step 1 : Simplify the series, parallel and Loop n/w.

step 2 : Simplify the summing points if adjuscent to eachother.

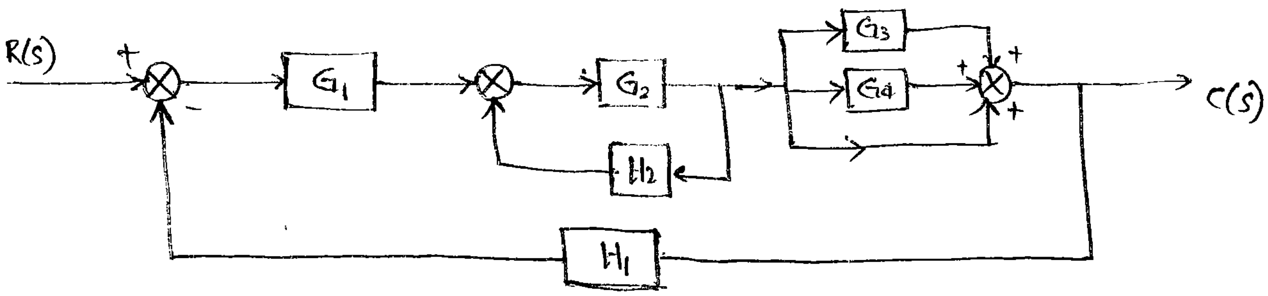
step 3 : Interchange the summing point such that, we get a loop.

step 4 : Then ~~take~~ adjust take up point such that we get a series, parallel or loop n/w.

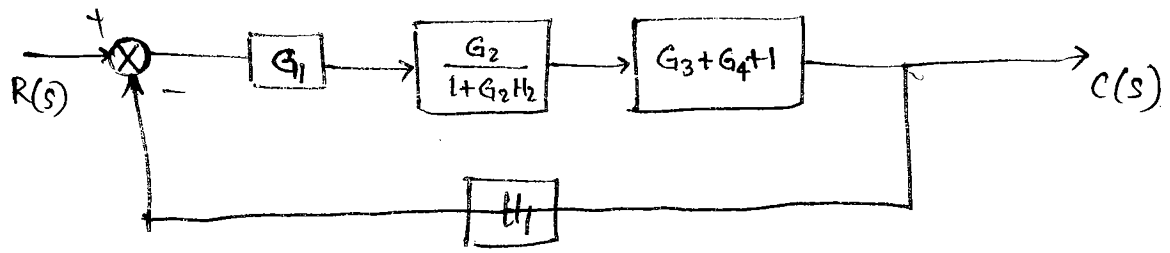
step 5 : shift or adjust the summing point across the blocks so that we get a series, parallel or loop n/w.

step 6 : shift or adjust the takeup point across the summing point.

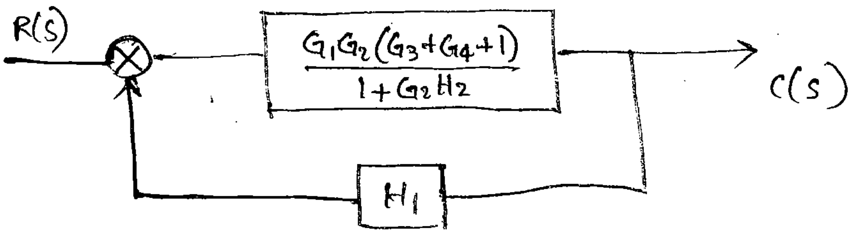
Q. Find the overall transfer fn to the given block diagram.



step 1



step 2

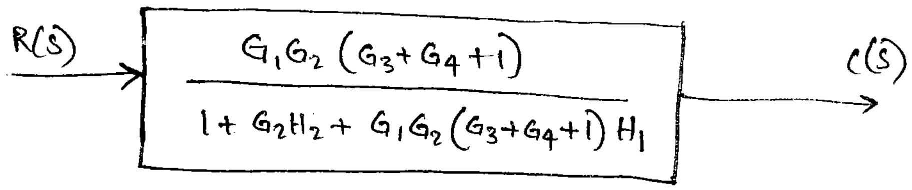


$$\frac{R(s)}{C(s)} = \frac{\dfrac{G_1G_2(G_3+G_4+1)}{1+G_2H_2}}{1+G_1G_2(G_3+G_4+1)\ldots}$$

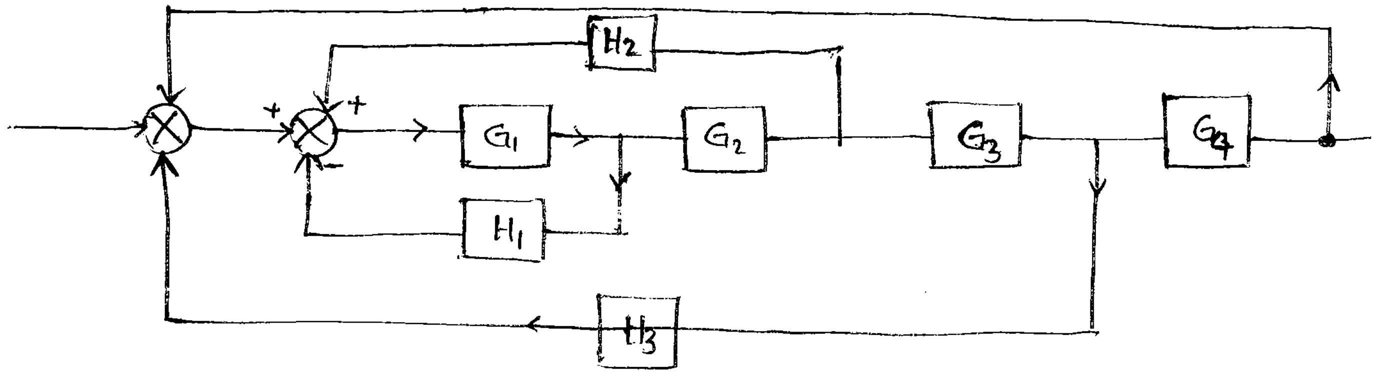
No need of this step for loop, take numerator as it is and take ~~num~~ denominator ± Numerator × Feedback

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 (G_3 + G_4 + 1)}{1 + G_2 H_2 + G_1 G_2 (G_3 + G_4 + 1) H_1}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_1 G_2 G_4 + G_1 G_2}{1 + G_2 H_2 + G_1 G_2 G_3 H_1 + G_1 G_2 G_4 H_1 + G_1 G_2 H_1}$$

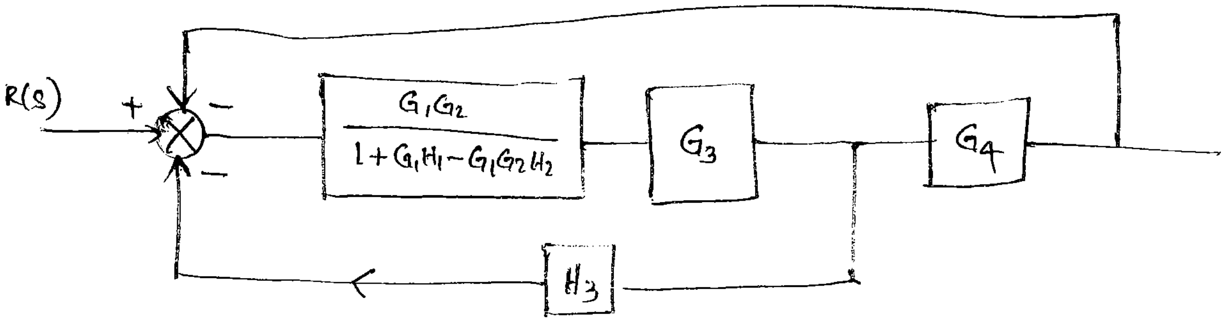


Q. Find the transfer function.

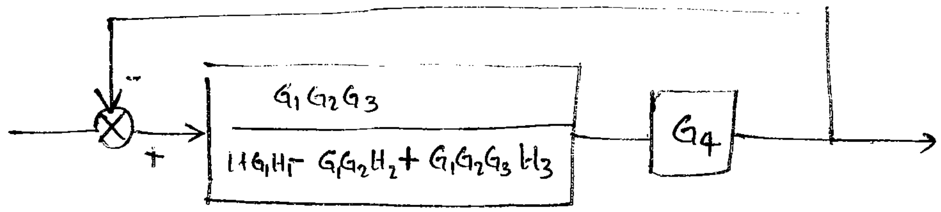


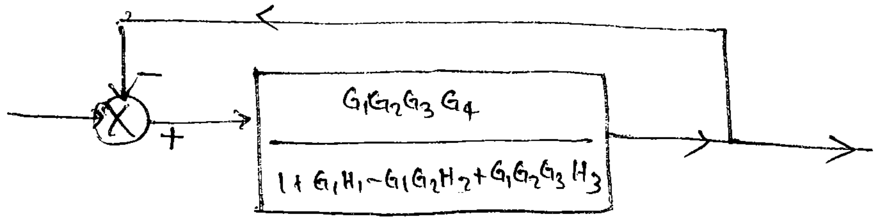
$$\frac{G_1}{1 + G_1 H_1} \times G_2$$

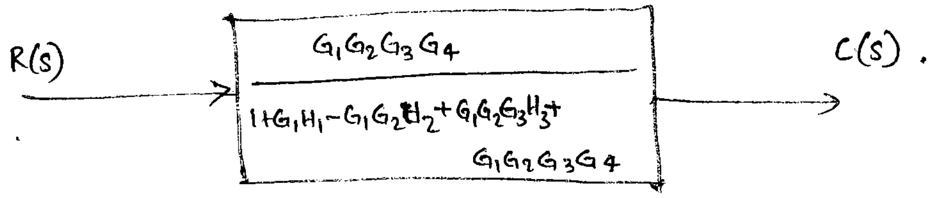
$$= \frac{G_1 G_2}{1 + G_1 H_1 - G_1 G_2 H_2}$$



$$\frac{G_1 G_2 G_3}{1 + G_1 H_1 - G_1 G_2 H_2}$$

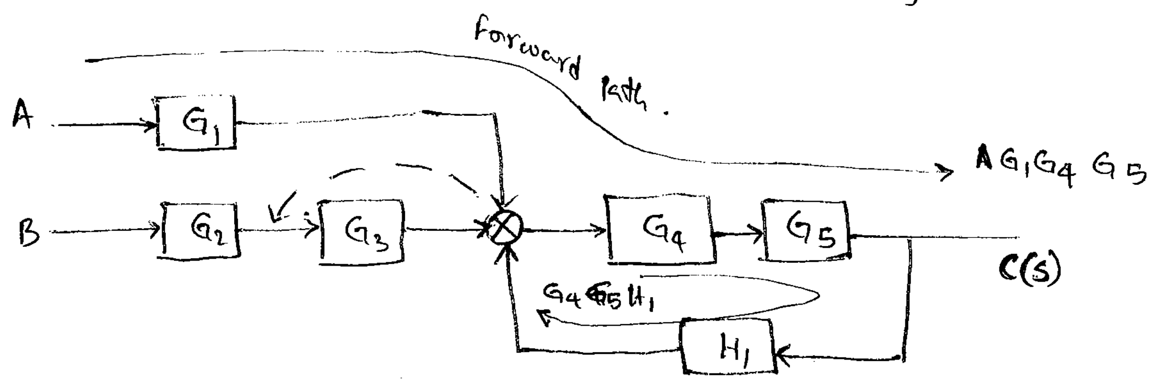


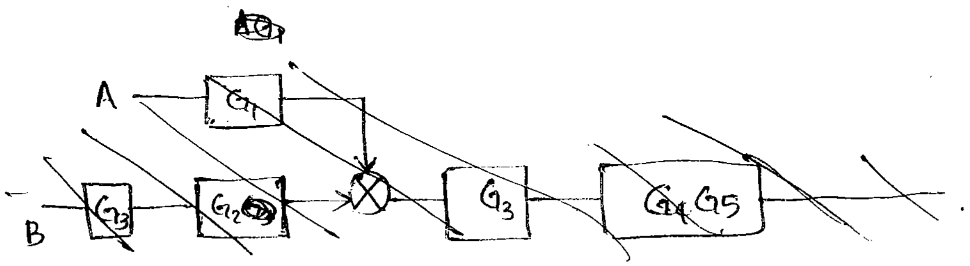




$$T\cdot F = \frac{C(S)}{R(S)} = \frac{G_1G_2G_3G_4}{1+G_1H_1-G_1G_2H_2+G_1G_2G_3H_3+G_1G_2G_3G_4}$$

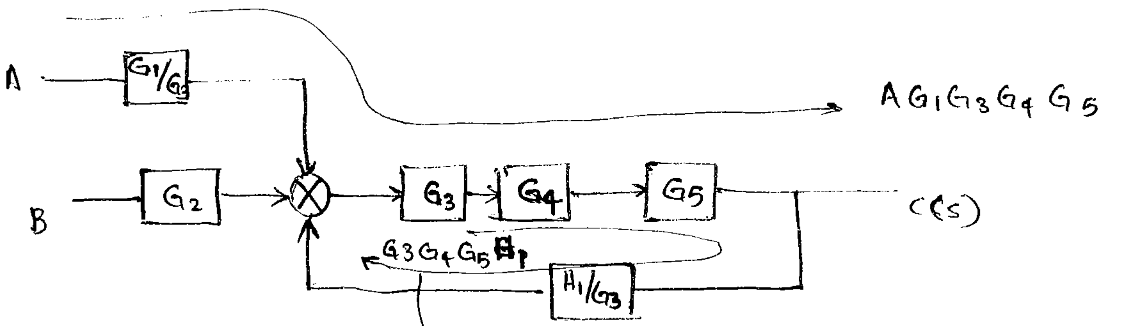
Q. Draw the equivalent block diagram to the following





$AG_1 + BG_1G_2$
$+ CH_1$

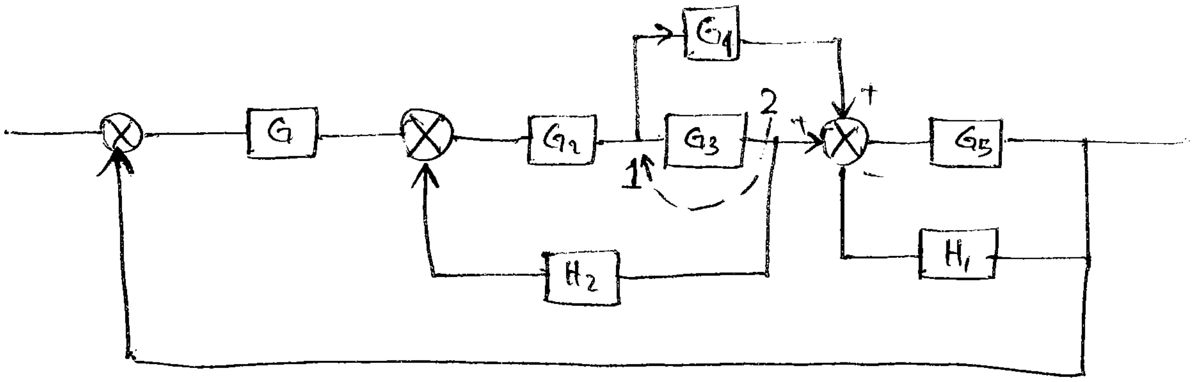
while shifting the summing ~~...~~ takeoff points the changes occurs only in additional forward path and feedback path. There is no change in main forward path.

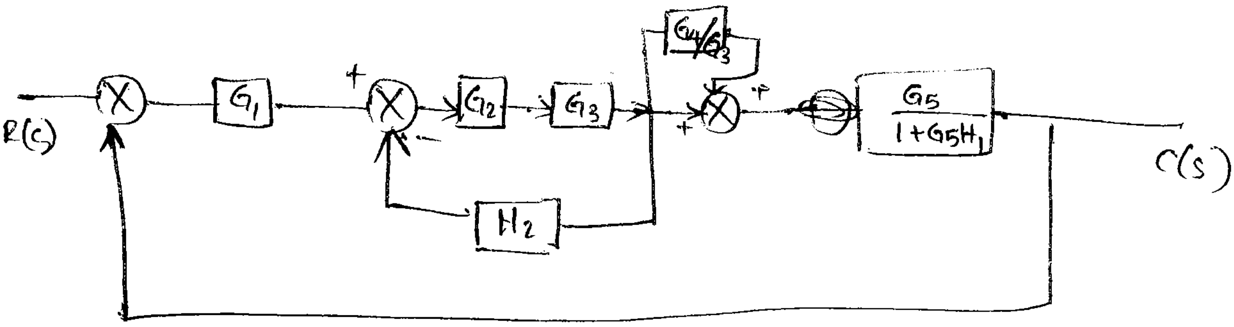


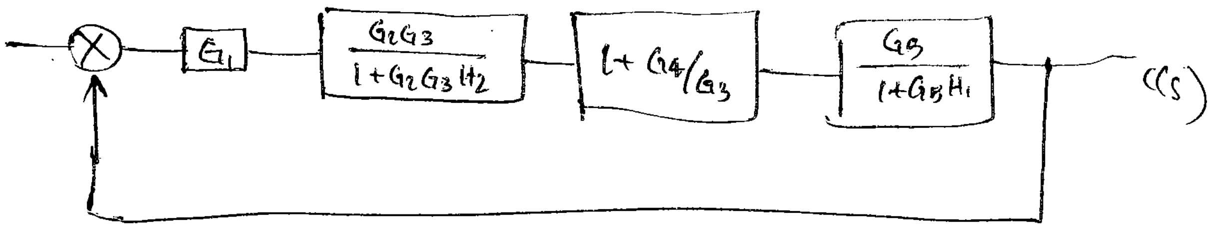
Here difference from before shifting is G_3. Hence divide G_3 from G_1

G_3 is extra from last figure, hence divide by G_3 from H_1

Q



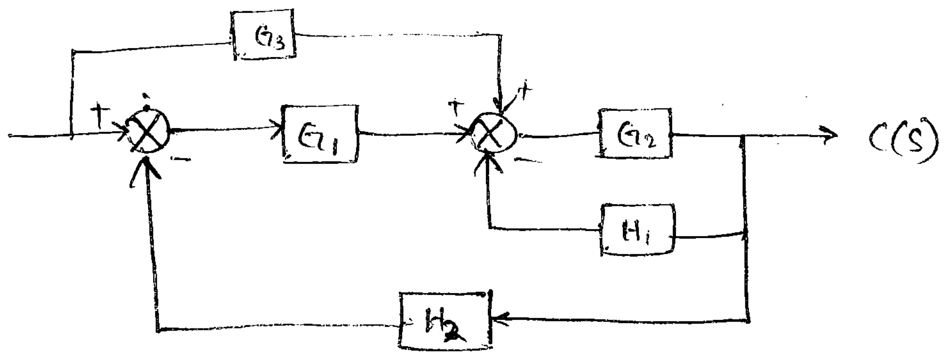




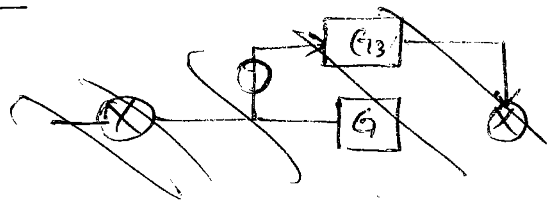
$$\frac{\frac{G_1G_2G_3}{(1+G_2G_3H_2)}\left(1+\frac{G_4}{G_3}\right)\frac{G_5}{(1+G_5H_1)}}{1+\frac{G_1G_2G_3G_5(G_3+G_4)}{(1+G_2G_3H_2)(1+G_5H_1)\,G_3}}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 (G_3 + G_4) G_5}{(1+G_2G_3H_2)(1+G_5H_1)G_3 + G_1G_2(G_3+G_4)G_5}$$

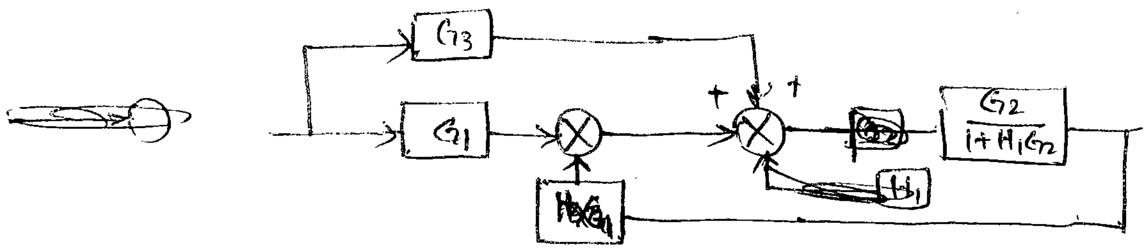
Q

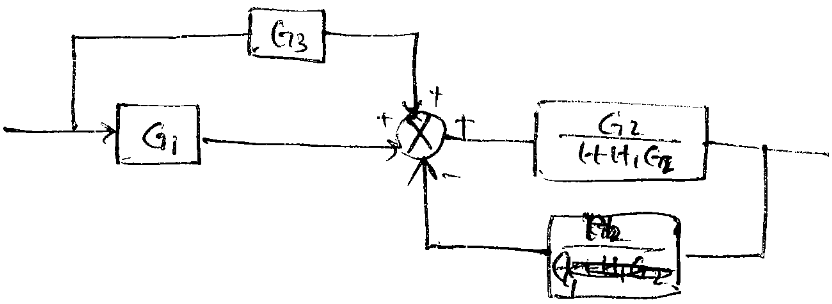


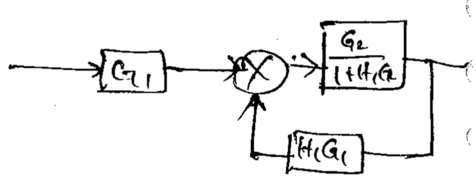
Method 1



~~Summing point~~ shifting summing point ahead of G.



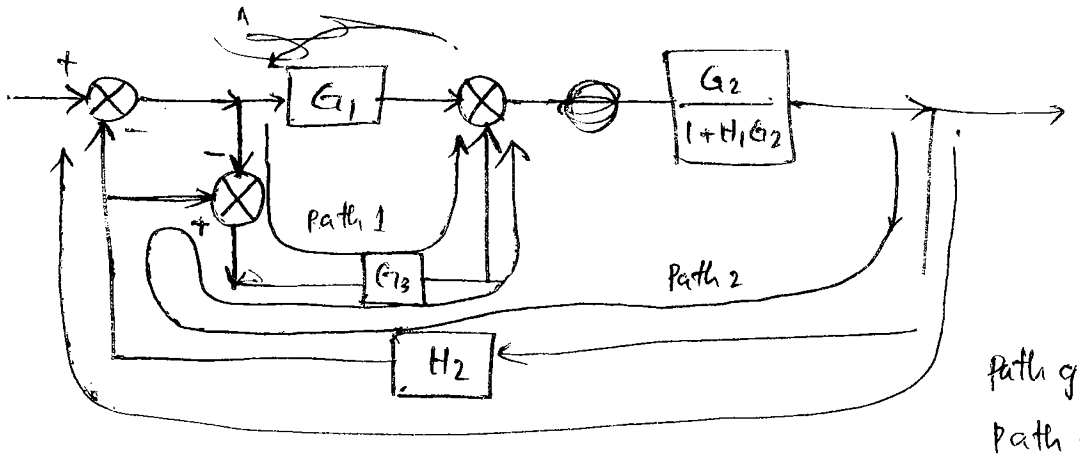




$$(G_1+G_3)\ \frac{G_2}{1+H_1G_1 + \frac{H_2}{G_1} \times G_2}$$

$$\frac{(G_1+G_3)G_2}{1+H_1G_2 + \frac{H_2}{G_1} \times G_2\ G_1}$$

method 2 Shifting the takeup point ahead of the summing point.



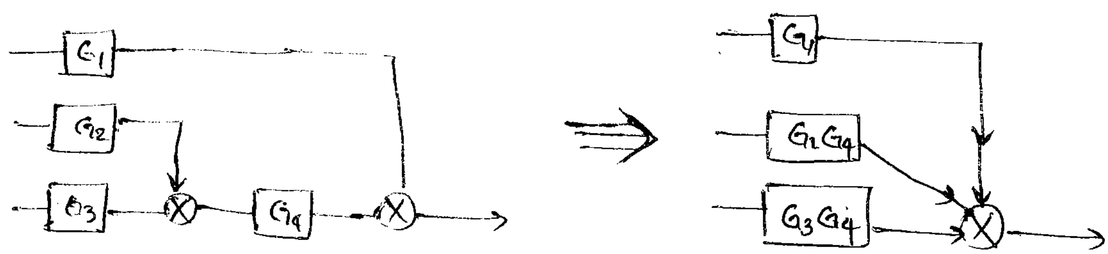
path gain 1 = G_3

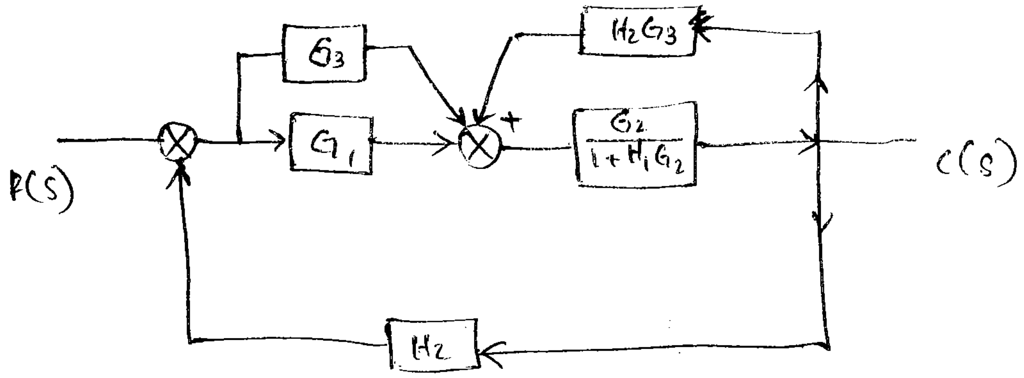
Path gain 2 = H_2G_3

Step 2 Eliminate the new summing point.

we can eliminate summing point only by considering the additional ~~su~~ forward path casing.

fore

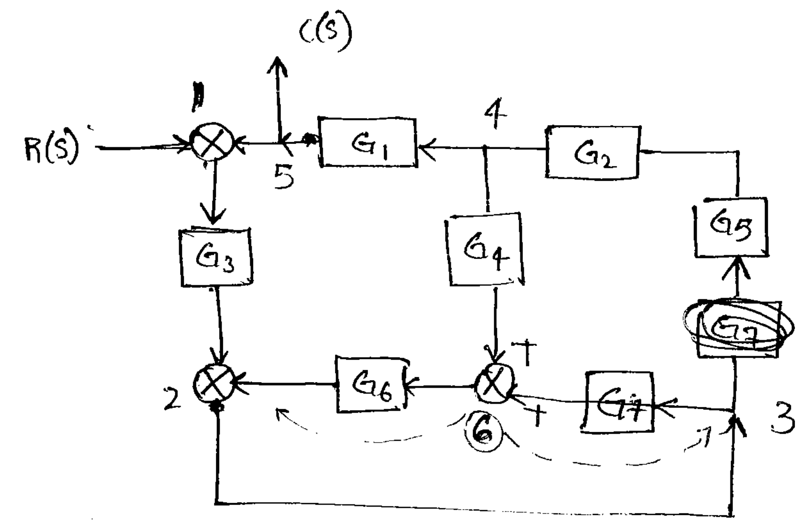


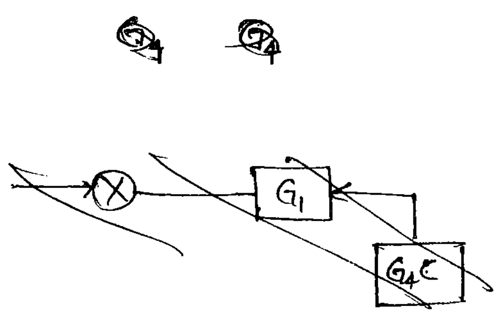


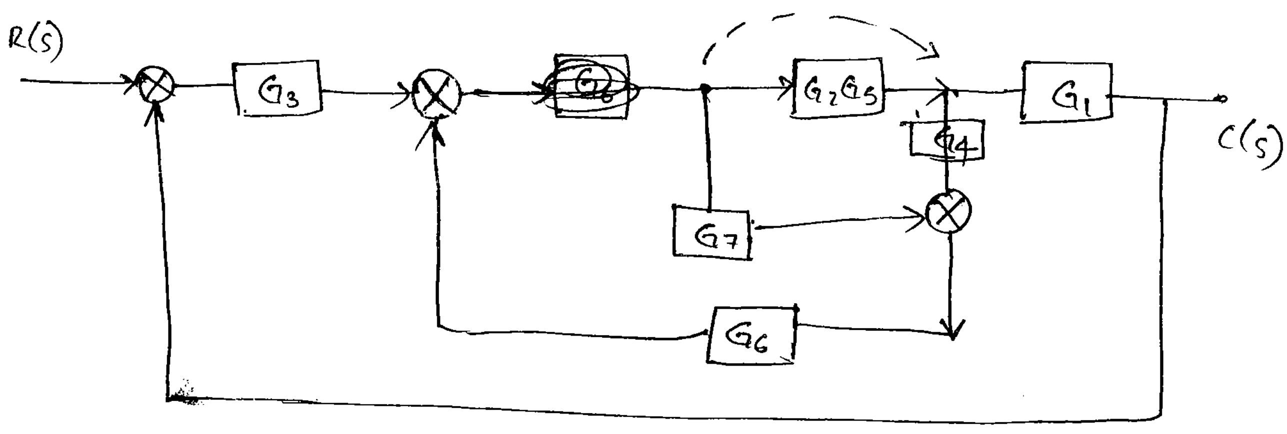
$$\boxed{\frac{(G_1+G_3)\,G_2}{-(G_1+G_3)G_2H_2 + 1+H_1G_2+H_2G_2G_3}} = \frac{(G_1+G_3)\,G_2}{1+H_1G_2 + \cancel{H_2G_2G_3} + G_1G_2H_2 - \cancel{G_3H_2G_3}}$$

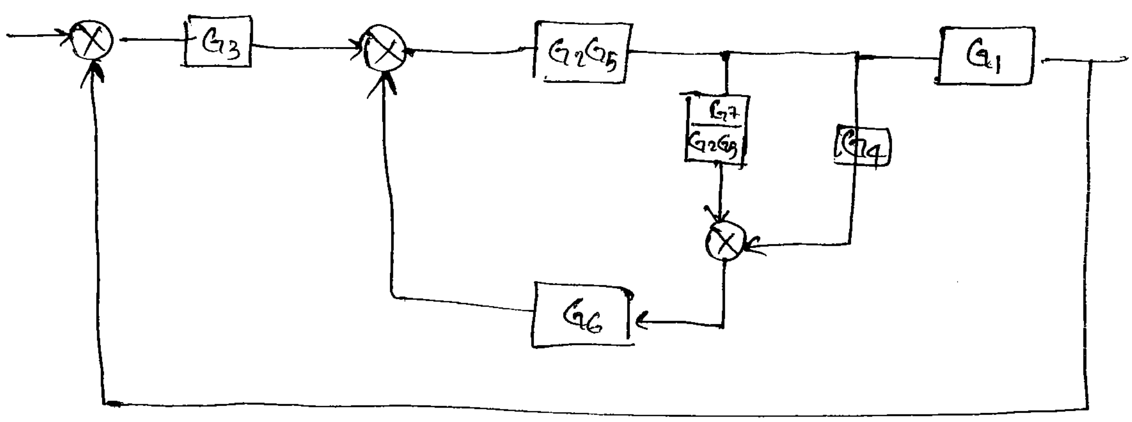
$$= \frac{(G_1+G_3)\,G_2}{1+H_1G_2+G_1G_2H_2}$$

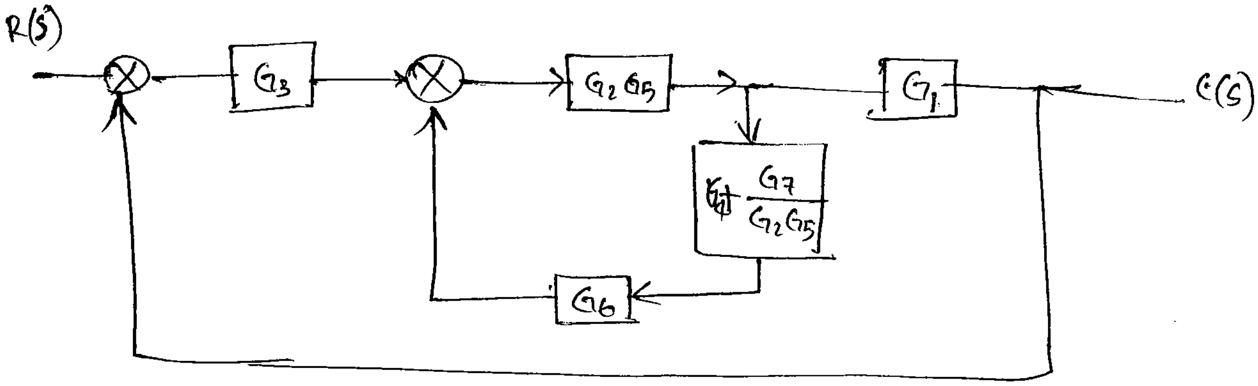
Q Find the transfer function.











$$\frac{G_2G_5G_1G_3}{1+\left(G_4+\frac{G_7}{G_2G_5}\right)G_6G_2G_5}$$

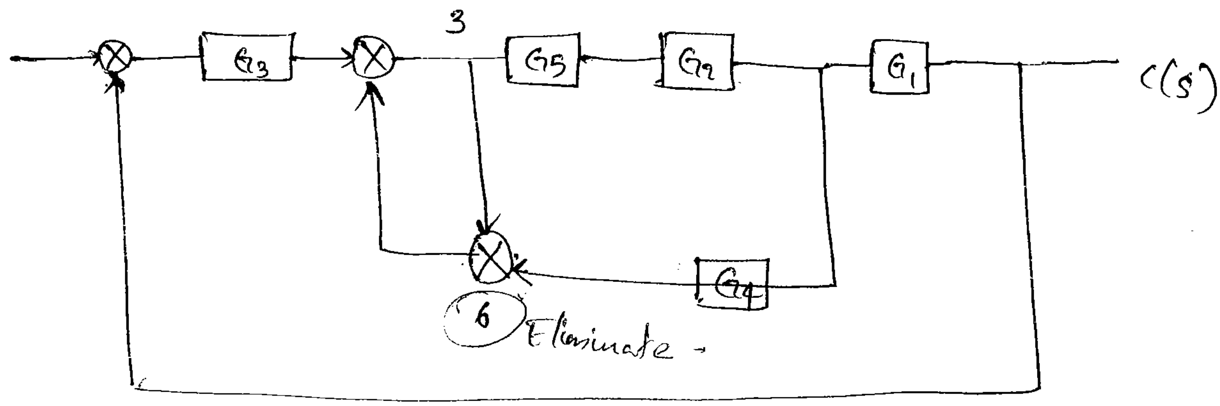
$$=\frac{G_2G_5G_1G_3}{\left(1+\left(G_4+\frac{G_7}{G_2G_5}\right)G_6G_2G_5\right)+G_2G_5G_1G_3}$$

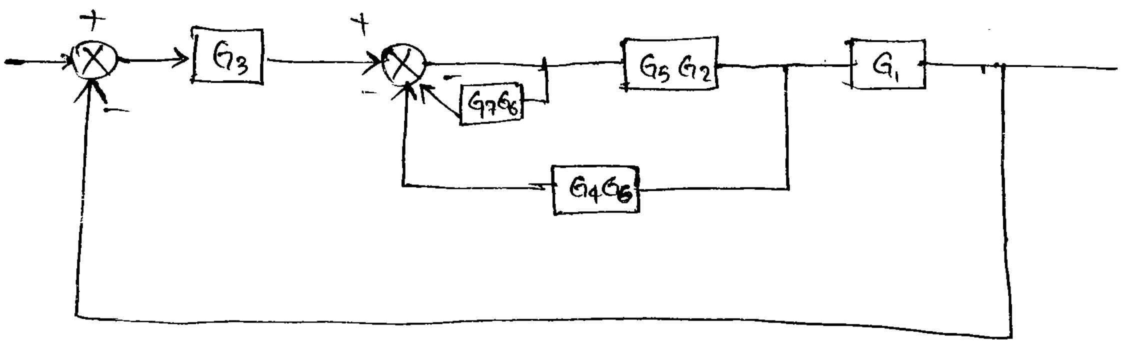
$$=\frac{G_2G_5G_1G_3}{1+\left(G_6G_2G_5G_4+\frac{G_7G_6\cancel{G_2G_5}}{\cancel{G_2G_5}}\right)+G_2G_5G_1G_3}$$

$$=\frac{G_1G_2G_3G_5}{1+G_2G_5G_6G_4+G_6G_7+G_1G_2G_3G_5}$$

Method 2

Eliminate the ~~additional~~ ~~four~~ summing point 6 by considering the additional path.





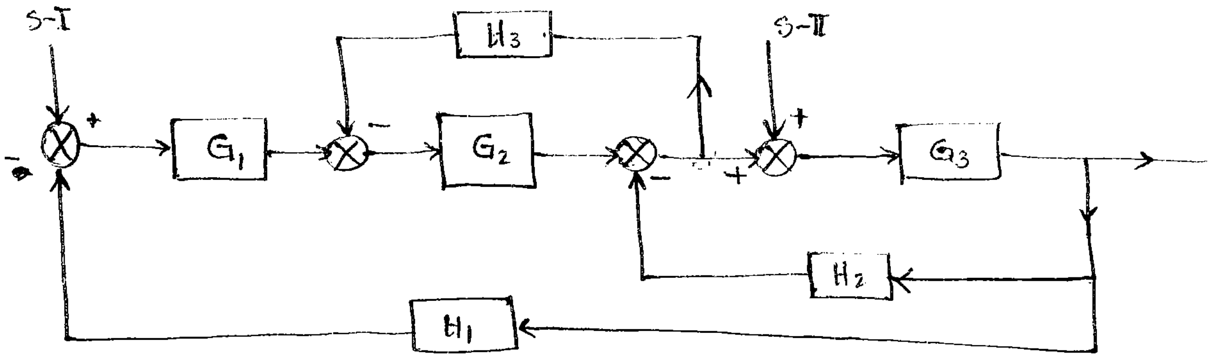
Loop 1 $\qquad \dfrac{G_2G_5}{1+G_7G_6}$

Loop 2 $\qquad \dfrac{G_2G_5\,G_7G_3}{1+G_6G_7+G_4G_6G_2G_5}$

Loop 3 $\qquad \dfrac{\cancel{G_2C}\quad G_1G_2G_3G_5}{1+G_6G_7+\cancel{G_2}G_4G_5G_6+G_1G_2G_3G_5}$

find the transfer function due to station 1

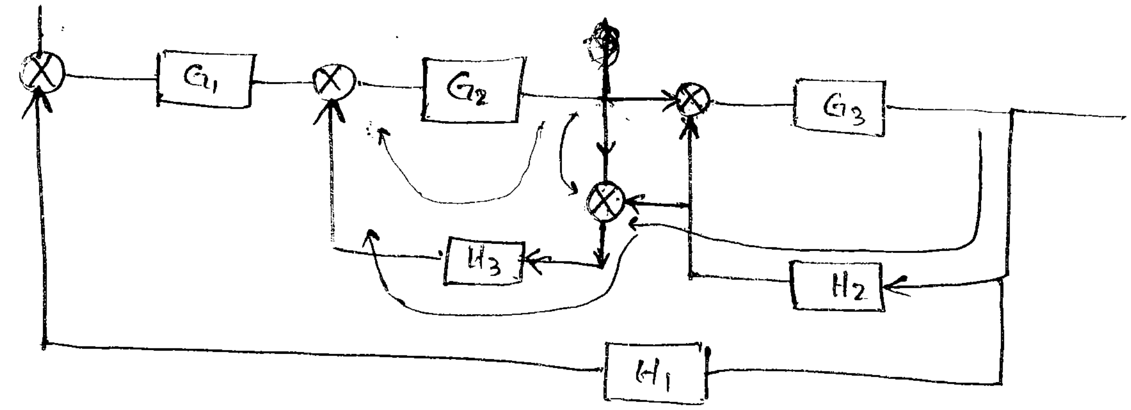
(ii) station 2.

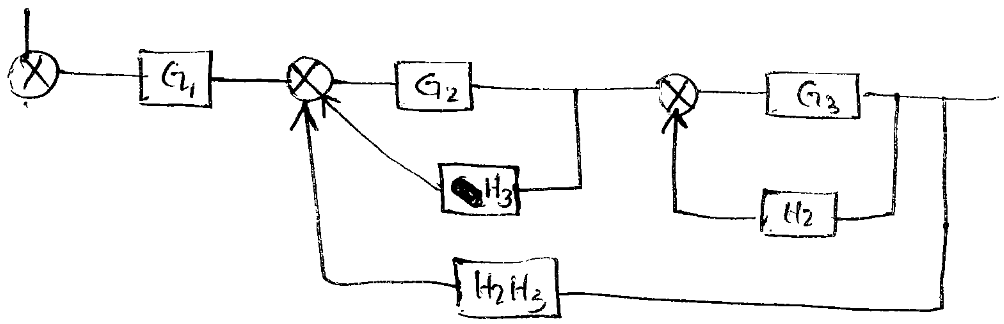


~~(i) station 2 = 0~~

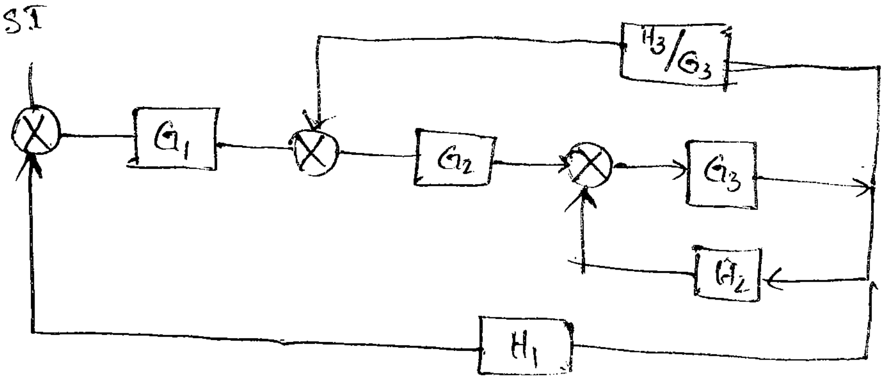
(i) station 1

station 2 = 0





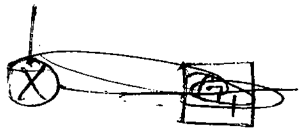
ST

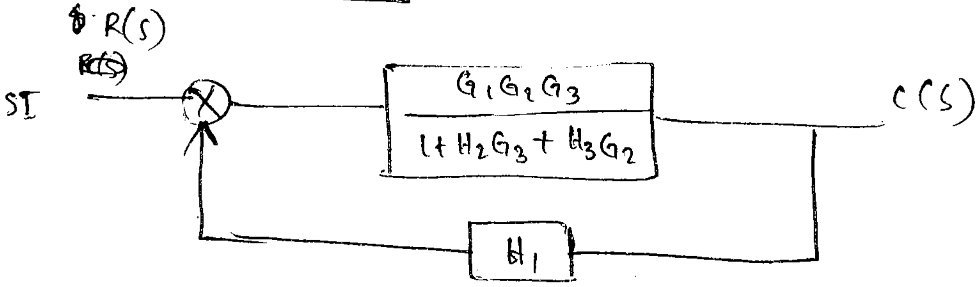


$$\frac{G_3G_2}{1+H_2G_3}$$

$$\frac{G_2G_3}{1+H_2G_3+\frac{H_3}{\cancel{G_3}}\times G_2\cancel{G_3}}$$

$$\frac{G_2G_3}{1+H_2G_3+H_3G_2}$$

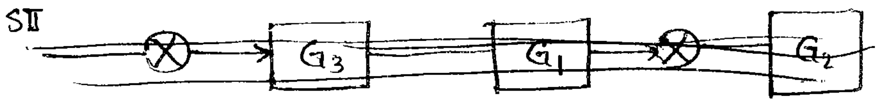


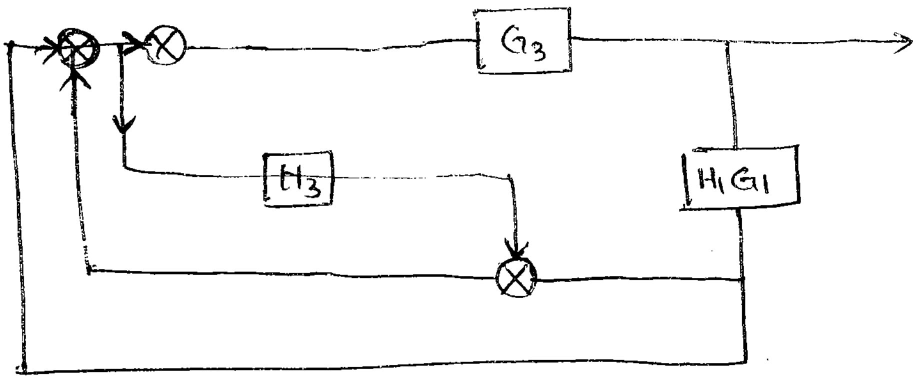


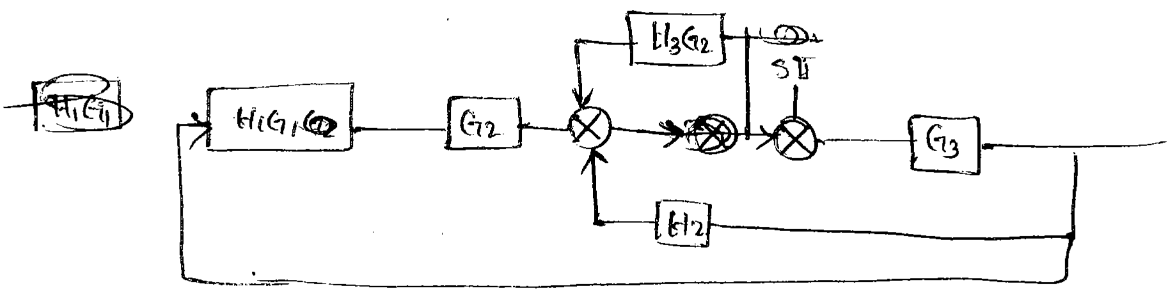
$$\frac{R(s)}{SI(s)} \quad \frac{G_1G_2G_3}{1+H_2G_3+H_3G_2+G_1G_2G_3H_1}$$

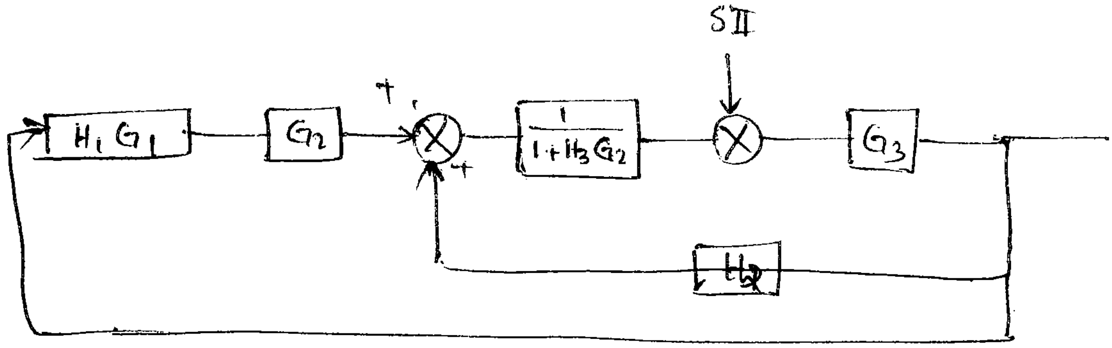
(ii) Station 2

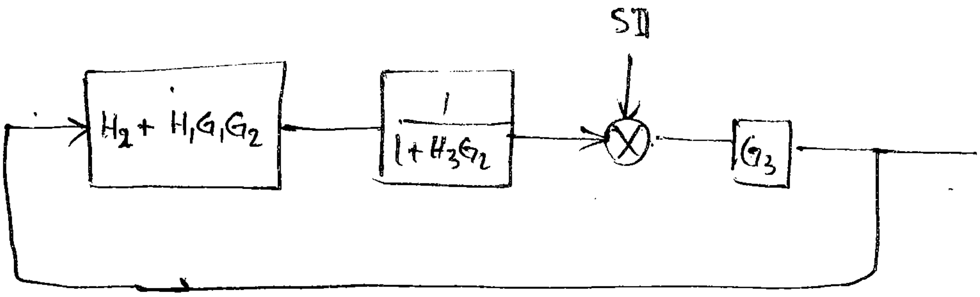
station 1 = 0









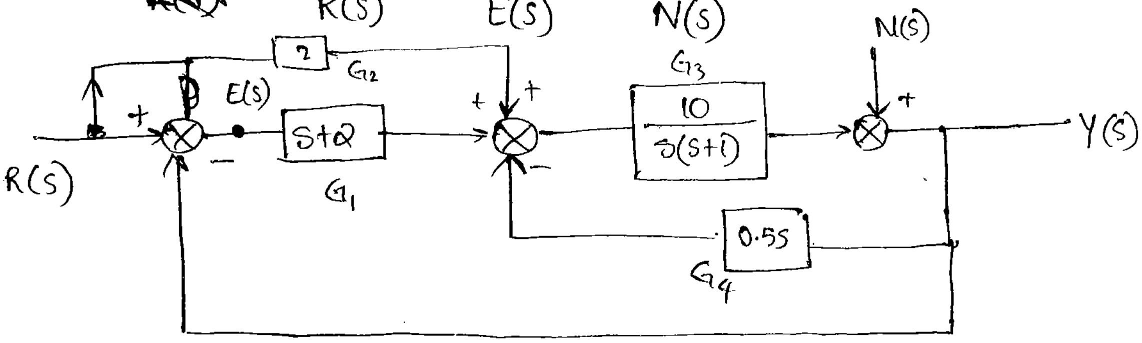


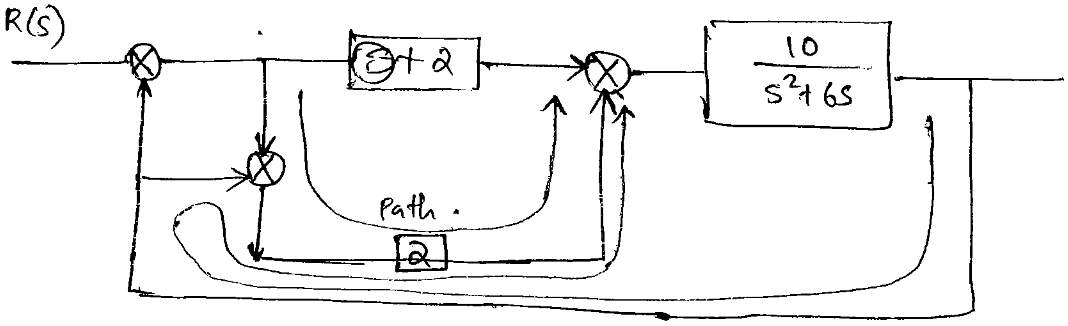
$$\frac{(H_2 + H_1G_1G_2)}{1+H_3G_2} \qquad G_3$$

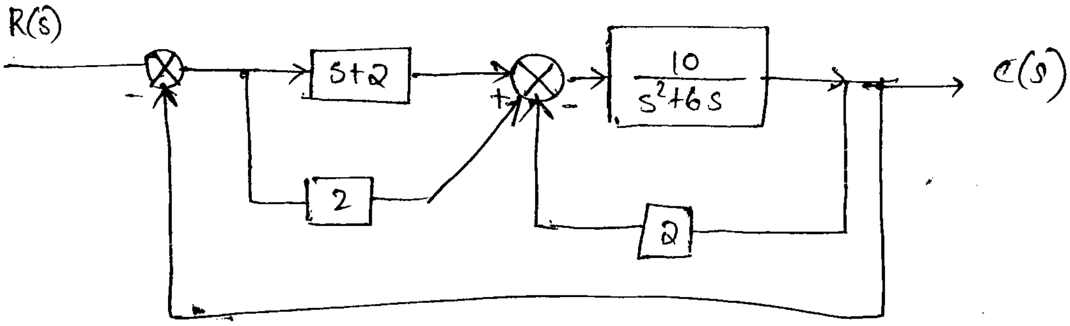
Loop $$\frac{G_3}{1+\left(\frac{H_2 + H_1G_1G_2G_3}{1+H_3G_2}\right)}$$

$$T.F = \frac{C(s)}{SII(s)} = \frac{G_3(1+H_3G_2)}{1+H_3G_2 + H_2H_1G_1G_2G_3 + H_2G_3}$$

Q. Find $\cancel{\frac{Y(s)}{E(s)}}$ $\frac{Y(s)}{R(s)}$, $\frac{Y(s)}{E(s)}$ & $\frac{Y(s)}{N(s)}$







$$\frac{10}{s(s+1)}$$

$$\frac{10}{s(s+1)+5s}$$

$$\frac{10}{s^2+s+5s}$$

$$\frac{10}{s^2+6s}$$

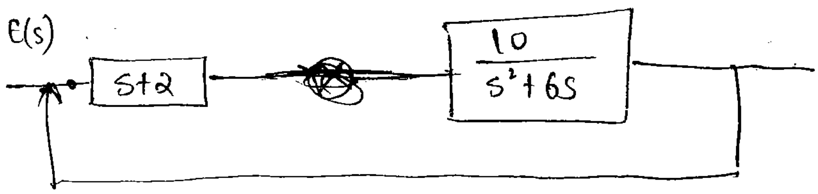
$$(s+4)\left(\frac{10}{s^2+6s\mp 20}\right)$$

$$= \frac{10(s+4)}{s^2+6s\mp 20+10(s+4)}$$

$$= \frac{10(s+4)}{s^2+6s-20+10s+40}$$

$$= \frac{10(s+4)}{s^2-4s+20}$$

$$\frac{C(s)}{E(s)}$$



$$\cancel{\frac{10(s+2)}{s^2+6s}} \qquad \cancel{\frac{10(s+2)}{s^2+6s-}}$$

$E(s) = R(s) - Y(s)$

$$\frac{Y(s)}{R(s)} = \frac{10(s+4)}{s^2+16s+20}$$

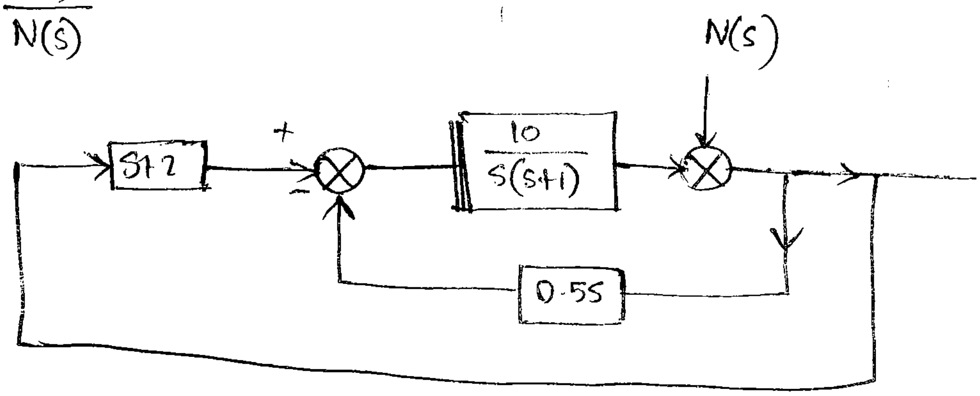
$$\frac{Y(s)+E(s)}{Y(s)} = \frac{s^2+16s+20}{10s+40}$$

$$\frac{E(s)}{Y(s)} = \frac{s^2+16s+20}{10s+40} - 1$$

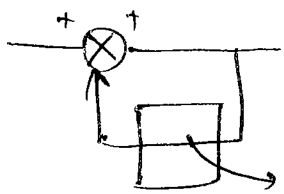
$$\frac{E(s)}{Y(s)} = \frac{s^2+16s+20-10s-40}{10s+40}$$

$$\frac{Y(s)}{E(s)} = \frac{10s+40}{s^2+6s-20}$$

(iii) $\frac{Y(s)}{N(s)}$



$$\left(0.5s + s+2\right)\frac{10}{s(s+1)} = \frac{10(1.5s+2)}{s(s+1)}$$



$$\boxed{\frac{15s+20}{s(s+1)}}$$

&

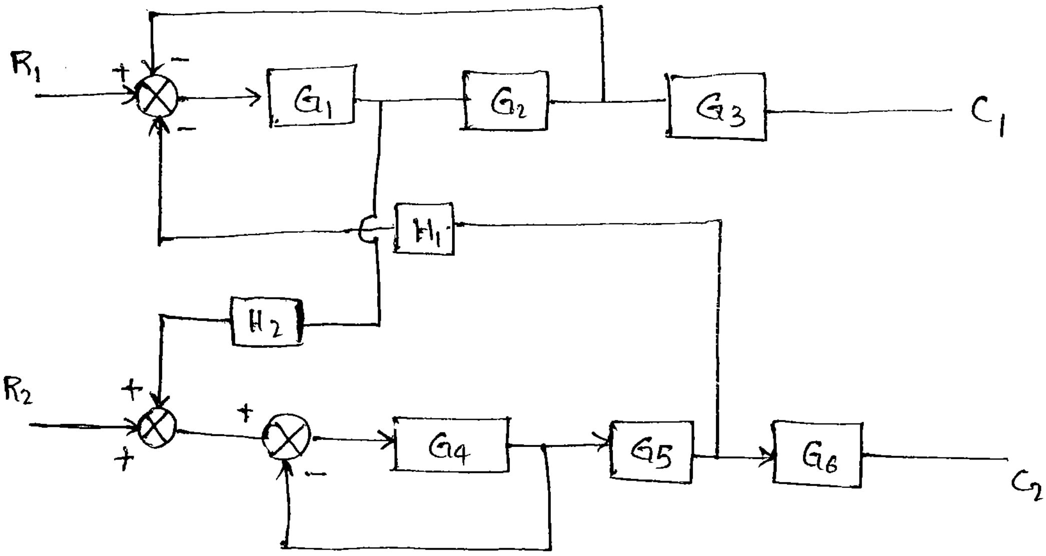
check

sign convention,

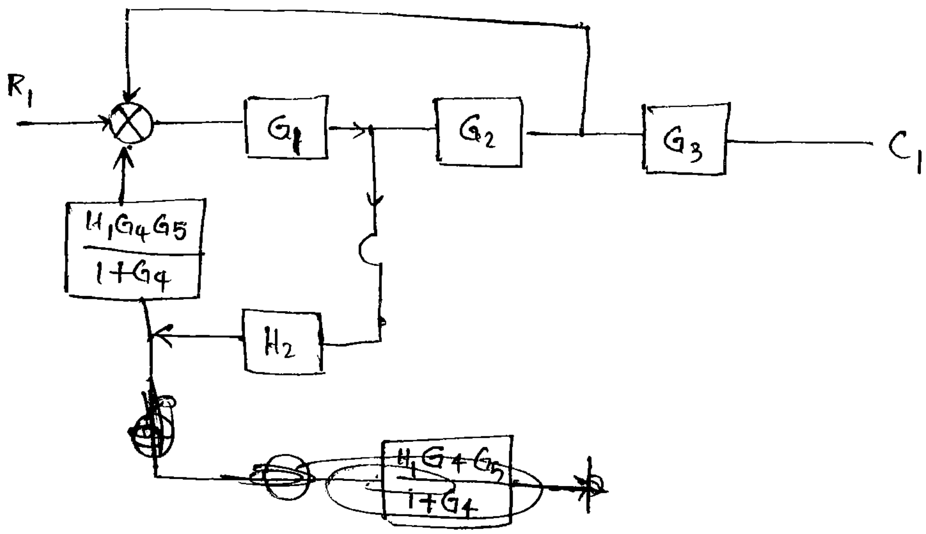
$$\frac{15s+20}{s(s+1)-15s-20} = \frac{15s+20}{s^2+s-15s-20}$$

$$= \frac{15s+20}{s^2-14s-20}$$

Q. Find $\frac{C_1}{R_1}$, $\frac{C_1}{R_2}$, $\frac{C_2}{R_1}$, $\frac{C_2}{R_2}$

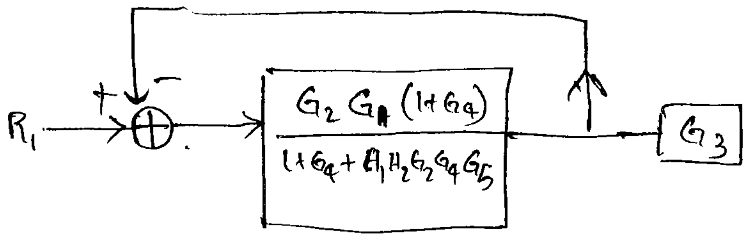


(i) $\frac{C_1}{R_1}$



$$\frac{G_2}{1+\frac{H_1H_2G_4G_5}{1+G_4}\times \cancel{G_2}}$$

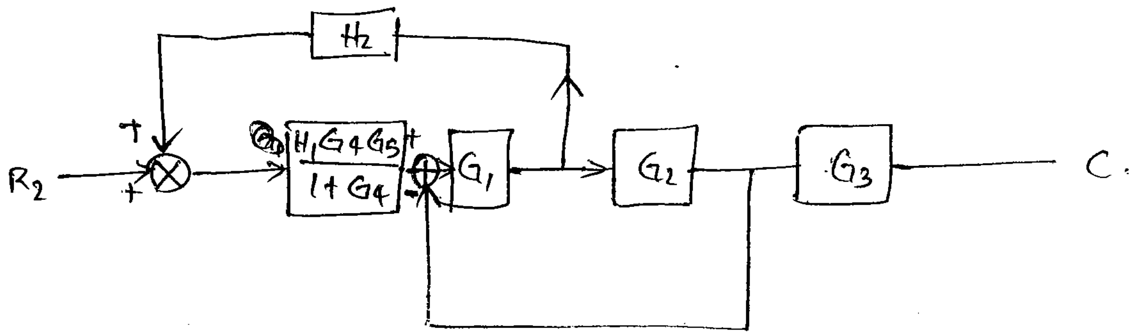
$$\frac{G_2(1+G_4)}{1+G_4+H_1H_2G_2G_4G_5}$$

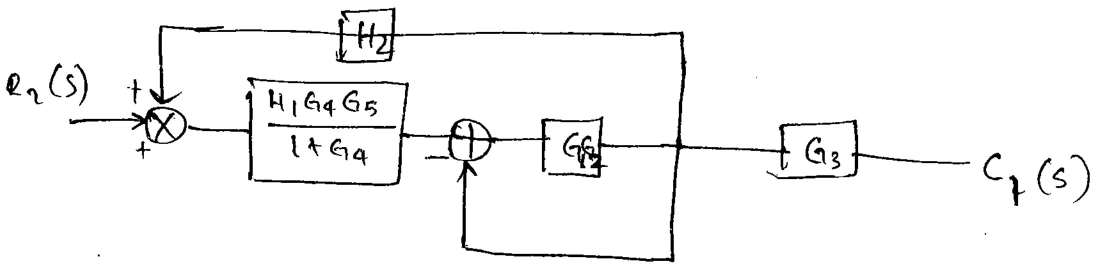


$$\frac{G_2G_1(1+G_4)\ G_3}{1+G_4+H_1H_2G_2G_4G_5+G_1G_2(1+G_4)}$$

$$= \frac{G_1G_1G_3(1+G_4)}{1+G_4+G_1G_2+G_1G_2G_4+G_2G_4G_5H_1H_2}$$

$\frac{C_1}{R_2}$



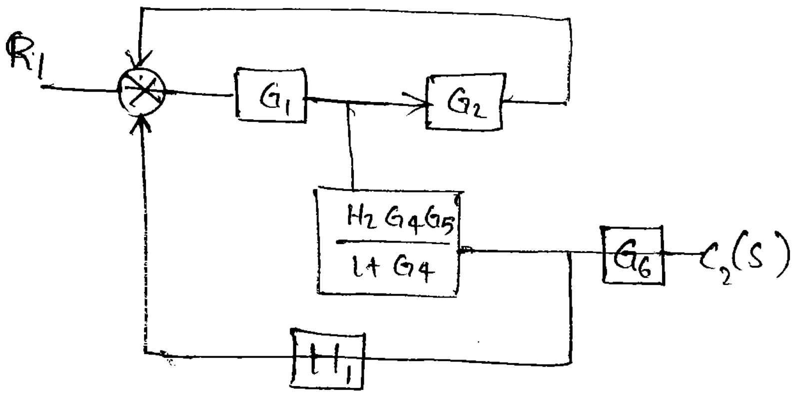


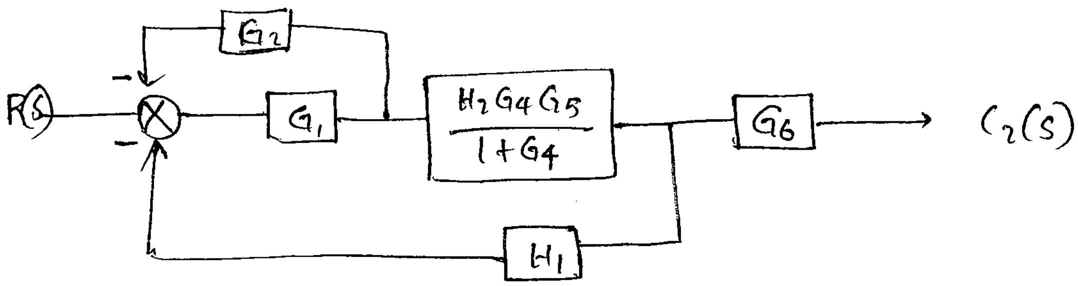
$$\frac{G_1G_2H_1G_4G_5}{(1+G_1G_2)(1+G_4)}$$

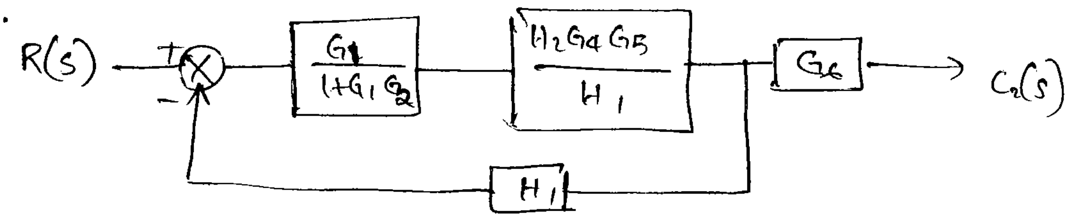
$$\frac{G_1G_2H_1G_4G_5}{(1+G_1G_2)(1+G_4)-H_2G_1G_2H_1G_4G_5}$$

$$= \frac{1}{1+G_4+G_1G_2+G_1G_2G_4-H_1H_2G_1G_2G_4G_5}$$

$\frac{C_2}{R_1}$





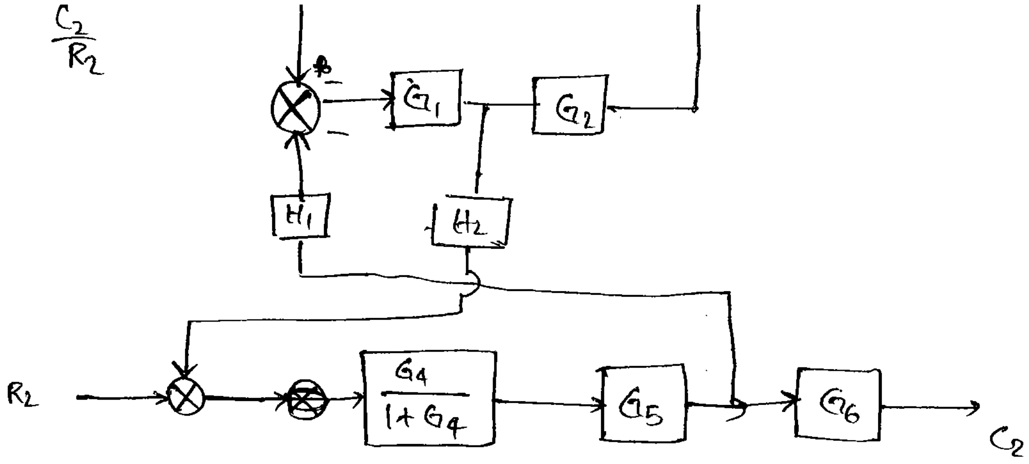


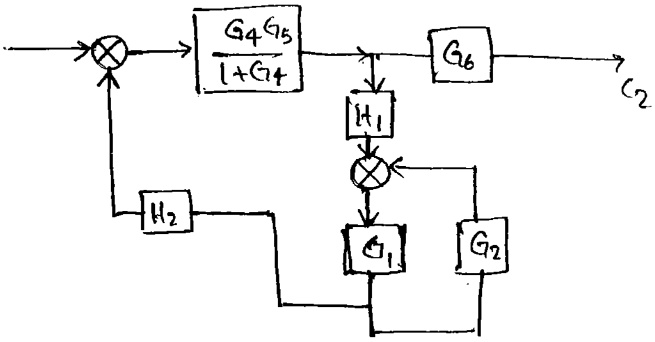
$$\frac{C_2(s)}{R_1(s)} = \frac{G_1 G_4 G_5 G_6 H_2}{H_1(1+G_1G_2) + H_1G_2G_4G_5} \quad \cancel{G_2 G_4}$$

$$\frac{G_1 G_4 G_5 G_6}{H_1G_2G_4G_5 + H_1(1+G_1G_2) + H_1G_1G_4G_5G_6H_2}$$

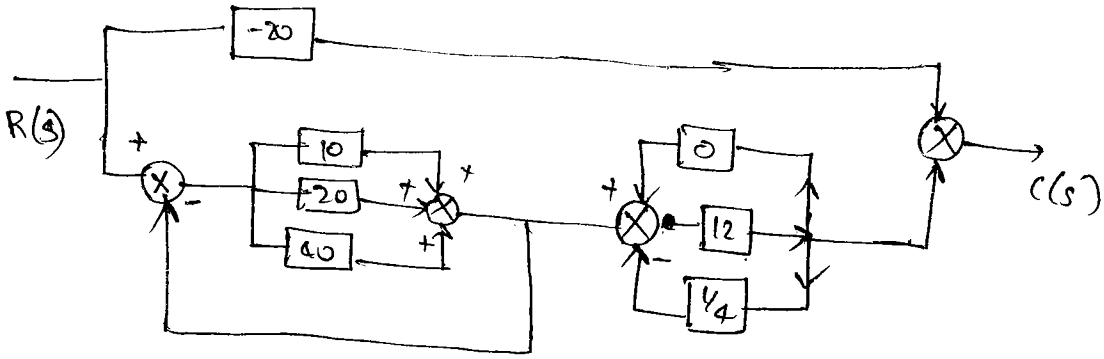
$$= \frac{G_1 G_4 G_5 G_6}{(1+G_1G_2)(1+G_4) + G\,G_4G_5H_2H_2}$$

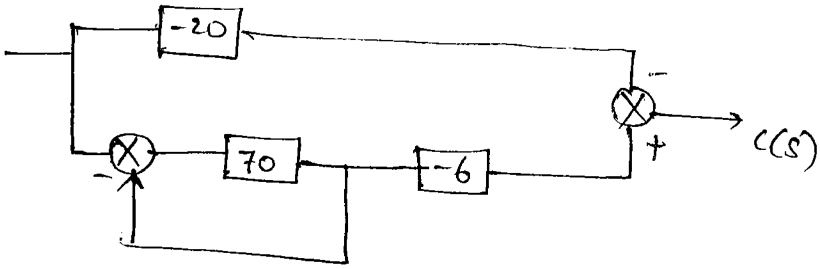
$\frac{C_2}{R_2}$





Q, Find the gain of the system below.





$\frac{12}{1+\cancel{4}\times 3}$ $\qquad \frac{70}{1+70}$ $\qquad \frac{12}{1+3} = \underline{\underline{3}}$

$= \frac{12}{-2} = \underline{\underline{-6}}$ $\qquad = \frac{70}{71} \times 6$ $\qquad \frac{70}{71} \times$

$-\frac{70}{71} \times 6 + 20 = -\frac{420}{71} + 20 = 23$

Q, The impulse response of a unity feedback system is

$c(t) = -t e^{-t} + 2e^{-t}$

The equivalent open loop transfer fn is

~~OLTF~~ $\quad \text{CLTF} = \frac{G}{1+G} = L\left[\text{Impulse response}\right]$

$\frac{G}{1+G}$ ~~OLTF~~ $= \frac{-1}{(s+1)^2} + \frac{2}{s+1}$

$\frac{G}{1+G} = \frac{-1+2(s+1)}{(s+1)^2}$

$\frac{G}{1+G} = \frac{2s+1}{s^2+2s+1}$ $\qquad G = \frac{\text{num}}{\text{deno} - \text{num}} = \underline{\underline{\frac{2s+1}{s^2}}}$

$$\frac{1+G}{G} = \frac{s^2+2s+1}{2s+1}$$

$$\frac{1}{G}+1 = \frac{s^2+2s+1}{2s+1}$$

$$\frac{1}{G} = \frac{s^2+\cancel{2s+1}-\cancel{(2s+1)}}{2s+1}$$

$$\frac{1}{G} = \frac{s^2}{2s+1}$$

Q, The impulse response of the system is $5e^{-5t}$. To produce a response of te^{-5t}, the input must be equal to

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{(s+5)^2}}{\frac{5}{(s+5)^2}}$$

~~Here not~~

Here not said anything about feedback

hence given OLTF

$$\frac{5}{(s+5)}$$

$$\text{Input} = \frac{1}{5(s+1)}$$

$$R(s) = \frac{1}{5(s+1)}$$

$$\frac{1}{(s+5)^2}$$

ILT,

$$\text{input} = 0.2\, e^{-t}$$

SIGNAL FLOW GRAPH

→ It is a graphical representation of set of linear algebraic equations, b/w input and output, that frames the system.

→ The set of linear algebraic equations represent the system.

→ The signal flow graph analysis is developed to avoid the mathematical calculations like solving the integro differential equation or linear algebraic equations.

The signal flow graph analysis is easier, as compared to mathamatical analysis.

CONSTRUCTION OF SIGNAL FLOW GRAPHS TO THE LINEAR ALGEBRAIC EQUATION

O/P node ← Q 1 → $y_2 = 10y_1$ → I/P node.

Q 2 → $y_4 = 10y_1 - 20y_2 + 30y_3$

Q 3 → $y_2 = 5y_1$

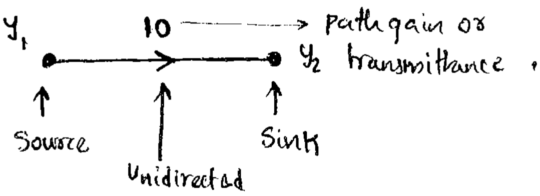
$y_3 = 10y_1$

$y_4 = 20y_1$

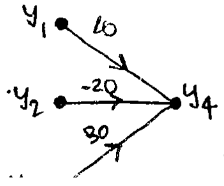
"The nodes in the signal flow graph are nothing but the variables that are currents and voltages." "The path gains are nothing but impedance or admittance of the system components"

→ nodes in the left ~~out~~ represent output node, nodes in right represent input node and the path gain is given by the constant.

Q 1

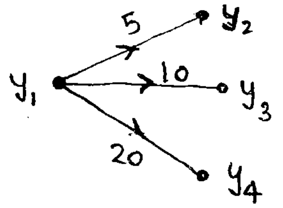


Q 2



Addition Rule (Many to One).

Q3



Transmission Rule (one to many).

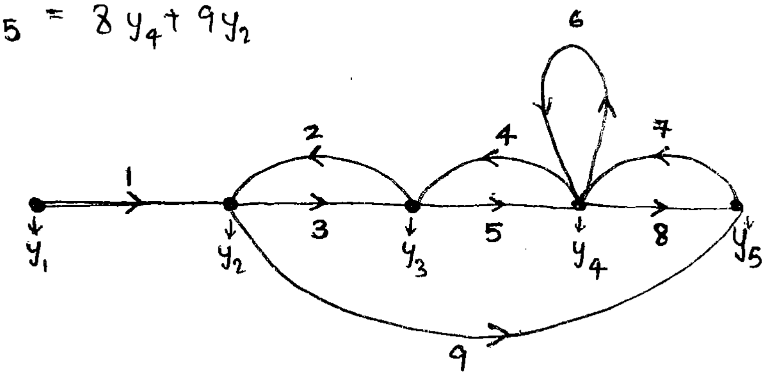
Q4 construct the signal flow graph to given set of linear algebraic eqns.

$$y_2 = y_1 + 2y_3$$

$$y_3 = 3y_2 + 4y_4$$

$$y_4 = 5y_3 + 6y_4 + 7y_5$$

$$y_5 = 8y_4 + 9y_2$$



Q Find the number of forward paths, individual loops, two non tuching loop to the above signal flow graph.

<u>FORWARD PATH</u> : It is a path from input to output.

<u>LOOP</u> : It is a path which terminates on the same node where it is started.

<u>NON TOUCHING LOOPS</u>: If there is no common node b/w two or more ~~loop~~ loops, then it is called non tuching loops.

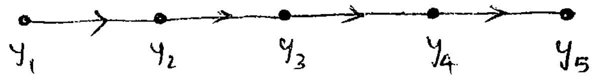
<u>INPUT NODE</u> : The node that have only outgoing branches.

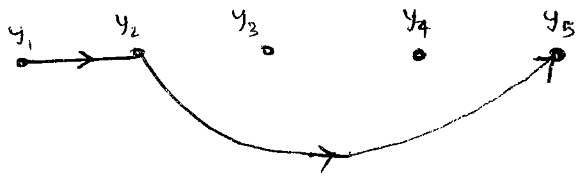
<u>OUTPUT NODE</u> : The node that have only incoming branches.

<u>CHAIN/LINK NODE</u> : The node which have both incoming and outgoing branche

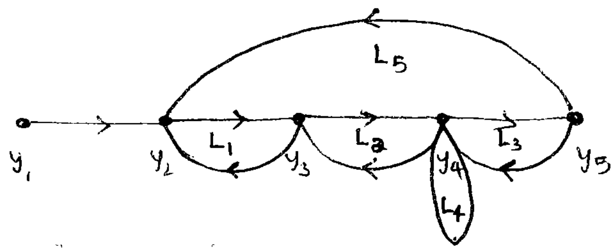
<u>Note</u>: while selecting a forward path or loop, each node should be tuched only once.

Forward paths





Individual loops



Non-Touching loops.

$L_1 \rightarrow L_2$ x, ~~L_3~~, ~~L_4~~, L_5 x

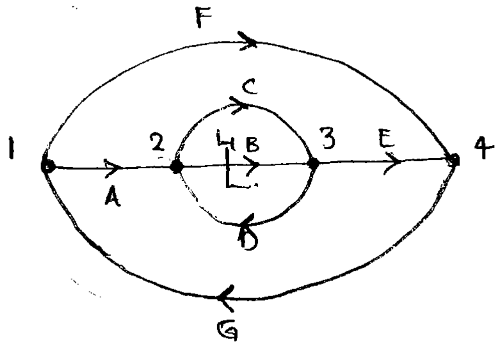
$L_2 \rightarrow L_3$ x, L_4 x, L_5 x

$L_3 \rightarrow L_4$ x, L_5 x

$L_4 L_5$ x

2 pairs, (i) L_1 and L_3 (ii) L_1 and L_4.

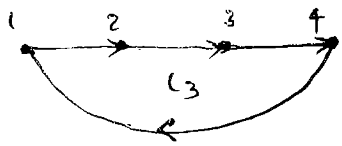
Q,

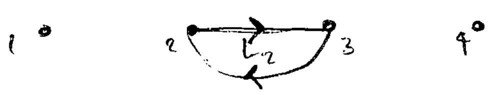


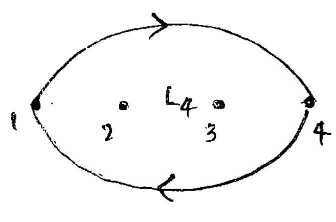
3 → Forward paths.

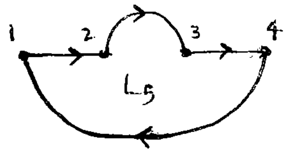
5 → Individual loops.

Loops:









$L_1 \rightarrow CD \longrightarrow 2,3$

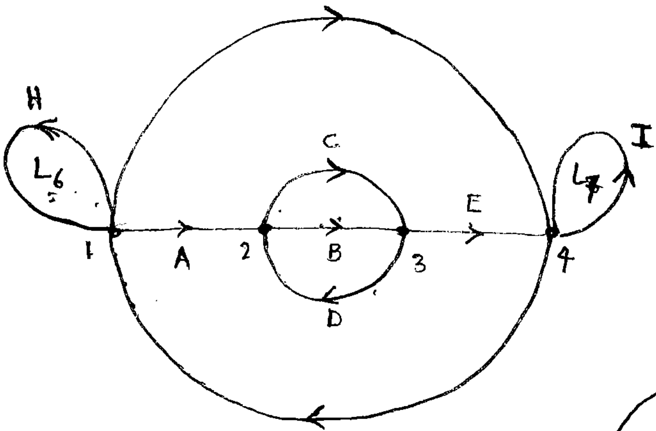
$L_2 \rightarrow BD \longrightarrow 2,3$

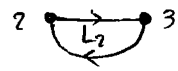
$L_3 \rightarrow ABEG \rightarrow 1,2,3,4$

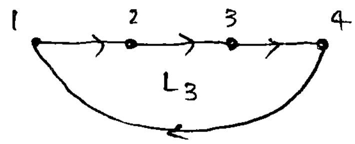
$L_4 \rightarrow ACEG \rightarrow 1,2,3,4$

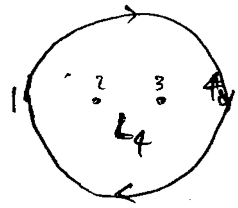
$L_5 \rightarrow FG \rightarrow 1,4$

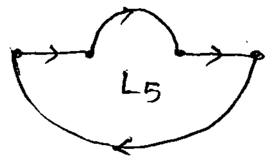
Find the ~~non~~ forward paths, no: of individual loops, 2 non-touching loops, 3 nontouching loops.











Forward Paths 3

~~Individual loops~~

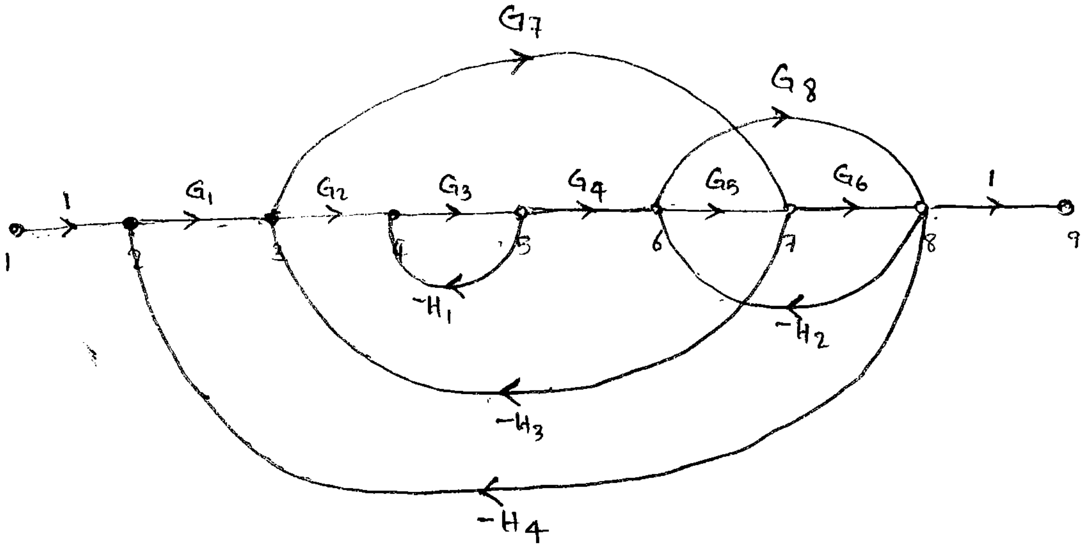
~~L_1 — (2,3)~~
~~L_2 — (2,3)~~
~~L_3 — (1,2,3,4)~~
~~L_4 — (1,2,3,4)~~
~~L_5 —~~
~~L_6 —~~
~~L_7 —~~

Individual loops — 7

2 nontouching loops — ~~4~~ 7 11

3 nontouching loops. — 2

Q Repeat the above problem



Forward path 3

(i) → L_1 (4, 5) → $-G_3H_1$ L_4 (3, 4, 5, 6, 7)

→ L_2 (6, 7, 8) → $-G_5G_6H_2$

→ L_3 (2, 3, 4, 5, 6, 7, 8) →

(ii) → L_5 (3, 7)

→ L_6 (2, 3, 7, 8)

(iii) → L_7 (6, 8)

L_8 (2, 3, 4, 5, 6, 8)

L_1: L_2 ✓, L_3 ×, L_4 ×, L_5 ✓, L_6 ✓, L_7 ✓, L_8 ×

L_2: L_3 ×, L_4 ×, L_5 ×, L_6 ×, L_7 ×, L_8 ×

L_3: L_4 ×, L_5 ×, L_6 ×, L_7 ×, L_8 ×

L_4: L_5 ×, L_6 ×, L_7 ×, L_8 ×

L_5: L_6 ✓, L_7 ✓, L_8 ×

L_6: L_7 ×, L_8 ×

L_7: L_8 ×

5 non touching loops,

3 non touching loops. ... loop.

L_1L_2 — $(4,5,6,7,8) \longrightarrow \times$

L_1L_5 — $(3,4,5,7) \longrightarrow L_7$

L_1L_6 — $(2,3,4,5,7,8) \longrightarrow \times$

L_1L_7 — $(4,5,6,8) \longrightarrow L_5$

L_5L_7 — $(3,6,7,8) \longrightarrow L_1$

$\underline{L_1L_5L_7}$ is a 3 non touching loop.

MASON'S GAIN FORMULA

PURPOSE: 1. To find the overall transfer function.

2. To find the ratio of any two nodes.

The overall transfer function equal to

$$\boxed{T.F = \sum_{K=1}^{i} \left(\frac{P_K \Delta_K}{\Delta}\right)}$$

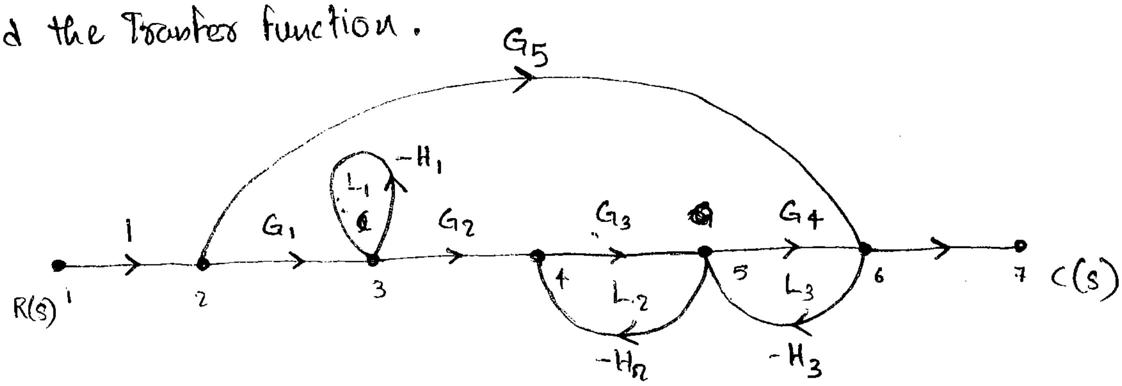
where $\boxed{P_K \rightarrow K^{th} \text{ forward path Gain.}}$

$$\Delta = 1 - \sum(\text{Individual Loop Gain}) + \sum(\text{Gain products of 2 non touching loops}) - \sum(\text{Gain products of 3 non touching loops}) + \ldots 4 \ldots - \ldots$$

Δ = 1 − sum of individual loop gains + sum of gain products of 2 non touching loops − sum of ~~individual~~ gain product of 3 non touching loops + ...

Δ_K is ~~the~~ obtained from Δ by removing the loops touching the K^{th} forward path.

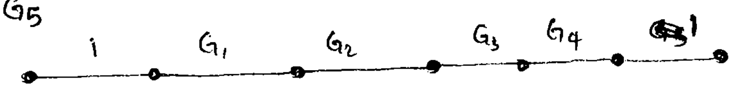
Q Find the Transfer function.



Forward paths

$P_1 = G_1G_2G_3G_4$

$P_2 = G_5$



Individual loops

$L_1 = -H_2$

$L_2 = -G_3H_2$

$L_3 = -G_4H_3$

2 - Non touching loops

$L_1L_2 = G_3H_1H_2$

$L_1L_3 = G_4H_1H_3$

$$\Delta = 1 - (L_1+L_2+L_3) + (L_1L_2+L_1L_3)$$

$\Delta_1 = \Delta$(remove all loops touching 1st forward path)

Here all loops touches 1st forward path.

$\therefore \Delta_1 = 1 - (0) + (0) - -$

$\Delta_1 = 1$

$\Delta_2 = \Delta$ (remove all loops touching 2nd forward path)

Here L_3 touches 2nd forward path.

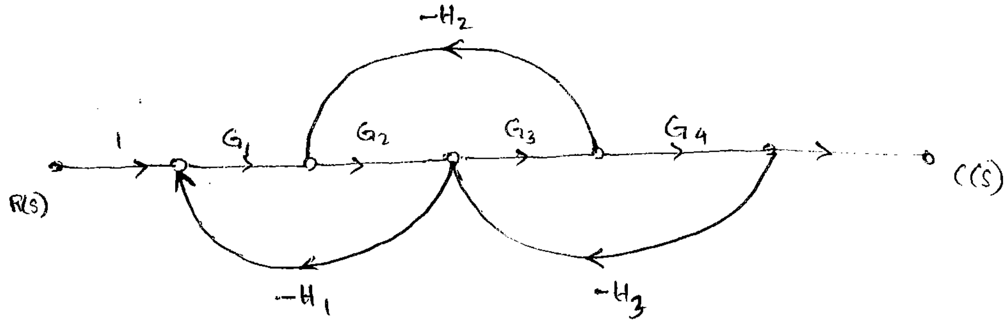
$\Delta_2 = 1 - (L_1+L_2) + (L_1L_2)$

$$T.F = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4 + G_5(1+H_1+G_3H_2)+(G_3H_1H_2)}{1+H_1+G_3H_2+G_4H_3+(H_1G_3H_2)+(G_4H_1H_3)}$$

~~select the first se~~

Note : In Δ or Δ_K, take the opposite sign for odd number of non touching loops and take the same sign for even number of non-touching loops.

Q Find the transfer function

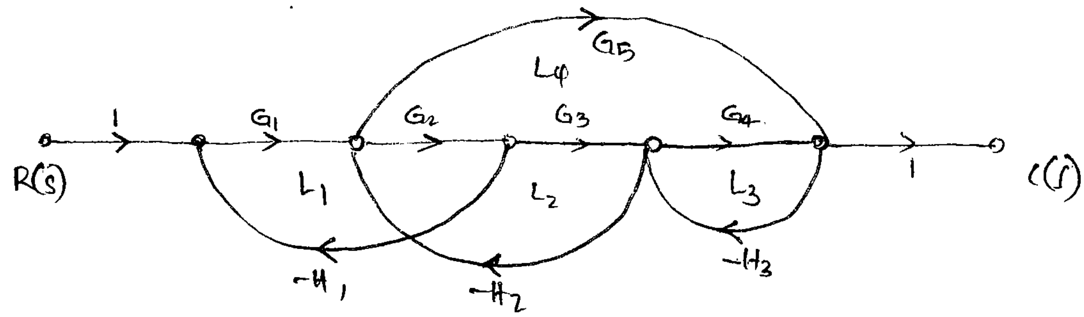


1 forward path.

$$P_1 = G_1G_2G_3G_4$$

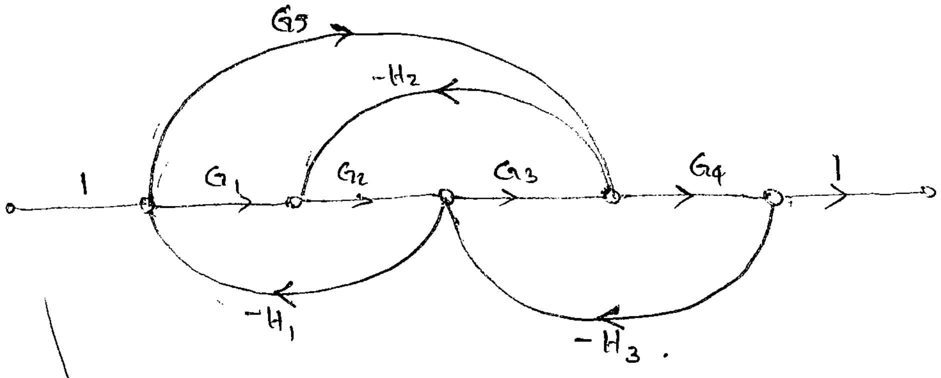
$$T \cdot F = \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_3G_4H_3}$$

Q



$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4 + G_1G_5}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_3 \cancel{G} + G_2G_3G_4H_2H_3 + \underbrace{G_1H_1G_4H_3}_{L_1L_3}}$$

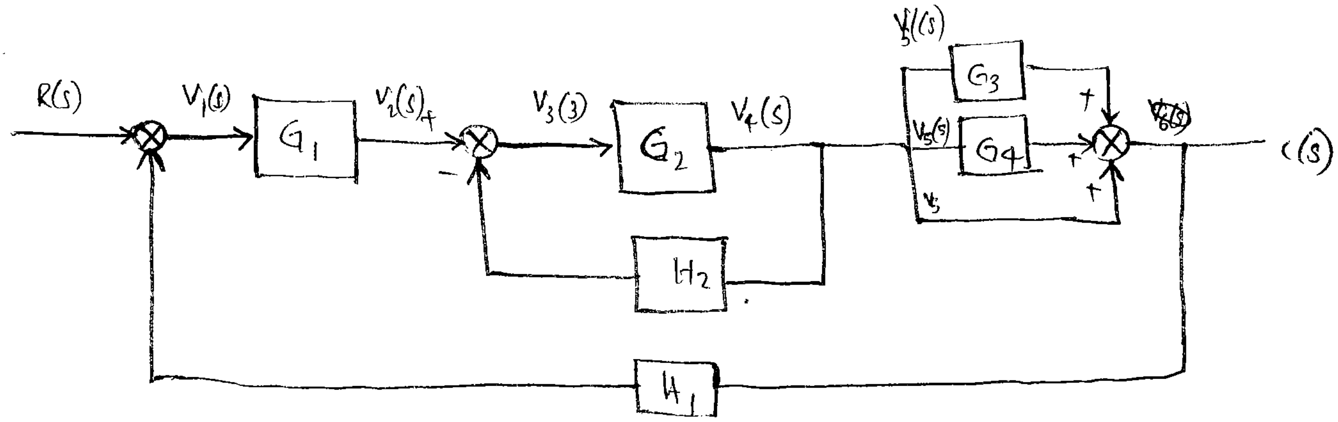
Q,



$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4 + \cancel{G}\cdot G_5G_4}{1 + G_1G_2H_1 + G_3G_4H_3 + G_2G_3H_2 \cancel{+ G_1H_4H_2H_3G_4} - G_5G_4H_1H_3 - G_5H_2G_2H_1}$$

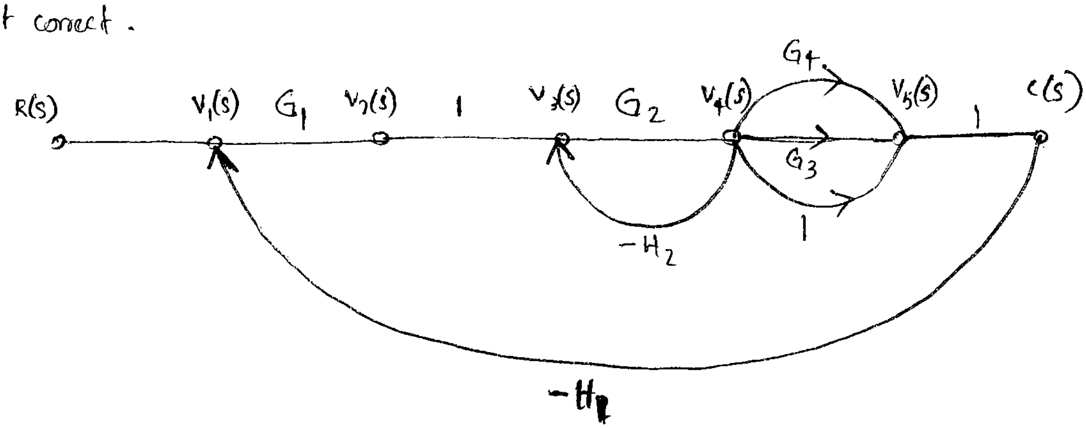
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4 + G_4G_5}{1 + G_1G_2H_1 + G_3G_4H_3 + G_2G_3H_2 - G_5G_4H_1H_3 - G_5G_2H_1H_2}$$

Q, Find the transfer function to the given ~~sto~~ block diagram using Mason's gain formula.

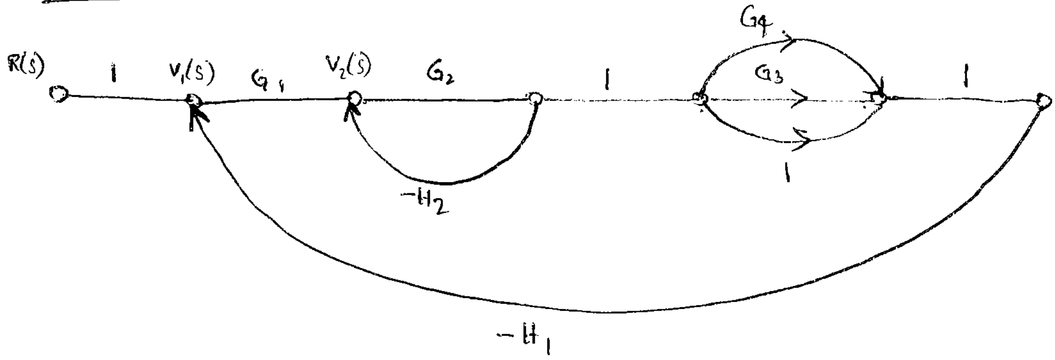


$$\frac{\cancel{C(s)}}{\cancel{R(s)}} = G_1 \cancel{G_2}$$

X Not correct.

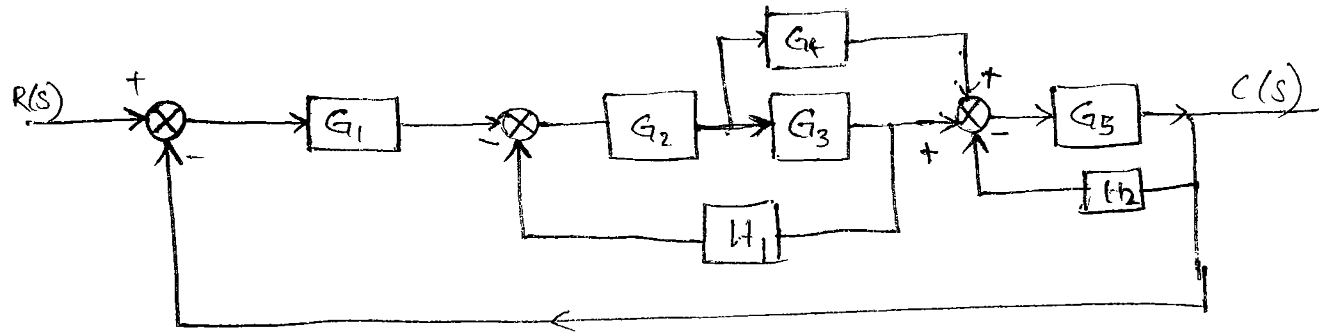


Signal flow graph



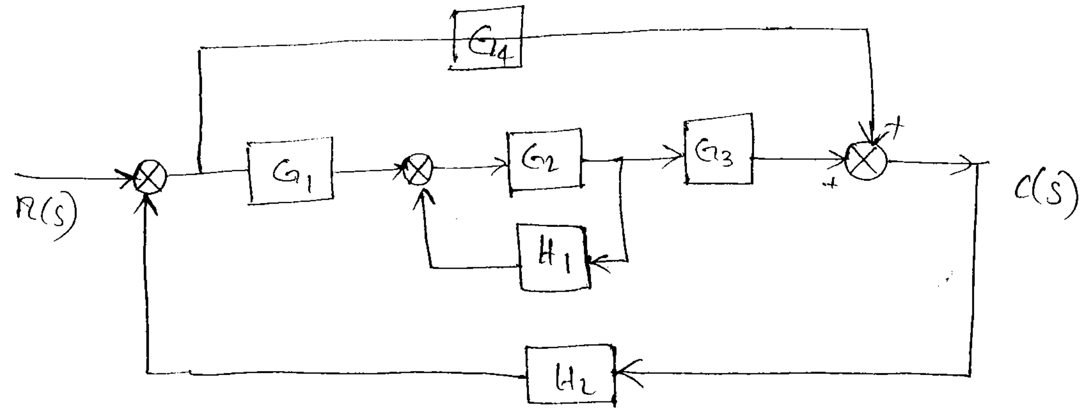
$$\frac{C(s)}{R(s)} = \frac{G_1G_2 + G_1G_2G_3 + G_1G_2G_4}{1 + G_2H_2 + G_1G_2(1+G_3+G_4)H_1}$$

Q



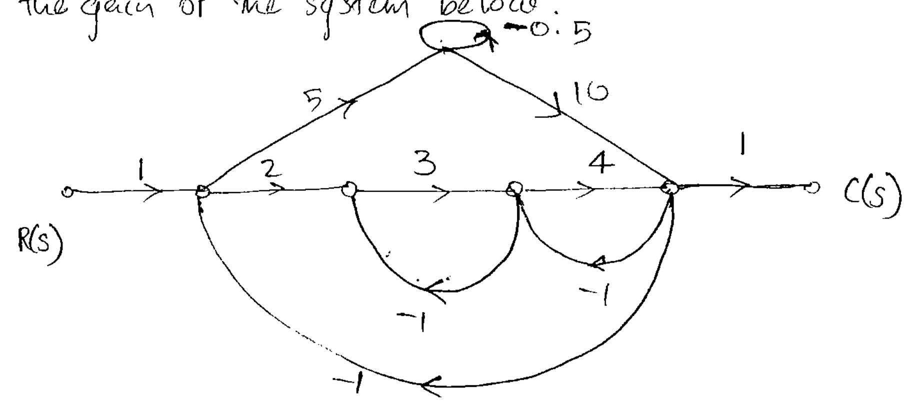
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_5 + G_1G_2G_4G_5}{1 + G_2G_3H_1 + G_5H_2 + G_1G_2G_3G_5 + G_1G_2G_4G_5 + G_2G_3G_5H_1H_2}$$

Q



$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_4(1+G_2H_1)}{1+G_2H_1+G_1G_2G_3H_2+G_4H_2+G_2G_4H_1H_2}$$

Q. Find the gain of the system below.



$$\frac{C(s)}{R(s)} = \frac{24(1+0.5)+50(1+3)}{1+3+4+24+0.5+50+1.5+2+12+150}$$

$$= \frac{24\times1.5+50\times4}{32+50.5+1.5+14+150}$$

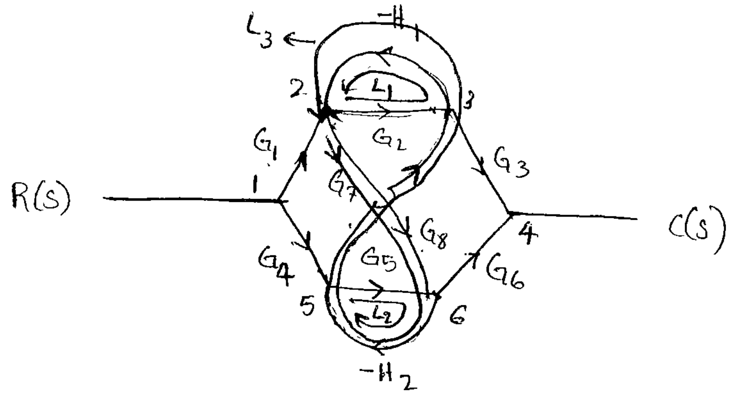
$$= \frac{236}{98+150}$$

0.95

$$\cancel{=2408} \quad = 0.9516$$

=

Q



$P_1 = 1, 2, 3, 4 \rightarrow G_1G_2G_3$

$P_2 = 1, 5, 6, 4 \rightarrow G_4G_5G_6$

$P_3 = 1, 2, 6, 4 \rightarrow G_1G_8G_6$

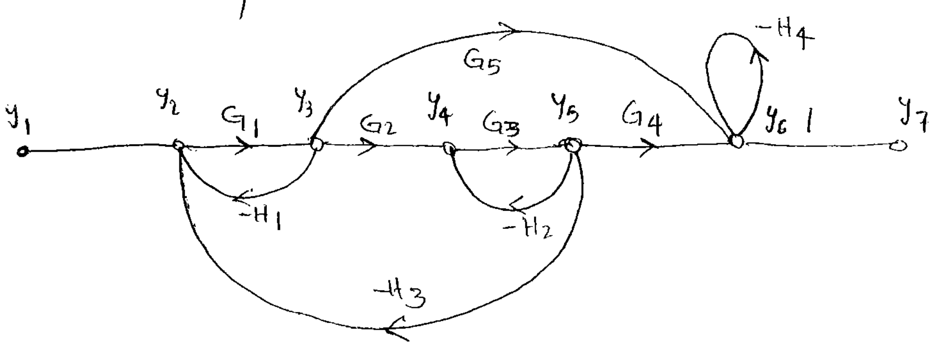
$P_4 = 1, 5, 3, 4 \rightarrow$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3(1+G_5H_2) + G_4G_5G_6(1+G_2H_1) + G_1G_8G_6 + G_4G_7G_3 - G_4G_7H_1G_8G_6 - G_1G_8H_2G_7G_3}{1+G_2H_1+G_5H_2+G_2G_5H_2H_1+G_7G_8H_1H_2}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3(1+G_5H_2) + G_4G_5G_6(1+G_2H_1) + G_1G_8G_6 + G_4G_7G_3 - G_4G_7H_1G_8G_6 - G_1G_8H_2G_7G_3}{1+G_2H_1+G_5H_2+G_2G_5H_2H_1+G_7G_8H_1H_2}$$

Q Find $\frac{Y_6}{Y_1}$, $\frac{Y_7}{Y_1}$, $\frac{Y_5}{Y_1}$, $\frac{Y_7}{Y_2}$, $\frac{Y_2}{Y_1}$, $\frac{Y_5}{Y_3}$, $\frac{Y_5}{Y_4}$, etc. . . .

Ratio ~~__~~ of any two nodes.



$$\frac{Y_6}{Y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{1+G_1H_1+G_3H_2+G_1G_2G_3H_3+H_4+G_1H_1H_4+G_3H_2H_4+G_1G_2G_3H_3H_4 + G_1G_3H_1H_2H_4 + G_1H_1G_3H_2}$$

$$\rightarrow \frac{Y_7}{Y_1} = \frac{Y_6}{Y_1}$$

$$\frac{Y_5}{Y_1} = \frac{G_1G_2G_3(1+H_4)}{}$$

$$\Delta = 1 + G_1H_1 + G_2H_2 + G_1G_2G_3H_3 + H_4 + G_1H_1H_4 + G_1H_2H_4 + G_1G_2G_3H_3H_4 + G_1G_3H_1H_2 + G_1G_3H_1H_2H_4$$

$$\rightarrow \cancel{\frac{Y_7}{Y_2} = \frac{Y_7}{Y_1}}$$

$$\rightarrow \cancel{\frac{Y_5}{Y_3} = \frac{G_2G_3(1+H_4)}{}}$$

$$\cancel{\Delta = 1 + G_1H_1 + G_2H_2 + G_1G_2G_3H_3 + H_4 + G_1H_1H_4 + G_1H_2H_4 + G_1G_2G_3H_4 + G_1G_3H_1H_2 + G_1G_3H_1H_2H_4}$$

$$\rightarrow \frac{Y_2}{Y_1} = \cancel{G_1G_2G_3}\ \frac{1(1+G_3H_2+H_4+G_3H_2H_4)}{\Delta}$$

$$\boxed{Y_2 \neq Y_1}$$

$$\rightarrow \frac{Y_7}{Y_2} = \frac{Y_7/Y_1}{Y_2/Y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{G_2G_3(1+H_4)}$$

whenever if ratio of intermediaded nodes are asked, ~~find~~ let $\frac{Y_n}{Y_m}$ is asked, find $\frac{Y_n}{Y_{input}}$ and $\frac{Y_m}{Y_{input}}$ and find their ratio. And also while finding $\frac{Y_n}{Y_{input}}$ & $\frac{Y_m}{Y_{input}}$ no need to write denominator Δ, they will cancel.

$$\cancel{\frac{Y_5}{Y_1} = G_1G_2G_3}$$

$$\frac{Y_3}{Y_1} = \frac{G_1(1+G_3H_2+H_4+G_3H_2H_4)}{\Delta}$$

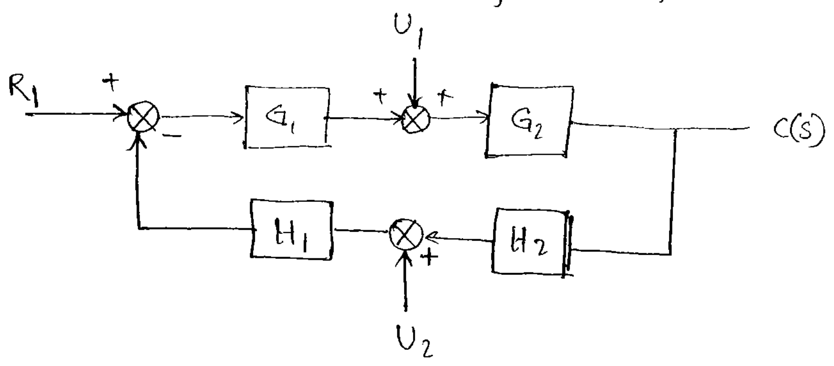
$$\frac{Y_5}{Y_3} = \frac{Y_5/Y_1}{Y_3/Y_1} = \frac{\dfrac{G_1G_2G_3(1+H_4)}{\cancel{\Delta}}}{\dfrac{G_1(1+G_3H_2+H_4+G_3H_2H_4)}{\cancel{\Delta}}}$$

$$= \frac{G_1G_2G_3(1+H_4)}{G_1(1+G_3H_2+H_4+G_3H_2H_4)}$$

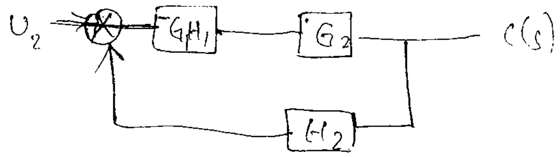
$$\frac{Y_4}{Y_1} = \frac{G_1G_2(1+H_4)}{\Delta}$$

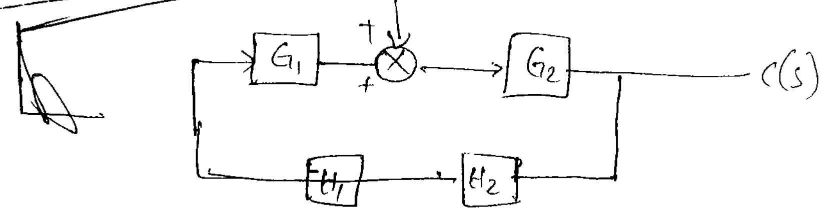
$$\cancel{\frac{Y_4}{Y_3}} \quad \frac{Y_5}{Y_4} = G_3$$

Q Find the output of the following block diagram having 3 inputs



$$\frac{C(s)}{R_1(s)} = \frac{G_1G_2}{1+H_1H_2G_1G_2}$$



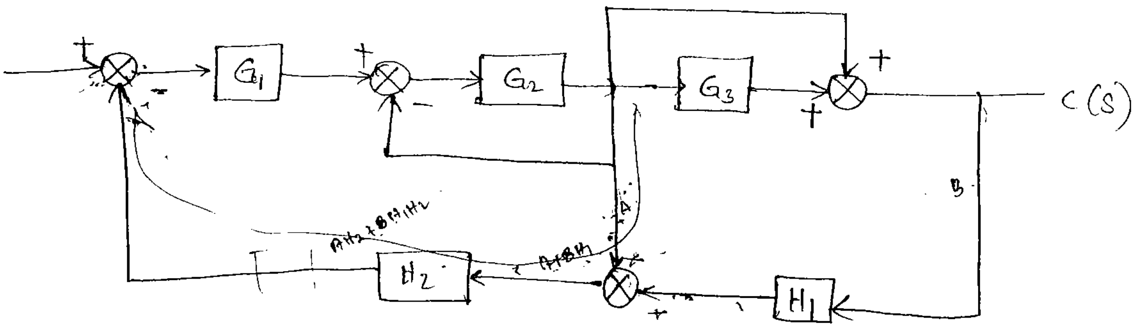


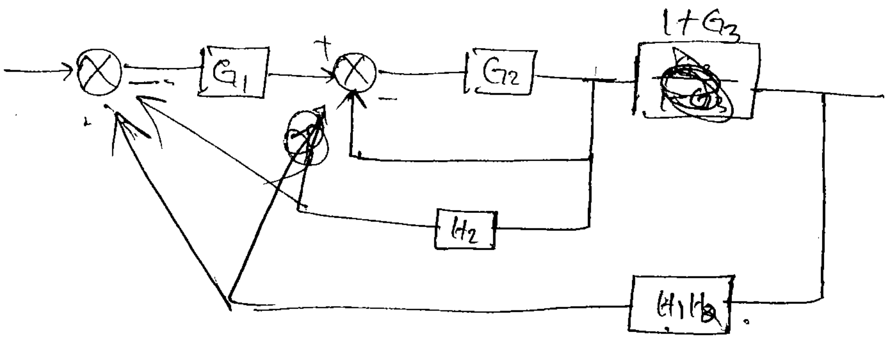
$$\frac{C(s)}{U_2(s)} = \frac{-G_1H_1G_2}{1+H_2H_1G_1G_2}$$

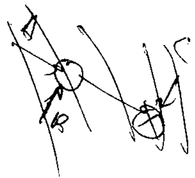
$$\frac{C(s)}{U_1(s)} = \frac{G_2}{1+G_1G_2H_2H_1}$$

Total output $= \dfrac{G_1G_2R_1 + G_2U_1 - G_1G_2H_1U_2}{1+G_1G_2H_1H_2}$.

Q. Obtain the transfer function by using ~~block~~ block diagram reduction technique for multi loop control s/m, shown in figure.







$$\cancel{\frac{G_2G_3}{(1+G_2)(1-G_3)}} \qquad \frac{G_1G_2}{1+G_2} \qquad = \frac{G_1G_2G_3}{(1-G_3)(1+G_2+H_2G_1G_2)}$$

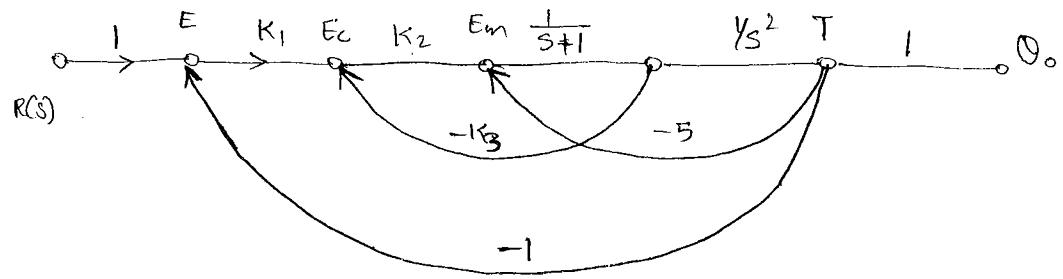
$$= \frac{G_1G_2(G_3+1)}{(1+G_2+H_2G_1G_2) + H_1H_2G_1G_2(G_3+1)}$$

$$= \frac{\cancel{G_1G_2G_3}}{\cancel{1+G_2+H_2G_1G_2 - G_3 - G_2G_3 - G_1G_2G_3H_2 + H_1H_2G_1G_2G_3}}$$

$$= \frac{G_1G_2(G_3+1)}{1+G_2+H_2G_1G_2+H_1H_2G_1G_2(G_3+1)}$$

Q, a) A system is represented by a signal flow graph, as shown in figure. The variable T is the torque. E is the error, determine the overall transfer fn. if $K_1 = 5$, $K_2 = 1$, $K_3 = 5$

b) The sensitivity of the system to change in K_2 at $\omega = 0$.



$$\frac{Q(s)}{R(s)} = \frac{T(s)}{R(s)} = \frac{K_1 K_2 \times \frac{1}{s+1} \times \frac{1}{s^2}}{1 + \frac{5}{s^2(s+1)} + \frac{K_2 K_3}{s+1} + \frac{K_1 K_2}{s^2(s+1)}}$$

$$= \frac{\frac{K_1 K_2}{\cancel{s^2(s+1)}}}{s^2(s+1) + 5 + s^2 K_2 K_3 + K_1 K_2} = \frac{K_1 K_2}{s^3 + s^2(1 + K_2 K_3) + (K_1 K_2 +}$$

$$= \frac{5}{s^2(s+1) + 5 + 5s^2 + 5}$$

$$\frac{Q_o(s)}{R(s)} = \frac{T(s)}{R(s)} = \frac{5}{s^3 + s^2 + 10 + 5s^2} = \frac{5}{s^3 + 6s^2 + 10}$$

b) Sensitivity of the transfer fn w.r.t K_2

$$S_{K_2}^{T} = \frac{(\partial T / T)}{(\partial K_2 / K_2)} = \left(\frac{K_2}{T}\right)\left(\frac{\partial T}{\partial K_2}\right) =$$

$$T\Big|_{\omega = 0} = \frac{K_1 K_2}{K_1 K_2 + 5}$$

$$\frac{\partial T}{\partial K_2} = \frac{K_1(K_1K_2+5) - K_1K_2K_1}{(K_1K_2+5)^2} = \frac{5K_1}{(K_1K_2+5)^2}$$

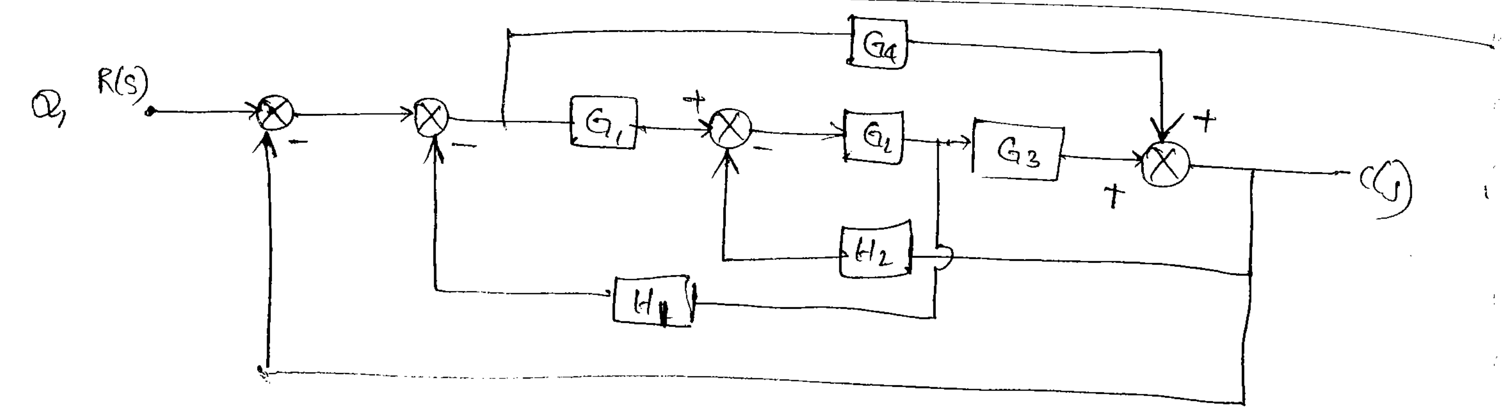
$$\therefore S^T_{K_2} = \frac{\partial T/T}{\partial K_2/K_2} = \frac{K_2}{T}\frac{\partial T}{\partial K_2} = \frac{\cancel{K_2}}{\frac{\cancel{K_1K_2}}{\cancel{K_1K_2+5}}} \times \frac{5\cancel{K_1}}{(K_1K_2+5)^{\cancel{2}}}$$

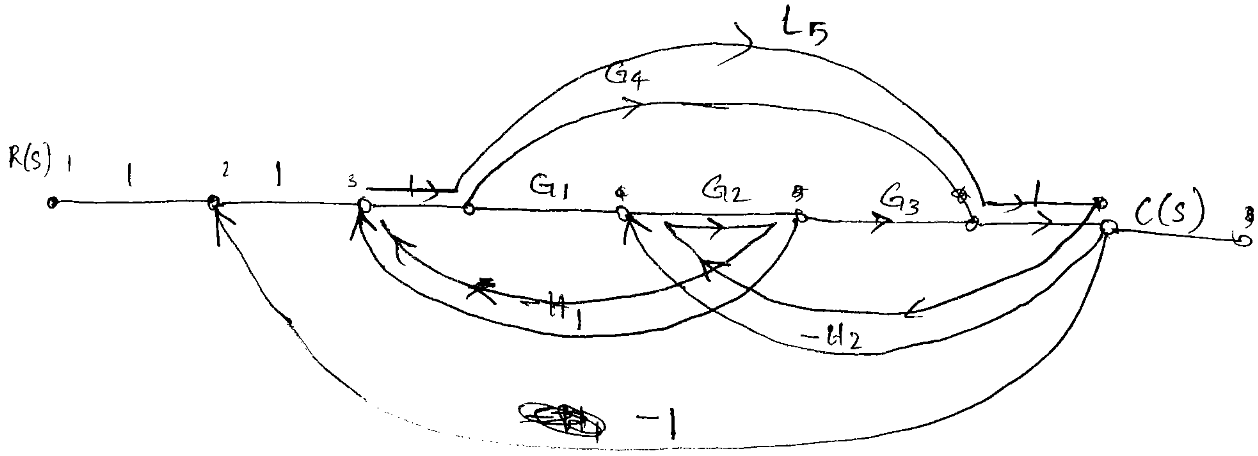
$$= \frac{5}{K_1K_2+5} = \frac{5}{5+5} = 0.5$$

Q. What is the effect of feedback on overall gain of the control s/m. Also explain the effect of f/b on gain sensitivity.

Q. Write the limitations of Mason's gain formula.

Q.



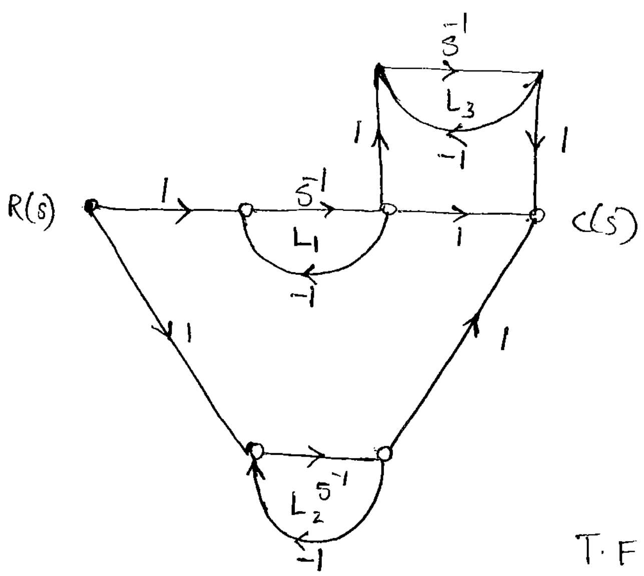


$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_4}{1 + G_2G_3H_2 + G_1G_2H_1 + G_1G_2G_3 + G_4 - G_4G_2H_2H_1}$$

Limitations of Mason's Gain Formula.

1. It is applicable to only LTI systems.
2. Mason's Gain ~~can to~~ Formula can only applicable to input and output.
3. It is complicated to impliment without making mistake, if signal flow graph consist many loops and nontouching loops.

Q. Give the basic properties of Signal flow graph and find the transfer function.



$P_1 \rightarrow s^{-1}$

$P_2 \rightarrow s^{-2}$

$P_3 \rightarrow s^{-1}$

$$T.F = \frac{s^{-1}\left(1+s^{-1}+s^{-1} + s^{-2}\right) + s^{-2}\left(1+s^{-1}\right) + s^{-1}\left(1+s^{-1}+s^{-1}+s^{-2}\right.}{1+s^{-1}+s^{-1}+s^{-1}+s^{-2}+s^{-2}+s^{-2}+s^{-3}}$$

~~$T.F = s^{-1}(1+s^{-1}+s^{-1}s^{-1}) + s^{-2}(1+s^{-1}) + s^{-1}(1+s$~~

$$T.F = \frac{\frac{1}{s}\left(1+\frac{2}{s}+\frac{1}{s^2}\right) + \frac{1}{s^2}\left(1+\frac{1}{s}\right) + \frac{1}{s}\left(1+\frac{2}{s}+\frac{1}{s^2}\right)}{1+\frac{3}{s}+\frac{3}{s^2}+\frac{1}{s^3}}$$

$$= \frac{2s^2\left(1+\frac{2}{s}+\frac{1}{s^2}\right) + s\left(1+\frac{1}{s}\right)}{s^3+3s^2+3s+1}$$

$$= \frac{2s^2+4s+2+s+1}{s^3+3s^2+3s+1} \quad // \quad = \frac{2s^2+5s+3}{s^3+3s^2+3s+1} \quad //$$

$\cancel{(s+2)(s+1)+(s+1)^2}$

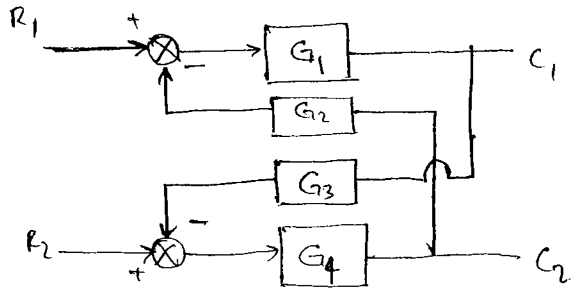
$\cancel{\frac{s+2+s+1}{(s+1)^2}} = \cancel{\frac{2s+3}{s^2+2s+1}}$ $\cancel{\frac{2s+3}{s^2+2s+1}}$

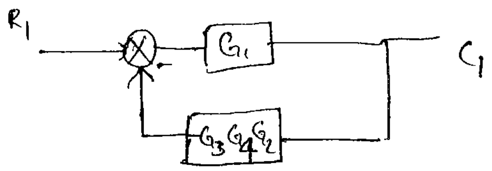
$\frac{1}{s+1}$

Properties of Signal Flow graph

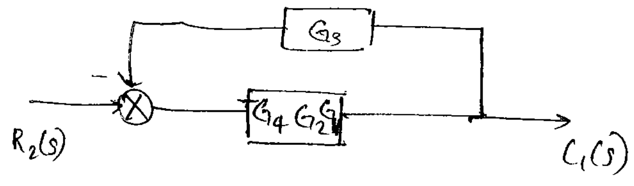
1. It is applicable to linear systems only.
2. The arrow indicates the flow of signals.
3. The signal flow graphs are unidirectional.
4. The algebraic sum of the signal enters into a particular node gives the value of a variable at that node.
5. A signal flow graph can be represented to a block diagram, not ~~not~~ reverse.
6. The nodes in a signal flow graph are variables, which are arranged from input to output (from left to right).
7. Using the mason's Gain formula, the overall Transfer function of the SFG can be obtained.

Q. Find the output C_1 and C_2 of the system given in the figure.



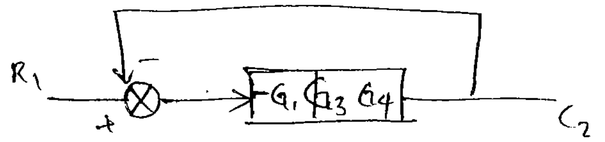


$$\frac{C_1}{R_1} = \frac{G_1}{1 - G_2G_3G_4G_1}$$



$$\frac{C_1(s)}{R_2(s)} = \frac{-G_4G_2G_1}{1-G_1G_2G_4G_3}$$

$$C_1(s) = \frac{G_1R_1 - G_4G_2G_1R_2}{1-G_1G_2G_3G_4}$$

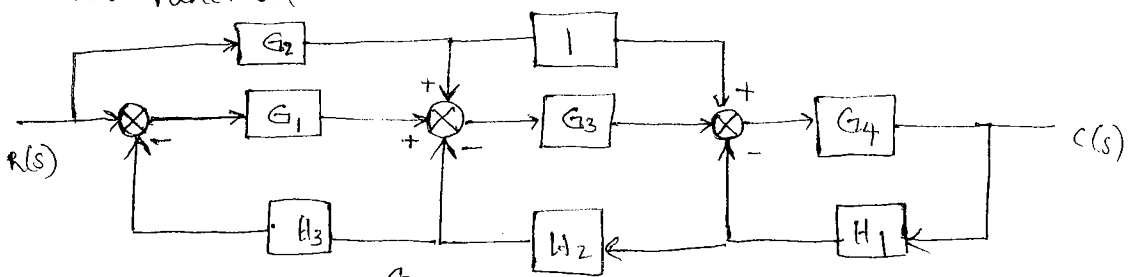


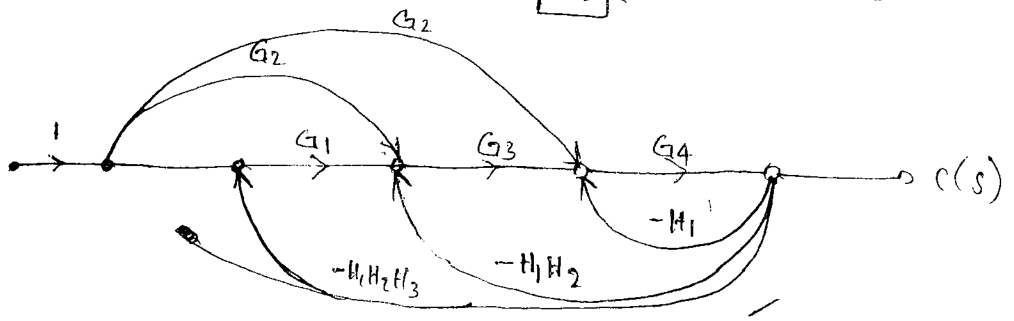
$$\frac{C_2}{R_1} = \frac{-G_1G_3G_4}{1-G_1G_2G_4G_3}$$

$$\frac{C_2}{R_2} = \frac{G_4}{1-G_1G_2G_3G_4}$$

$$C_2 = \frac{G_4R_2 + G_1G_3G_4R_1}{1-G_1G_2G_3G_4}$$

Q Figure is a block diagram of a linear control system. obtain a signal flow graph for the system, and hence calculate the transfer function.



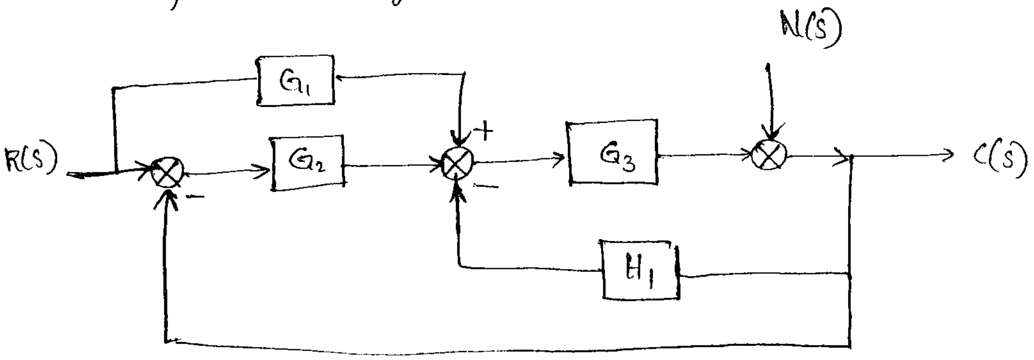


$$\frac{C(s)}{R(s)} = \frac{G_1G_3G_4 + G_2G_3G_4 + G_2G_4}{1+G_4H_1 + G_3G_4H_1H_2 + G_1G_3G_4H_1H_2H_3}$$

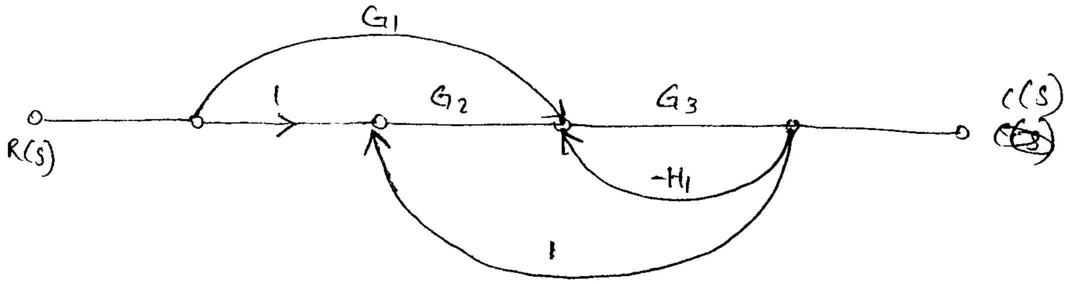
Q. i) obtain the transfer function $\frac{C(s)}{R(s)}$ by making $N(s) = 0$

ii) $\frac{C(s)}{N(s)}$ by making $R(s) = 0$

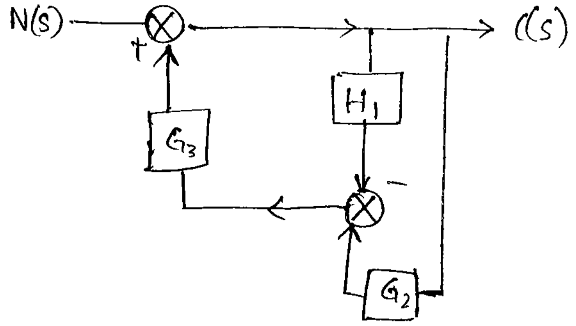
Using mason's gain formula.

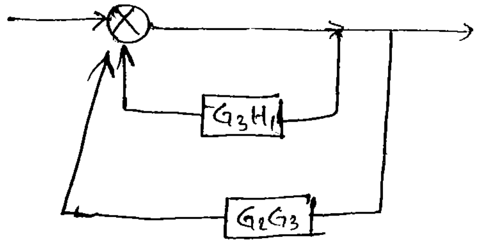


$$\frac{C(s)}{R(s)} = \frac{G_1G_3 + G_2G_3}{1+ G_3H_1 + G_2G_3}$$



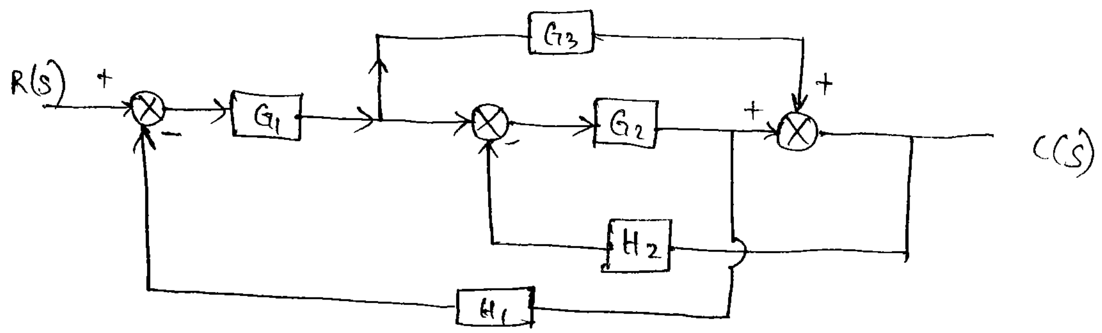
$$\frac{C(s)}{R(s)} = \frac{G_2G_3 + G_1G_3}{1+G_3H_1 + G_2G_3}$$

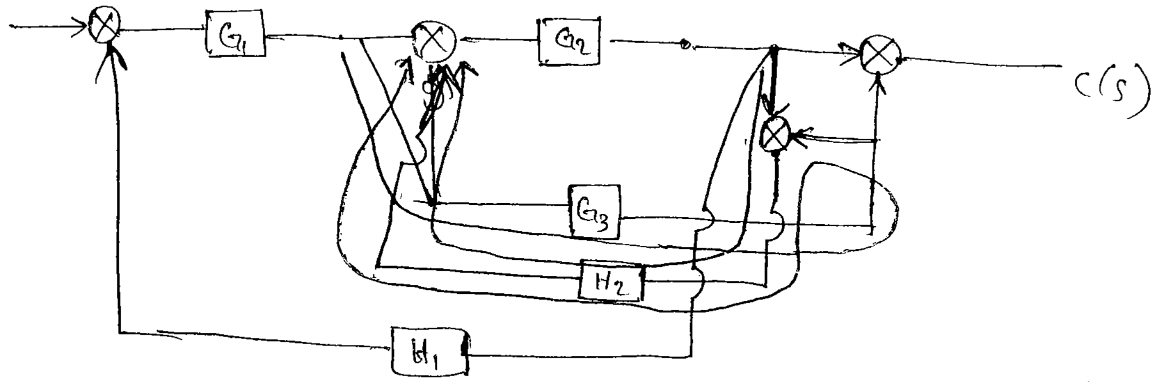


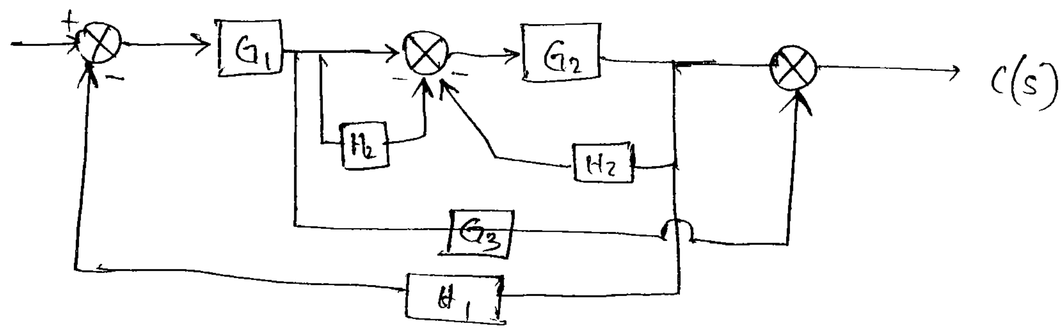


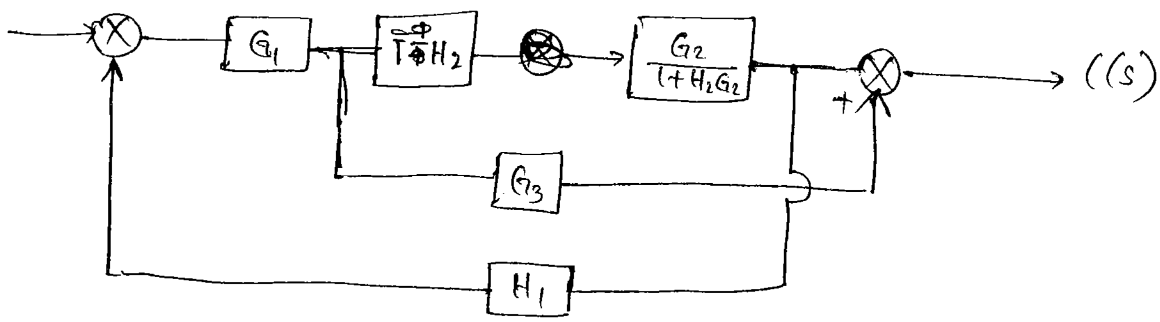
$$= \frac{1}{1+G_3H_1+G_2G_3}$$

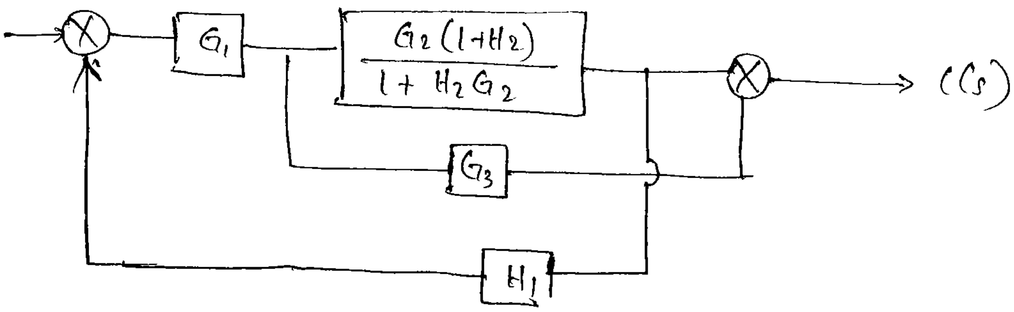
Q, Find the transfer function using block diagram reduction formula.

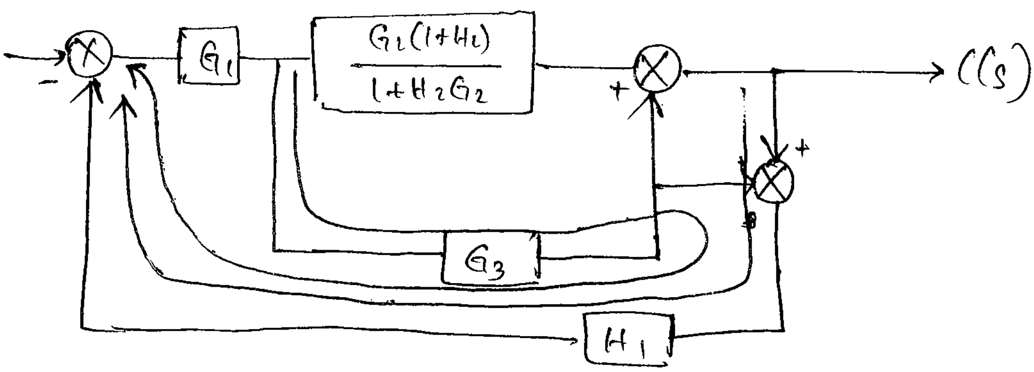


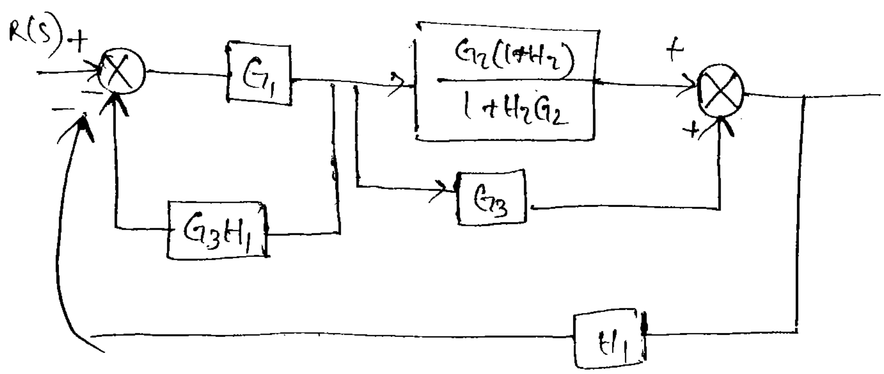


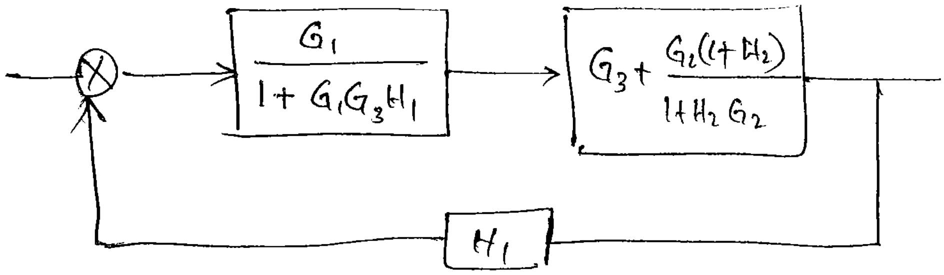

+
−
G_1
G_2
H_2
H_2
G_3
H_1
$C(s)$


G_1
$1+H_2$
$\frac{G_2}{1+H_2G_2}$
+
G_3
H_1
$C(s)$


G_1
$\frac{G_2(1+H_2)}{1+H_2G_2}$
G_3
H_1
$C(s)$


−
G_1
$\frac{G_2(1+H_2)}{1+H_2G_2}$
+
+
G_3
H_1
$C(s)$


$R(s)$ +
−
G_1
$\frac{G_2(1+H_2)}{1+H_2G_2}$
+
+
G_3
G_3H_1
H_1



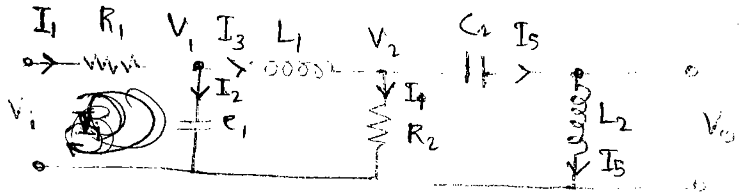
$$\frac{G_1\left[G_3(1+H_2G_2) + G_2(1+H_2)\right]}{(1+G_1G_3H_1)(1+H_2G_2)}$$

$$\frac{C(s)}{R(s)} = \frac{G_1\left[G_3(1+H_2G_2) + G_2(1+H_2)\right]}{(1+G_1G_3H_1)(1+H_2G_2) + H_1G_1\left[G_3(1+H_2G_2) + G_2(1+H_2)\right]}$$

$$= \frac{G_1G_3 + \cancel{G_1G_2G_3H_2} + G_1G_2 - \cancel{G_1G_2H_2G_3}}{1 + \cancel{G_1G_3H_1} + H_2G_2 + \cancel{G_1G_2G_3H_1H_2} + \cancel{H_1G_1G_3} + \cancel{G_1G_2G_3H_1H_2} + H_1G_1G_2 - H_1G_1G_2H_2}$$

$$= \frac{G_1(G_2+G_3)}{1 + G_2H_2 + G_1G_2H_1 - G_1G_2G_3H_1H_2}$$

CONSTRUCTION OF SIGNAL FLOW GRAPH TO ELECTRICAL NETWORKS



Step 1 : select the branch currents and node voltages.

$I_1, I_2, I_3, I_4, I_5 \rightarrow$ selecting current.

$V_1, V_2 \rightarrow$ selecting voltages.

Step 2 : Apply laplace transform to n/w variables and elements.

$I_1(s)\ I_2(s)\ I_3(s),\ I_4(s),\ I_5(s)$

$V_1(s),\ V_2(s)$

$R_1,\ \frac{1}{SC_1},\ SL_1,\ R_2,\ \frac{1}{SC_2},\ SL_2$

Step 3 : write the equations for unknown currents and voltages.

$$I_1(s) = \frac{V_i(s) - V_1(s)}{R_1} \longrightarrow (1)$$

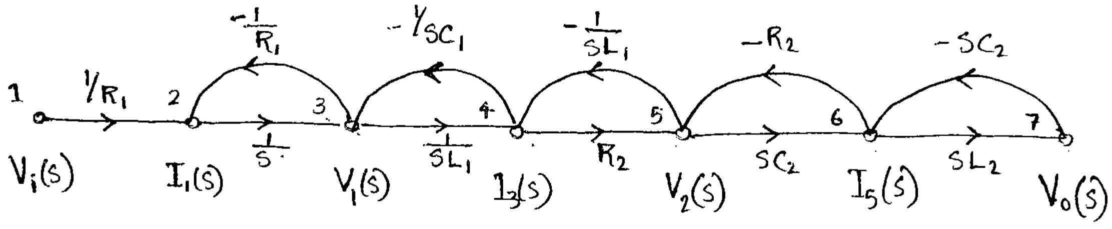
$$V_1(s) = \frac{I_2(s)}{SC_1} = \frac{I_1(s) - I_3(s)}{SC_1} \longrightarrow (2)$$

$$I_3(s) = \frac{V_1(s) - V_2(s)}{SL_1} \longrightarrow (3)$$

$$V_2(s) = I_4(s) R_2 = \left(I_3(s) - I_5(s)\right) R_2 \longrightarrow (4)$$

$$I_5(s) = \frac{V_2(s) - V_0(s)}{1/SC_2} = SC_2\left(V_2(s) - V_0(s)\right) \longrightarrow (5)$$

$$V_0(s) = I_5(s)\, SL_2 \longrightarrow (6)$$

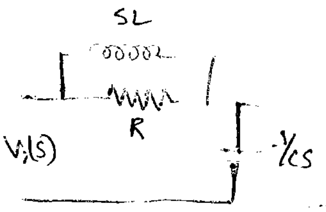


* Nodes in the signal flow ~~is th~~ are the variables in the upper path of the electrical n/w. That's why $I_2(S)$ and $I_4(S)$ are avoided and represented in terms of other variables.

$$\boxed{T.F_{E-N/W} = T.F_{BD\ (or)\ SFG}}$$

(For the gate point of view, never go for conventional method, only verify the transfer fn of electrical n/w and that of ~~Bode~~ block diagram or signal flow graph)

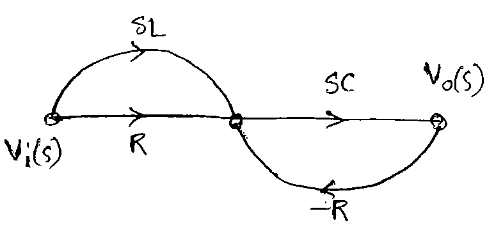
Q. Identify the equivalent signal flow graph to the given electrical n/w.



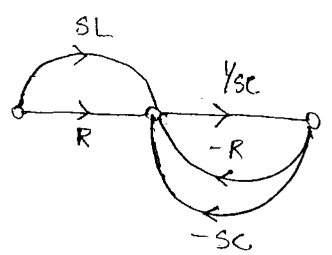
$$\longrightarrow T.F = \frac{1/CS}{\frac{1}{CS} + \frac{RSL}{R+SL}} = \frac{R+SL}{R+SL+RLCS^2} \quad \frac{RSL}{R+SL}$$

options (a)

$=$



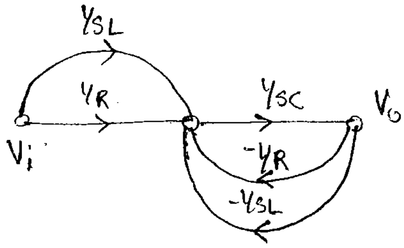
(b)



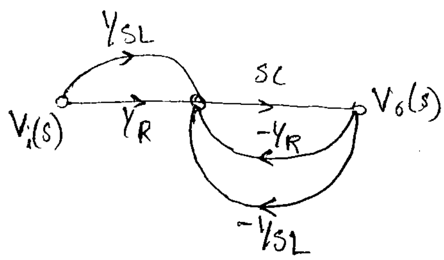
~~$\frac{1}{RC} + \frac{1}{RSC}$~~

~~$\frac{RSC + S^2LC}{S^3LCR}$~~

(c)



(d)



(b) $\dfrac{\frac{R}{SC} + \frac{SL}{SC}}{1 + \frac{R}{SC} + \cancel{\frac{SC}{SC}}\,1} = \dfrac{R+SL}{SC(2 + R/SC)}$

$= \dfrac{R+SL}{2SC + R}$

(c) $\dfrac{\frac{1}{SRC} + \frac{1}{SLSC}}{1 + \frac{1}{SRC} + \frac{1}{S^2LC}} = \dfrac{\left[\frac{1}{SRC} + \frac{1}{S^2LC}\right]}{1 + \frac{1}{SRC} + \frac{1}{S^2LC}}$

$= \dfrac{S^2RLC}{S^2RLC} \dfrac{\left(\frac{1}{SRC} + \frac{1}{S^2LC}\right)}{\left(1 + \frac{1}{SRC} + \frac{1}{S^2LC}\right)}$

$= \dfrac{SL + R}{S^2RLC + SL + R}$

(d)

MECHANICAL SYSTEMS (only for IES)

→ Based on the type of the motion, the mechanical systems are classified into two ways.

i, Translational Motion.
ii, Rotational Motion.

TRANSLATIONAL MOTION	ROTATIONAL MOTION
It is defined as a motion that take place along a straight line (Linear motion). The variables that are used to describe translational motion are i acceleration. (a) ii Velocity (V) iii displacement (x)	The rotational motion of a body is defined as a motion about the fixed axis. The variables that are used to describe the rotational motion are i Angular Acceleration (α) ii Angular velocity (ω) iii Angular Displacement (θ)

NEWTON'S LAWS OF MOTION

It states that the algebraic sum of the forces acting on a rigid body in a given direction equal to the product of mass of the body and its acceleration, in the same direction.

ie, $\boxed{\sum \text{Forces} = m.a.}$

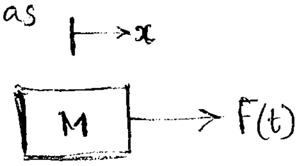
→ Here involed three elements

i Mass
ii Friction
iii Linear spring.

i) MASS

Mass is considered as the Property of an element that stores the kinetic energy of the translational. If ~~the~~ W is the weight of the body, then mass $= \frac{W}{g}$.
where $g = 9.8\ m/s^2$

→ when a force acting on a body with mass m as shown in figure, then the force equation can be written as



$$\boxed{\text{Force } F = Ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}}$$

ii) FRICTION

whenever there exist a motion, there exist a friction. The friction may be b/w moving elements or moving element & fixed

NEWTON'S LAWS OF ROTATIONAL MOTION

It states that the algebraic sum of the torques about a fixed axis equal to the product of Inertia and angular acceleration, about the axis.

$$\boxed{\sum \text{Torques} = J.\alpha}$$

In rotational system also involved three elements.

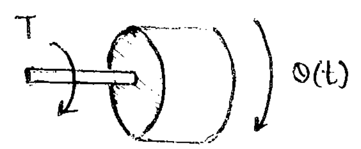
i Inertia
ii Friction
iii Torsional spring.

i) INERTIA

It is considered to be the property of an element that stores the kinetic energy for rotational motion.
The inertia of circular disc or shaft about its geometric axis is given by

$$\boxed{J = \frac{1}{2} m r^2}$$

→ when torque is applied to a body with a ~~inte~~ inertia J as shown in figure, then the torque can be written as



$$\boxed{T = J\alpha = J\frac{d\omega}{dt} = J\frac{d^2\theta}{dt^2}}$$

ii) FRICTION

Friction same as in translational motion.

support.

→ There are 3 types of friction.

a) Viscous Friction.

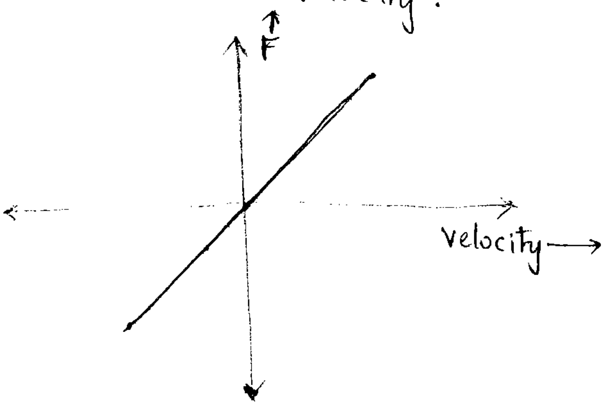
b) static Friction

c) Coloumb Friction.

→ The friction is a retarding force which is opposite direction to the motion.

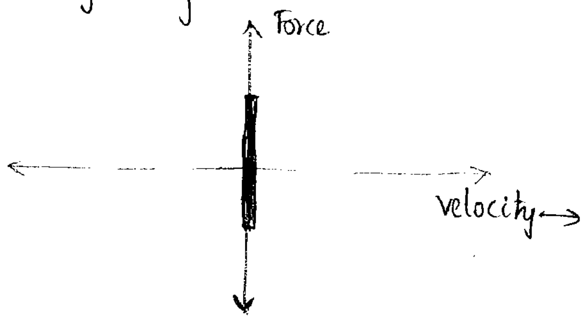
a) VISCOUS FRICTION

It represents the retarding Force that has linear relationship b/w force and velocity.



b) STATIC FRICTION

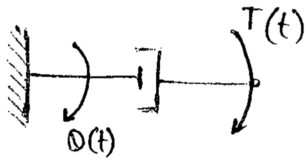
It is a retarding force that tends to prevent the motion from the begining.



c) COLOUMB FRICTION

It is a retarting force that has constant amplitude, with change in velocity.

But instead of force, torque will be there.

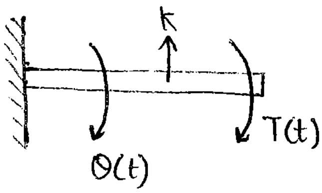


$$\boxed{T(t) = D\omega = D\frac{d\theta}{dt}}$$

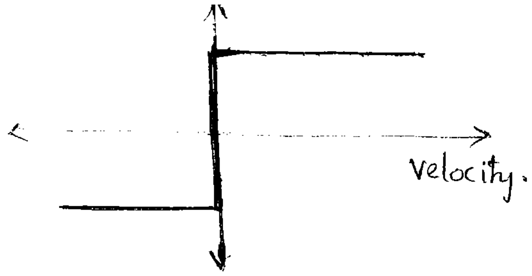
TORSIONAL SPRING

It is represented as ROD or SHAFT

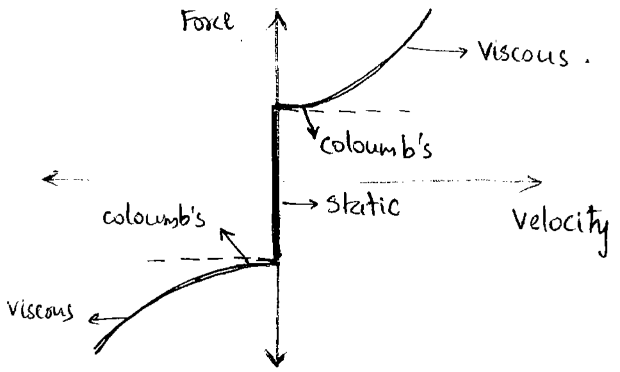
when a torque is applied on torsional spring, then the torque equation can be written as



K → Represents stiffness of rod.



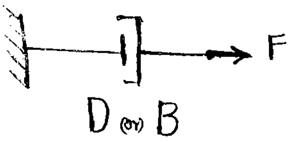
→ The resultant of three frictional forces is ~~called~~ given as.



~~→ Viscous frictiona~~

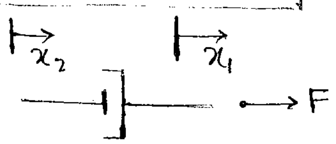
→ Among these 3 frictional forces, viscous friction is more. (more dominating). Hence static and coloumb's frictions are neglected.

→ The friction is represented by <u>Damper</u> or <u>Dash-Pot</u>.



when a force is acting on a body as shown, then the ~~friction equation~~ frictional force is represented as

$$\boxed{F = Dv = D\frac{dx}{dt}}$$



If not fixed →

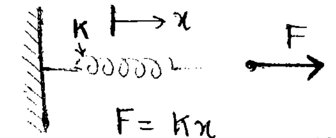
$$\boxed{F = D(V_1 - V_2) = D\frac{d(x_1 - x_2)}{dt}}$$

LINEAR SPRING

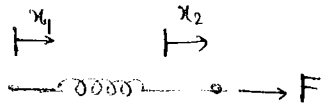
A linear spring is considered to be an element that stores the potential energy.

when spring is subjected to a force as shown in figure, then the elastic deformation occurs.

The force equation for the spring is



$F = Kx$

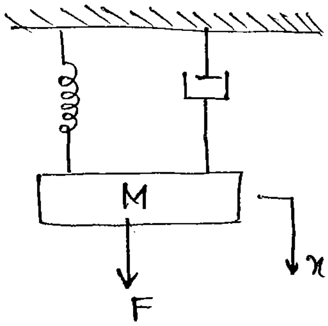


$$\boxed{F = K(x_2 - x_1)}$$

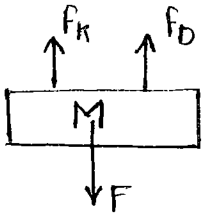
Q, For the mechanical translational system shown in Fig:

obtain i) Differential equation.

ii) Transfer function.



Step 1 : Draw the free body diagram. (FBD)



$$\cancel{m\frac{d^2x}{dt^2} = f_K + f_D}$$

Step 2 :

According to the Newton's Law, the algebraic sum of forces = m x a.

$$F - f_D - f_K = ma$$

$$F = ma + f_D + f_K$$

$$\boxed{F = m\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx}$$

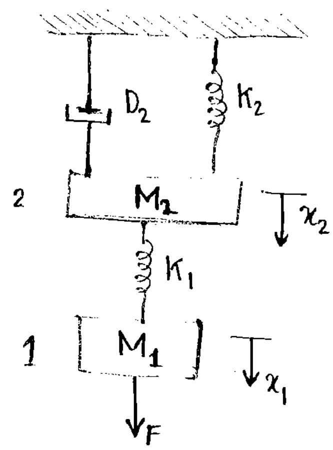
(ii) T·F

$$F(s) = (ms^2 + Ds + K)\, X(s)$$

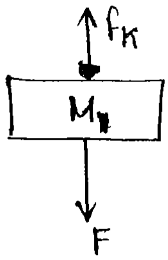
$$\cancel{\frac{F(s)}{X(s)} = m}$$

$$\boxed{T\cdot F = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + Ds + K}}$$

Q. Find $\dfrac{X_1(s)}{F(s)}$, $\dfrac{X_2(s)}{F(s)}$, $\dfrac{X_2(s)}{X_1(s)}$, to the following translational system.



At node 1



$\cancel{F = m}$

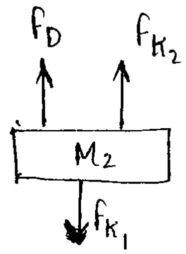
$$F - f_K = ma$$

$$F = m_1 a + f_K$$

$$F = m_1 \frac{d^2 x_1}{dt^2} + K_1(x_1 - x_2)$$

$$F(s) = (m_1 s^2 + K)\, X_1(s) - K_1 X_2(s)$$

At node 2



$$0 = m_2 \frac{d^2 x_2}{dt^2} + D_2 \frac{dx_2}{dt} + K_2 x_2 + K_1 (x_2 - x_1)$$

$$0 = (m_2 s^2 + D_2 s + K_2 + K_1)\, X_2(s) - K_1 X_1(s)$$

$$K_1 X_1(s) = (m_2 s^2 + D_2 s + K_2 + K_1)\, X_2(s)$$

$$X_1(s) = \frac{(M_2 s^2 + D_2 s + K_1 + K_2)\, X_2(s)}{K_1}$$

$$F(s) = \left[\frac{(M_1 s^2 + K)(M_2 s^2 + D_2 s + K_1 + K_2)}{K_1} - K_1\right] X_2(s)$$

$$\frac{F(s)}{X_2(s)} = \left[\frac{(M_1 s^2 + K)(M_2 s^2 + D_2 s + K_1 + K_2) - K_1^2}{K_1}\right]$$

$$\frac{X_2(s)}{F(s)} = \frac{K_1}{(M_1 s^2 + K_1)(M_2 s^2 + D_2 s + K_1 + K_2) - K_1^2}$$

$$X_2(s) = \frac{K_1 X_1(s)}{M_2 s^2 + D_2 s + K_1 + K_2}$$

$$F(s) = \left[(M_1 s^2 + K_1) - \frac{K_1^2}{M_2 s^2 + D_2 s + (K_1 + K_2)}\right] I_1(s)$$

$$\frac{F(s)}{I(s)} = \left[\frac{(M_1 s^2 + K_1)(M_2 s^2 + D_2 s + (K_1 + K_2)) - K_1^2}{M_2 s^2 + D_2 s + (K_1 + K_2)}\right]$$

$$\frac{I_1(s)}{F(s)} = \left[\frac{M_2 s^2 + D_2 s + (K_1 + K_2)}{(M_1 s^2 + K_1)(M_2 s^2 + D_2 s + (K_1 + K_2)) - K_1^2}\right]$$

Alternate method $\rightarrow$ Cramer's Rule

$$\begin{bmatrix} F(s) \\ 0 \end{bmatrix} = \begin{bmatrix} M_1 s^2 + K_1 & -K_1 \\ -K_1 & M_2 s^2 + D_2 s + K_1 + K_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

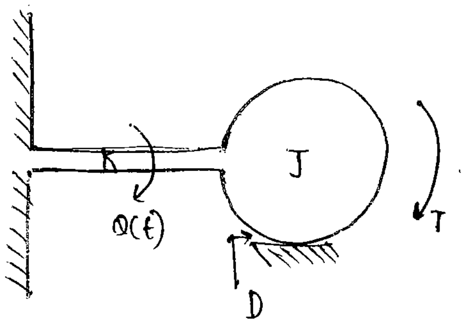
$$\cancel{X_2(s) = \begin{bmatrix} M_1s^2+K_1 & F(s) \\ -K_1 & M_2s^2+D_2s+K_1+K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$$

$$X_2(s) = \frac{\begin{bmatrix} M_1s^2+K_1 & F(s) \\ -K_1 & 0 \end{bmatrix}}{\begin{bmatrix} M_1s^2+K_1 & -K_1 \\ -K_1 & M_2s^2+D_2s+K_1+K_2 \end{bmatrix}} = \frac{K_1F(s)}{(M_1s^2+K_1)(M_2s^2+D_2s+K_1+K_2)-K_1^2}$$

$$X_1(s) = \frac{\begin{bmatrix} F(s) & -K_1 \\ 0 & M_2s^2+D_2s+K_1+K_2 \end{bmatrix}}{\begin{bmatrix} M_1s^2+K_1 & -K_1 \\ -K_1 & M_2s^2+D_2s+K_1+K_2 \end{bmatrix}} = \frac{(M_2s^2+D_2s+K_1+K_2)\,F(s)}{(M_1s^2+K_1)(M_2s^2+D_2s+K_1+K_2)-K_1^2}$$

$$\frac{X_2(s)}{X_1(s)} = \frac{\frac{X_2(s)}{F(s)}}{\frac{X_1(s)}{F(s)}} = \frac{K_1}{M_2s^2+D_2s+K_1+K_2}$$

Find the transfer function to the given rotational system.



According to Newton's Law of Rotational Motion, the applied torque equal to $J\alpha$

$$T - K\theta(t) - D\frac{d\theta}{dt} = J\alpha$$

$$\boxed{T = J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\,\theta(t)} \longrightarrow ①$$

$$T(s) = \left[Js^2 + Ds + K\right]\theta(s)$$

$$T.F = \frac{\theta(s)}{T(s)} = \frac{1}{Js^2+Ds+K}$$

ANALOGOUS SYSTEMS

In b/w electrical and mechanical system, there exist a fixed analogy. and there exist a similarity b/w equillibrium equation. Due to this, it is possible to draw an equivalent electrical system which behaves exactly ~~to the~~ similar to the given mechanical system. This is called electrical analogous to the given mechanical system.

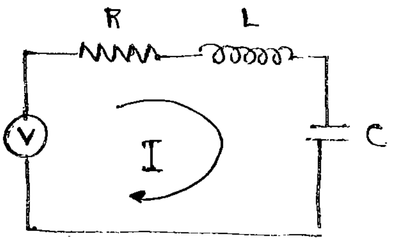
The analogous systems are basically two types,

i) Force-Torque-Voltage Analogy

ii) Force - Torque - Current Analogy

(i) Force-Torque-Voltage Analogy [Loop Analysis]

Consider the electrical n/w as shown in figure. In this method the voltage is treated as a analogous quantity to the force in the mechanical system.



Applying KVL

$$V = IR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt.$$

$$i = \frac{dQ}{dt}$$

$$V = \frac{dQ}{dt}R + L\frac{d^2Q}{dt^2} + \frac{Q}{C}$$

Equivalent Translational System Equ

$$\boxed{F = M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx}$$

Equivalent Rotational System Equ.

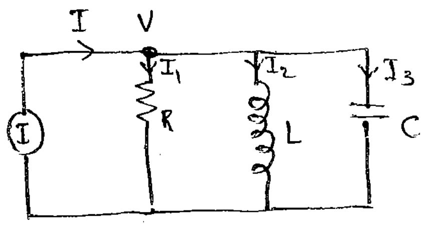
$$\boxed{T = J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta}$$

ELECTRICAL SYSTEM	TRANSALATIONAL SYSTEM	ROTATIONAL SYSTEM
Voltage (V) →	Force (F) →	Torque. (T)
Inductor (L) →	Mass (M) →	Inertia. (J)
Resistor (R) →	Damper (D) →	Damper. (D)
Capacitor $\left(\frac{1}{C}\right)$ →	Spring (K) →	Torsional Spring (K)
charge (Q) →	Displacement (x) →	Angular Displacement (θ)

(ii) Force - Torque - Current Analogy [Nodal Analysis]

In this method, the current is treated as a analogous quantity to the Force to the mechanical system.

Consider the electrical n/w as shown in figure.



Applying KCL

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt}$$

But $V = \frac{d\phi}{dt} \Rightarrow$

$$\boxed{I = C\frac{d^2\phi}{dt} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi}$$

Equivalent Translational System Equ.	Equivalent Rotational system Eqn.
$F = M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx$	$T =$

ELECTRICAL SYSTEM	TRANSLATIONAL SYSTEM	ROTATIONAL SYSTEM
Current (I) →	Force (F) →	Torque (T)
Capacitor (C) →	Mass (M) →	Inertia (J)
Resistance $\left(\frac{1}{R}\right)$ →	Damper (D) →	Damper (D)
Inductor $\left(\frac{1}{L}\right)$ →	Spring (K) →	Torsional Spring (K)
Flux (ϕ) →	Displacement (x) →	Angular Displacement (θ)

NODAL METHOD FOR OBTAINING DIFFERENTIAL EQN FOR MECHANICAL SYSTEMS

Step 1 : Identify the number of nodes.

$\boxed{\text{The number of nodes = Number of displacements}}$

Step 2 : Take one reference node independent of main node. This is an additional node.

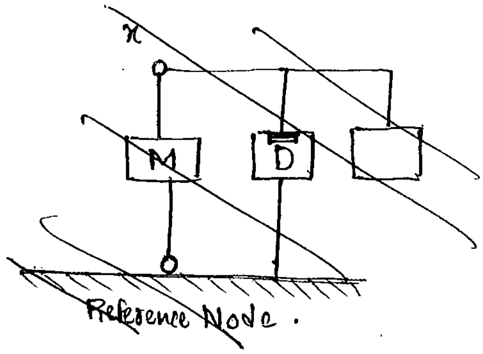
Step 3 : The mass and Inertia elements are connected b/w their main nodes and their reference node, irrespective of their position and placement.

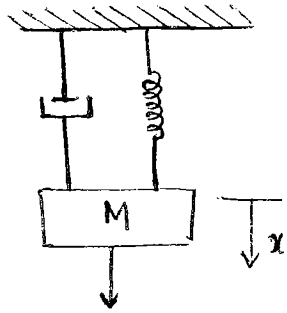
Step 4 : The spring and Damper elements are connected b/w main nodes or main node and reference node ~~bas~~ depends on position or placement on the mechanical system.

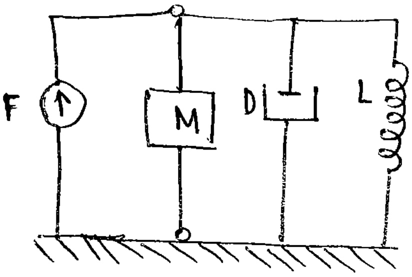
Step 5 : The driving Force or Torque connected at the appropriate node.

Q. Obtain the differential equation to the given mechanical system by

(a) using nodal method.

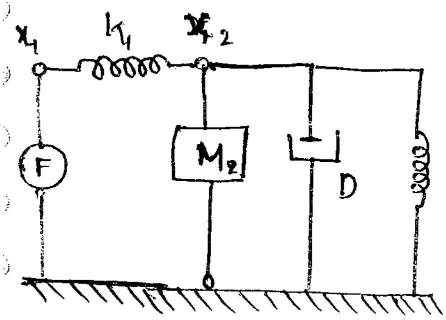


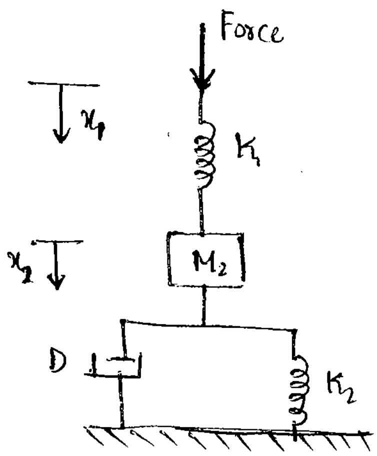




$$F = M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx$$

Q. Draw the nodal diagram and write the differential eqn.





$$F = K_1(x_1 - x_2) \longrightarrow (1)$$

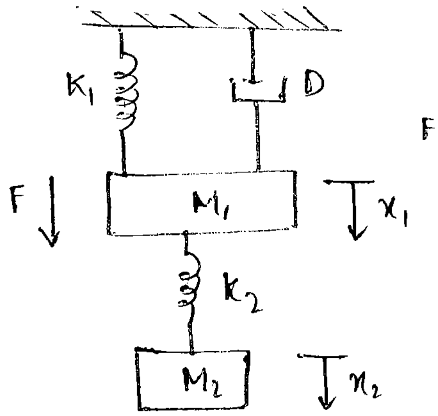
$$0 = M_2\frac{d^2x}{dt^2} + D\frac{dx}{dt} + K_2x_2 + K_1(x_2 - x_1) \longrightarrow (2)$$

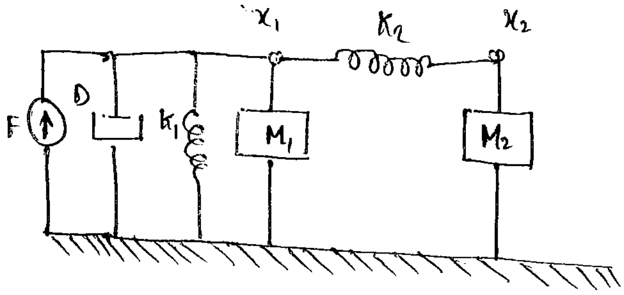
Q, a) Draw the equivalent mechanical system of the given system and then write the set of equillibrium equation.

b) Obtain the electrical anologous ckt for the given mechanical system by using
  i) Force Voltage anology.
  ii) Force current anology.

Soln.

a)

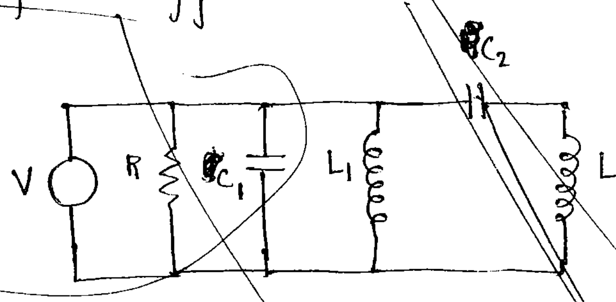




$$F = M_1 \frac{d^2x_1}{dt^2} + D\frac{dx_1}{dt} + K_1 x_1 + K_2(x_1 - x_2) \longrightarrow (1)$$

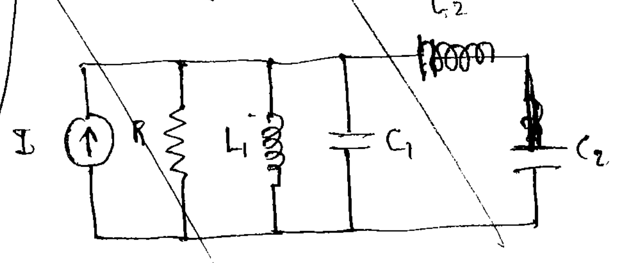
$$0 = M_2 \frac{d^2x_2}{dt^2} + K_2(x_2 - x_1) \longrightarrow (2)$$

~~b) Force voltage Analogy.~~



$R = D$
$C_1 = \frac{1}{K_1}$
$C_2 = \frac{1}{K_2}$
$L_1 = M_1$
$L_2 = M_2$

Force current Analogy



$R = \frac{1}{D}$
$L_1 = \frac{1}{K_1}$, $L_2 = \frac{1}{K_2}$
$C_1 = M_1$ $C_2 = M_2$

b) Force voltage equivalent eqn.

~~Force current equivalent eqn.~~

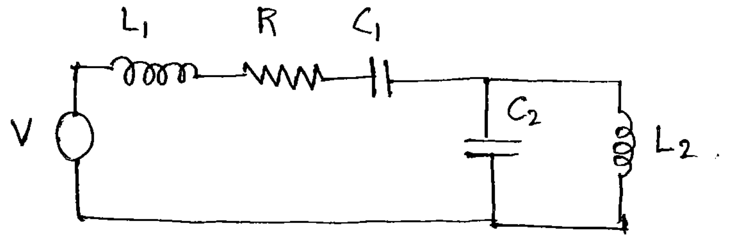
$$V = L_1 \frac{d^2 Q}{dt^2} + R\frac{dQ_1}{dt} + \frac{Q_1}{C_1} + \frac{1}{C_2}(Q_1 - Q_2) \rightarrow (3)$$

$$0 = L_2 \frac{d^2 Q_2}{dt^2} + \frac{1}{C_2}(Q_2 - Q_1) \longrightarrow (4)$$

$$I = \frac{dQ}{dt}, \quad q = \int I\, dt$$

$$V = L_1 \frac{dI_1}{dt} + RI_1 + \frac{1}{C_1}\int I_1 dt + \frac{1}{C_2}\int (I_1 - I_2)\, dt \rightarrow (5)$$

$$0 = L_2 \frac{dI_2}{dt} + \frac{1}{C_2}\int (I_2 - I_1)\, dt. \longrightarrow (6)$$



Force current equivalent eqn.

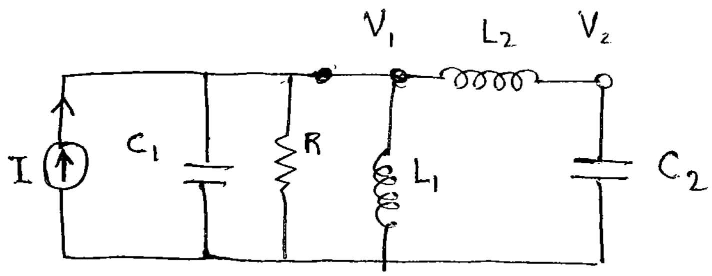
$$I = C_1 \frac{d^2\phi_1}{dt^2} + \frac{1}{R}\frac{d\phi_1}{dt} + \frac{1}{L_1}\phi_1 + \frac{1}{L_2}(\phi_1 - \phi_2)$$

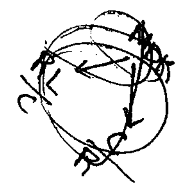
$$V = \frac{d\phi}{dt} \qquad \phi = \int V dt.$$

$$0 = C_2 \frac{d^2\phi_2}{dt^2} + \frac{1}{L_2}(\phi_2 - \phi_1)$$

$$I = C_1 \frac{dV_1}{dt} + \frac{1}{R}V_1 + \frac{1}{L_1}\int V_1 dt + \frac{1}{L_2}\int (V_1 - V_2)\, dt.$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int (V_2 - V_1)\, dt.$$

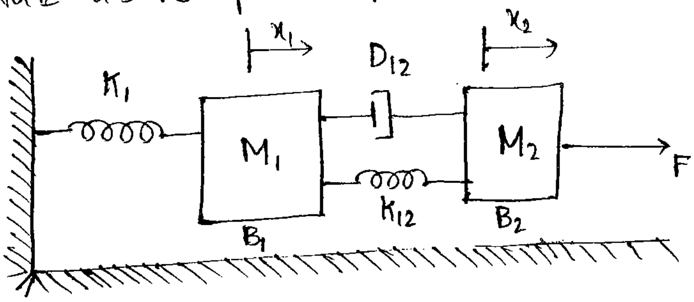


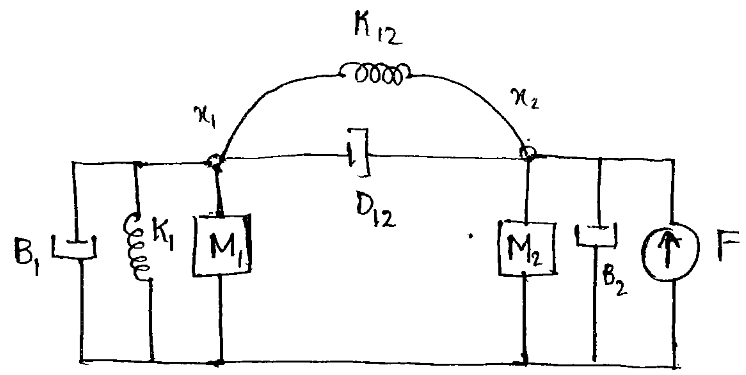


MDK

$$\frac{LR}{C} \qquad \frac{C}{RL}$$

Q. Repeat the above problem for





$$0 = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d x_1}{dt} + D_{12} \frac{d(x_1 - x_2)}{dt} + K_1 x_1 + K_{12}(x_1 - x_2)$$

$$F = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{d x_2}{dt} + D_{12} \frac{d(x_2 - x_1)}{dt} + K_{12}(x_2 - x_1)$$

Force voltage Analogy

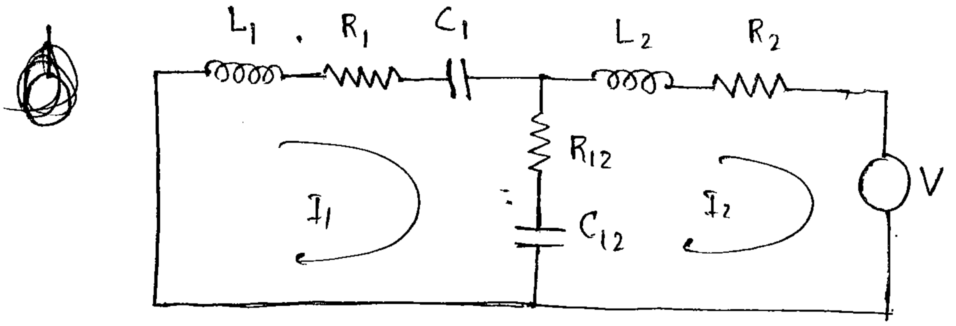
$$0 = L_1 \frac{d^2 Q_1}{dt^2} + R_1 \frac{dQ_1}{dt} + R_{12} \frac{d(Q_1 - Q_2)}{dt} + \frac{1}{C_1} Q_1 + \frac{1}{C_{12}} (Q_1 - Q_2)$$

$$F = L_2 \frac{d^2 Q_2}{dt^2} + R_2 \frac{dQ_2}{dt} + R_{12} \frac{d(Q_2 - Q_1)}{dt} + \frac{1}{C_{12}} (Q_2 - Q_1)$$

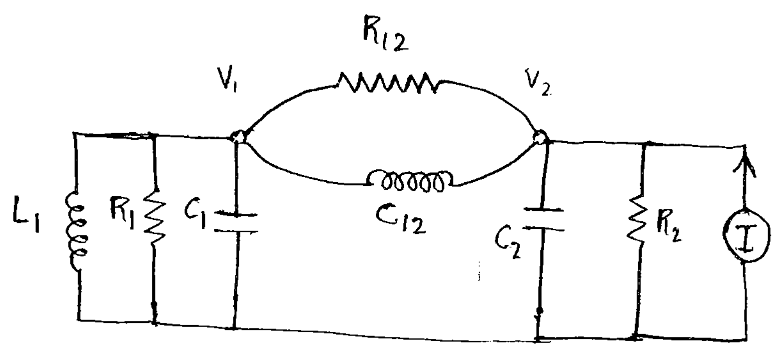
$$I = \frac{dQ}{dt} \qquad Q = \int I\,dt.$$

$$0 = L_1 \frac{dI_1}{dt} + R_1 I_1 + R_{12}(I_1 - I_2) + \frac{1}{C_1}\int I_1\,dt + \frac{1}{C_{12}}\int (I_1 - I_2)\,dt.$$

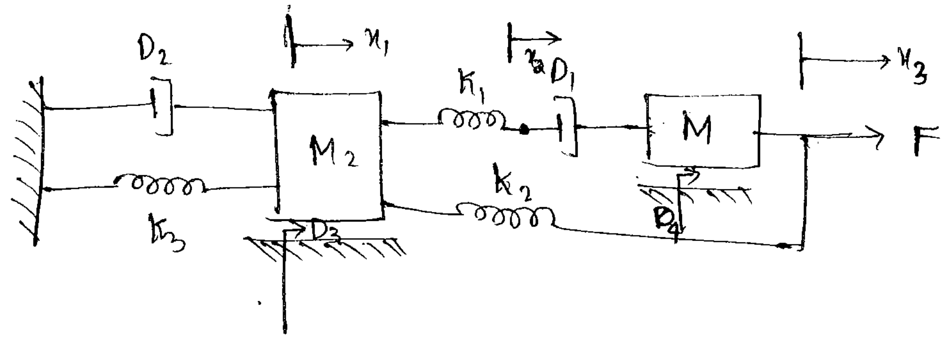
$$F = L_2 \frac{dI_2}{dt} + R_2 I_2 + R_{12}(I_2 - I_1) + \frac{1}{C_{12}}\int (I_2 - I_1)\,dt.$$

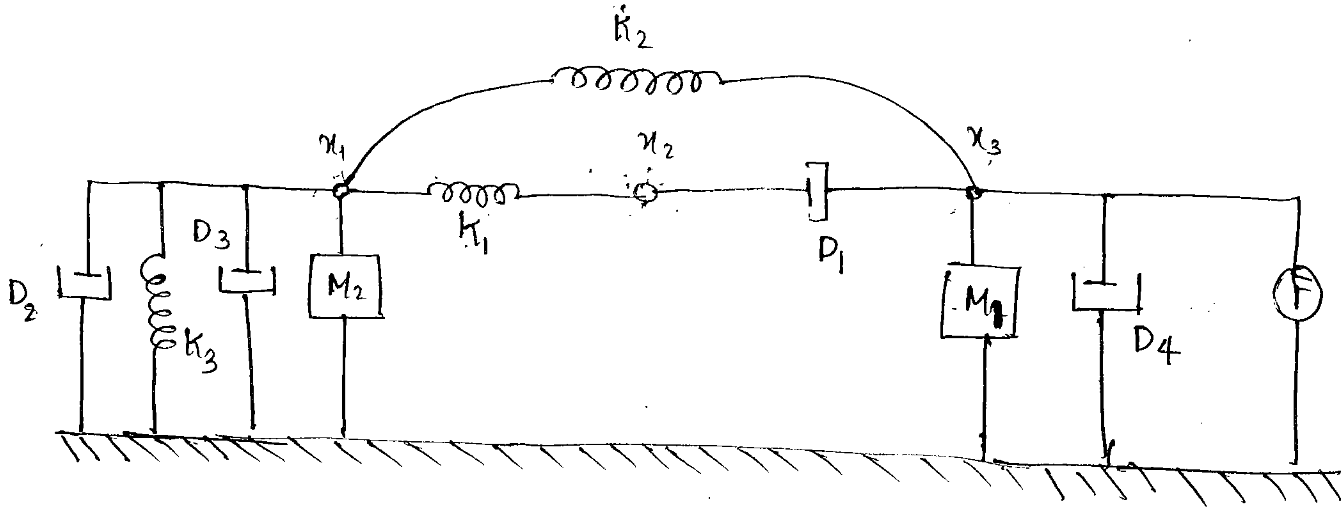


Force Current Analogy.



Q





$$0 = M_2 \frac{d^2 x_1}{dt^2} + D_3 \frac{dx_1}{dt^2} + K_3 x_1 + D_2 \frac{dx_1}{dt^2} + K_1(x_1 - x_2) + K_2(x_1 - x_3)$$

$$\cancel{F = M_1 \frac{d^2 x_2}{d}}$$

$$0 = \cancel{F} - K_1(x_2 - x_1) + D_1(x_2 - x_3)$$

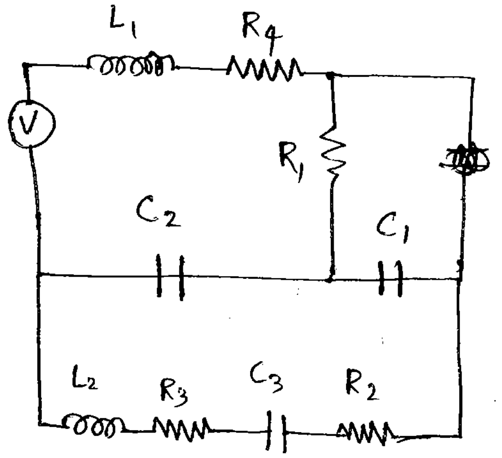
$$F = M_1 \frac{d^2 x_3}{dt^2} + D_4 \frac{dx_3}{dt} + D_1 \frac{d(x_3 - x_2)}{dt} + K_2(x_3 - x_1)$$

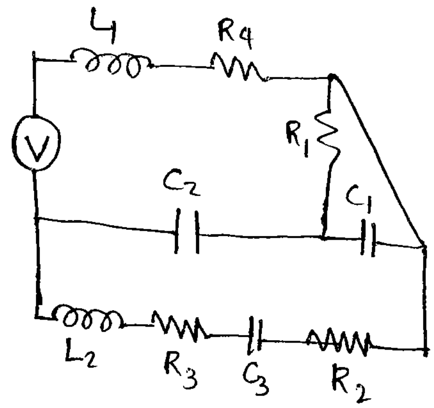
~~voltage~~ Force voltage Analogy

$$0 = L_2 \frac{dI_1}{dt} + R_3 I_1 + \frac{1}{C_3}\int I_1\,dt + R_2 I_1 + \frac{1}{C_1}\int (I_1 - I_2)\,dt + \frac{1}{C_2}\int (I_1 - I_3)\,dt.$$

$$0 = \frac{1}{C_1}\int (I_2 - I_1)\,dt + R_1 (I_2 - I_3)$$

$$V = L_1 \frac{dI_3}{dt} + R_4 I_3 + R_1 (I_3 - I_2) + \frac{1}{C_2}\int (I_3 - I_1)\,dt.$$



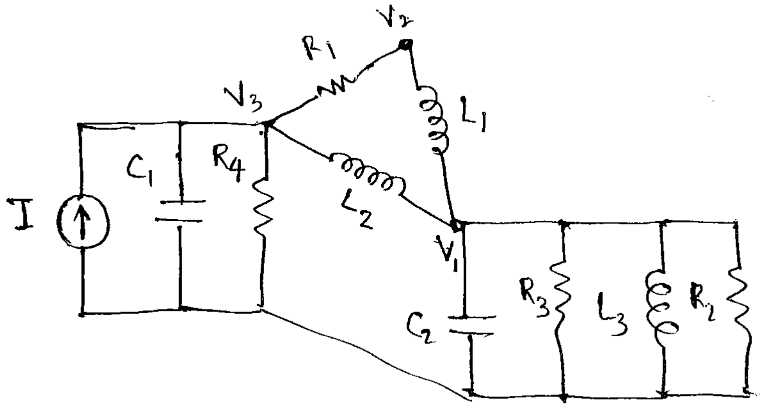


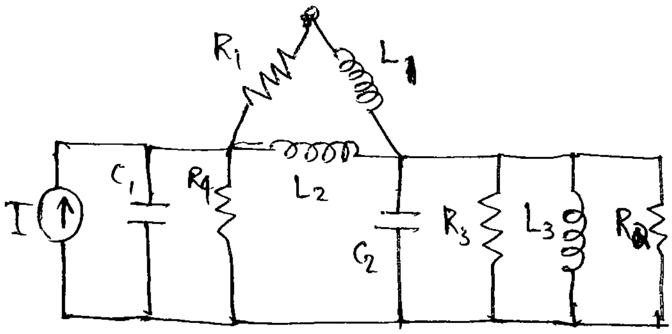
Force current Analogy.

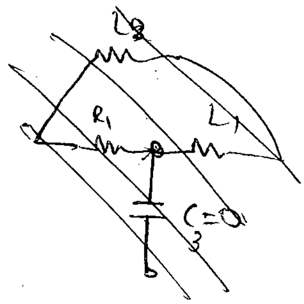
$$0 = C_2 \frac{dV_1}{dt} + \frac{1}{R_3} V_1 + \frac{1}{L_3}\int V_1\, dt + \frac{1}{R_2} V_1 + \frac{1}{L_1}\int (V_1 - V_2)\, dt + \frac{1}{L_2}\int (V_1 - V_3)\, d$$

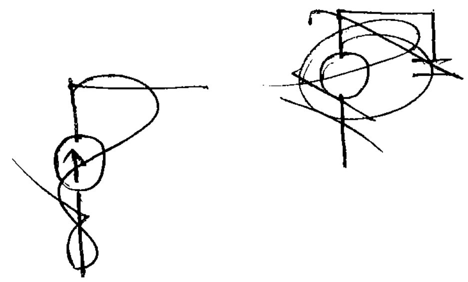
$$0 = \frac{1}{L_1}\int (V_2 - V_1)\, dt + \frac{1}{R_1}(V_2 - V_3)\, dt$$

$$I = C_1 \frac{dV_3}{dt} + \frac{1}{R_4} V_3 + \frac{1}{R_1}(V_3 - V_2) + \frac{1}{L_2}\int (V_3 - V_1)\, dt$$



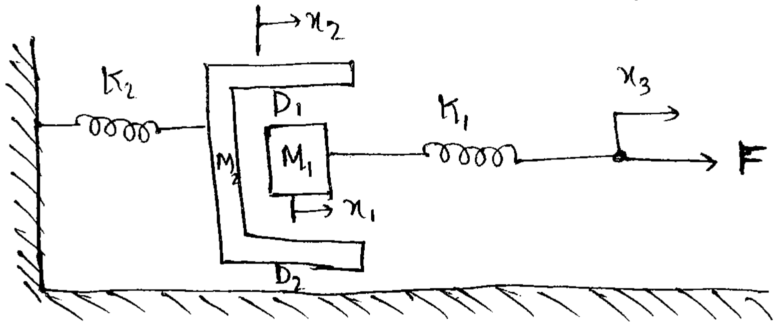


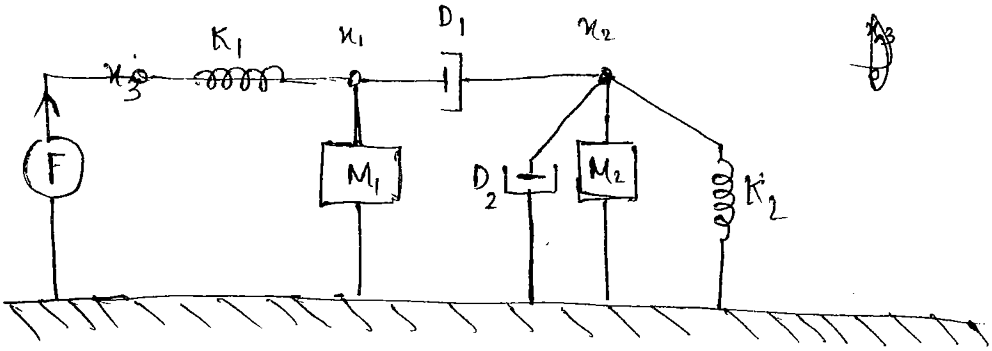




H.W

Q write the time domain system equation and represent force voltage and force current anology.





$$F = K_1(x_3 - x_1)$$

$$0 = K_1(x_1 - x_3) + M_1 \frac{d^2 x_1}{dt^2} + D_1 \frac{d}{dt}(x_1 - x_2)$$

$$0 = D_1 \frac{d}{dt}(x_2 - x_1) + D_2 \frac{d x_2}{dt} + M_2 \frac{d^2 x_2}{dt^2} + K_2 x_2$$

Force voltage Analogy

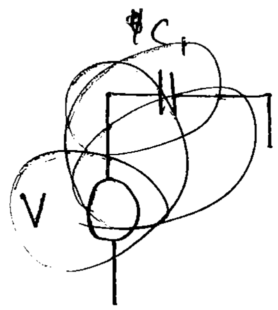
MDK

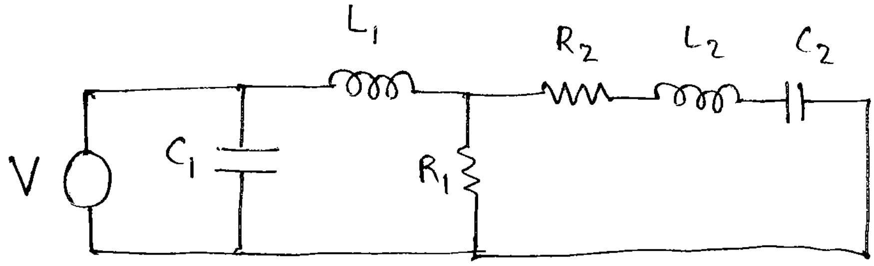
$\frac{LR}{C}$

$$V = \frac{1}{C_1}\int (I_3 - I_1)\, dt$$

$$0 = \frac{1}{C_1}\int (I_1 - I_3)\, dt + L_1 \frac{dI_1}{dt} + R_1(I_1 - I_2)$$

$$0 = R_1(I_2 - I_1) + R_2 I_2 + L_2 \frac{dI_2}{dt} + \frac{1}{C_2}\int I_2\, dt .$$





Force current Analogy

MDK

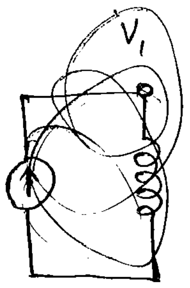
↓

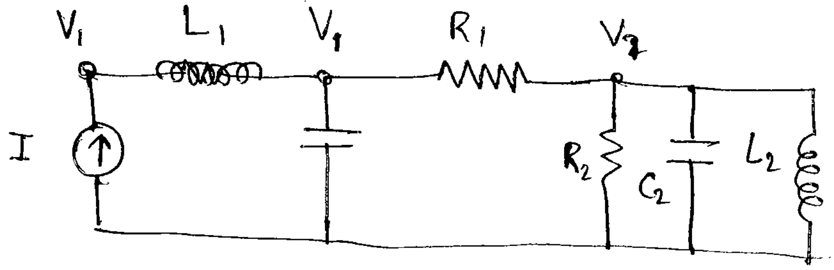
$\frac{C}{RL}$

$$I = \frac{1}{L_1}\int (V_3 - V_1)\, dt .$$

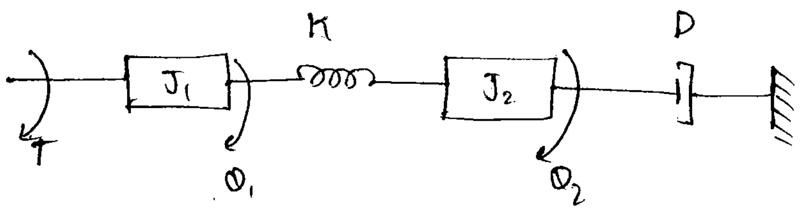
$$0 = \frac{1}{L_1}\int (V_1 - V_3)\, dt + C_1 \frac{dV_1}{dt} + \frac{1}{R_1}(V_1 - V_2)$$

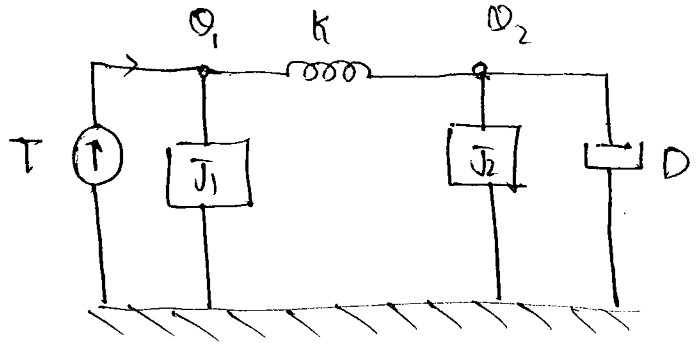
$$0 = \frac{1}{R_1}(V_2 - V_1) + \frac{1}{R_2} V_2 + C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int V_2\, dt$$





Q Find the time domain equations and represent torque-voltage, torque current anology.





$$T = J_1 \frac{d^2\theta}{dt^2} + K(\theta_1 - \theta_2)$$

$$0 = K(\theta_2 - \theta_1) + J_2 \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt}.$$

(i) Torque voltage Analogy.

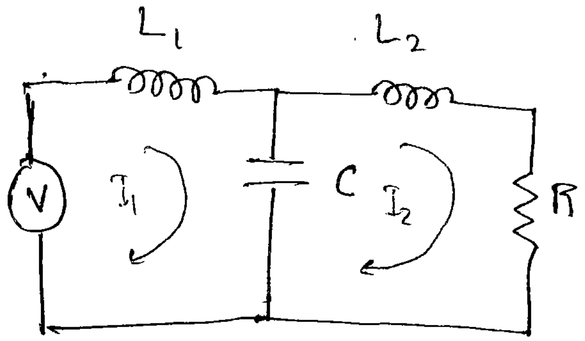
$$V = L_1 \frac{d^2 Q_1}{dt^2} + \frac{1}{C}(Q_1 - Q_2)$$

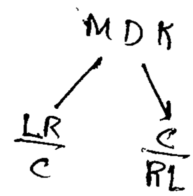
$$0 = L_2 \frac{d^2 Q_2}{dt^2} + \frac{1}{C}(Q_2 - Q_1) + R\frac{dQ}{dt}$$

$$I = \frac{dQ}{dt} \qquad Q = \int I\,dt.$$

$$V = L_1 \frac{dI_1}{dt} + \frac{1}{C}\int (I_1 - I_2)\,dt.$$

$$0 = L_2 \frac{dI_2}{dt} + \frac{1}{C}\int (I_2 - I_1)\,dt + RI_2$$





(ii) Torque Current Analogy.

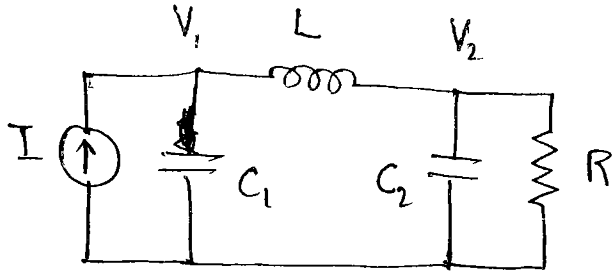
$$I = C_1 \frac{d^2 \phi_1}{dt^2} + \frac{1}{L}(\phi_1 - \phi_2)$$

$$0 = C_2 \frac{d^2 \phi_1}{dt^2} + \frac{1}{L}(\phi_1 - \phi_2) + \frac{1}{R}\frac{d\phi_2}{dt}.$$

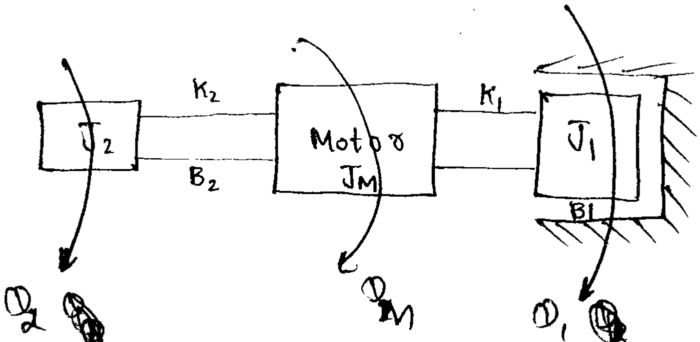
$$V = \frac{d\phi}{dt} \qquad \phi = \int V\,dt.$$

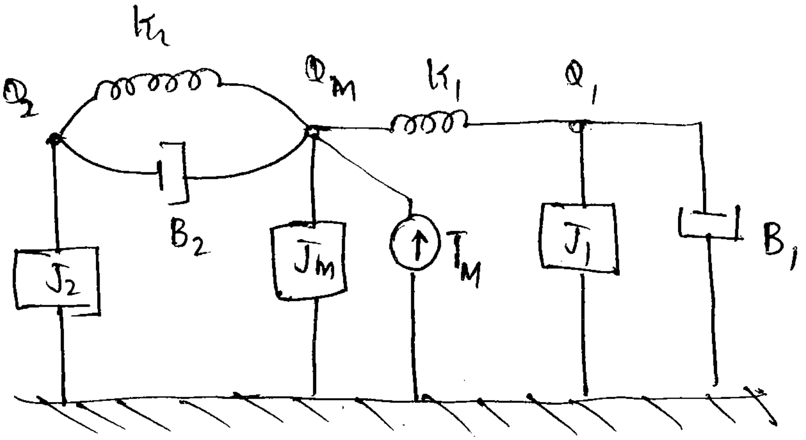
$$I = C_1 \frac{dV_1}{dt} + \frac{1}{L} \int (V_1 - V_2)\, dt .$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L} \int (V_2 - V_1)\, dt + \frac{1}{R} V_2 .$$



Q





MDC

$\frac{LR}{C}$

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + K_2(\theta_2 - \theta_M) + B_2\left(\frac{d(\theta_2 - \theta_M)}{dt}\right)$$

$$T_M = J_M \frac{d^2\theta_M}{dt^2} + K_2(\theta_M - \theta_2) + B_2\left(\frac{d}{dt}(\theta_M - \theta_2)\right) + K_1(\theta_M - \theta_1)$$

$$0 = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2)$$

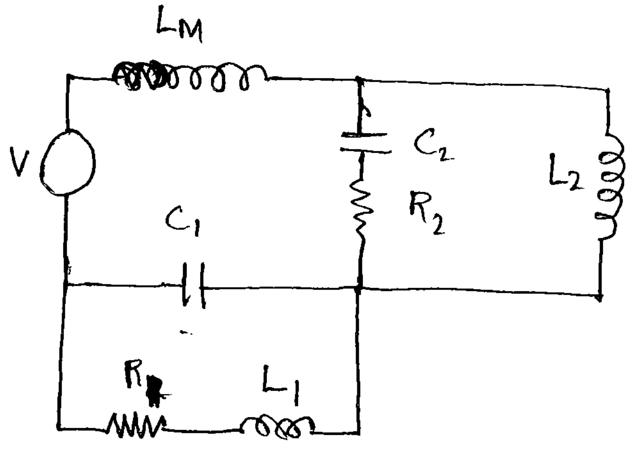
(i) Force voltage Analogy.

$$0 = L_2 \frac{d^2 Q_2}{dt^2} + \frac{1}{C_2}(Q_2 - Q_M) + R_2 \frac{d}{dt}(Q_2 - Q_M)$$

$$0 = L_2 \frac{dI_2}{dt} + \frac{1}{C_2}\int (I_2 - I_M)\,dt + R_2(I_2 - I_M) \longrightarrow (1)$$

$$V = L_M \frac{dI_M}{dt} + \frac{1}{C_2}\int (I_M - I_2)\,dt + R_2(I_M - I_2) + \frac{1}{C_1}\int (I_M - I_1) \longrightarrow (2)$$

$$0 = L_1 \frac{dI_1}{dt} + R_1 I_1 + \frac{1}{C_1}\int (I_1 - I_M)\,dt \longrightarrow (3)$$



MDK

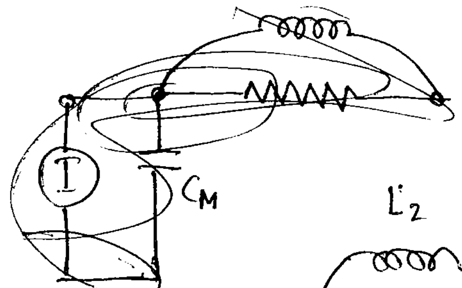
$\frac{LR}{C}$ $\frac{C}{RL}$

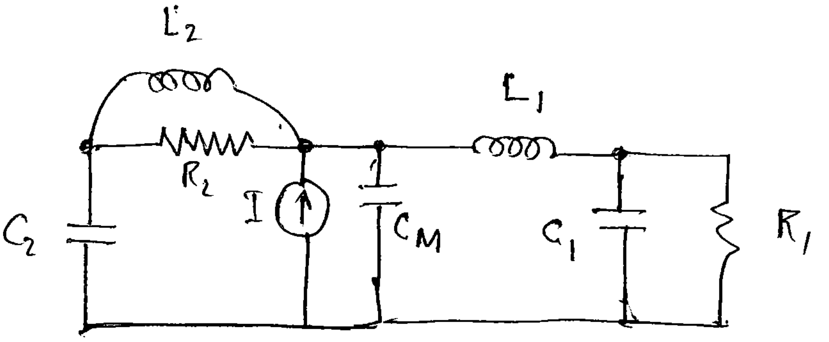
(ii) Force Current Analogy.

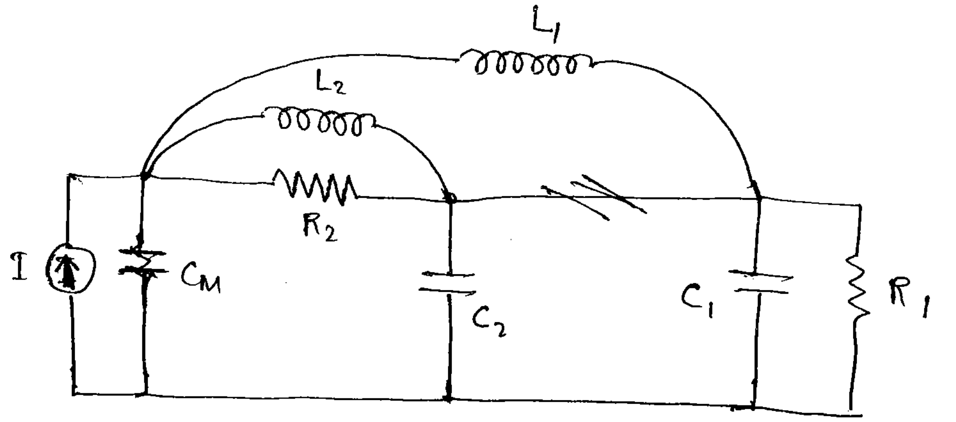
$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int (V_2 - V_M)\,dt + \frac{1}{R_2}(V_2 - V_M) \longrightarrow (4)$$

$$I = C_M \frac{dI_M}{dt} + \frac{1}{L_2}\int (V_M - V_2)\,dt + \frac{1}{R_2}(V_M - V_2) + \frac{1}{L_1}\int (V_M - V_1)\,d \longrightarrow (5)$$

$$0 = C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int (V_1 - V_M)\, dt .$$



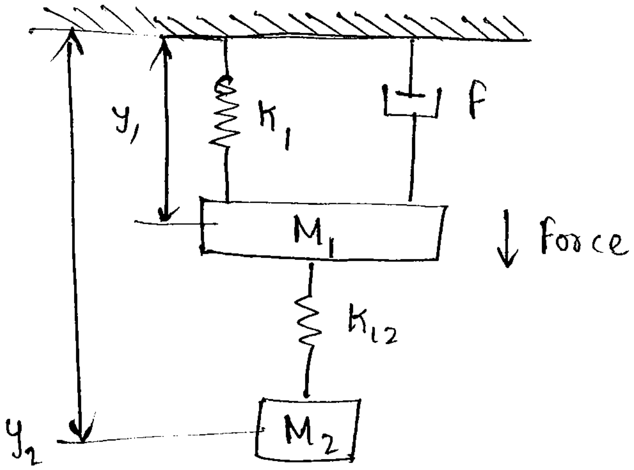


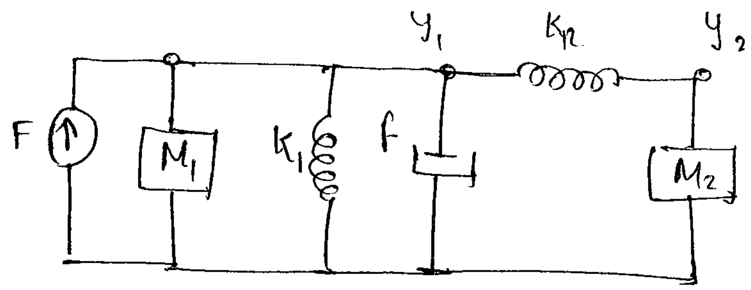


Q. A dynamic vibration absorber is shown in figure. The system is seeing many situations involving machine containing several unbalanced components. The parameters M_2, K_{12} may be chosen such that the main mass M_1 does not vibrate when $F(t) = A \sin \omega t$. Obtain the differential equation to discribe the system. ~~Draw the an~~

(b) Draw the analogous electrical ckt based on force current analogy.

(c) What is the condition for mass M_1 does not vibrate to the given input.





(a)

$$F = M_1 \frac{d^2 y_1}{dt^2} + K_1 y_1 + f \frac{d y_1}{dt} + K_{12}(y_1 - y_2) \longrightarrow (1)$$

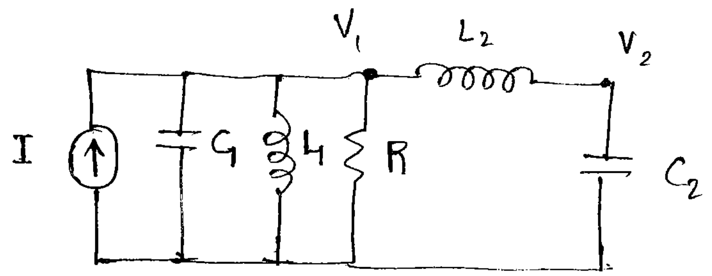
$$0 = M_2 \frac{d^2 y_2}{dt^2} + K_{12}(y_2 - y_1) \longrightarrow (2)$$

(b)

$$A \sin \omega t = C_1 \frac{dV_1}{dt} + \frac{1}{L_1}\int V_1\, dt + \frac{1}{R} V_1 + \frac{1}{L_2}\int (V_1 - V_2)\, dt$$

MDK → $\frac{C}{RL}$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int (V_2 - V_1)\, dt$$



~~A sinωt =~~

Put $y_1 = 0$ in ~~②~~

~~$K_{12} y_1 = 0$~~

① becomes ~~$A\sin\omega t = M_1 \frac{d^2 y_1}{dt^2} + r$~~

$$A\sin\omega t = -K_{12} y_2$$

$$K_2 y_2 = -M_2 \frac{d^2 y_2}{dt^2}$$

$$A\sin\omega t = M_2 \frac{d^2 y_2}{dt^2}$$

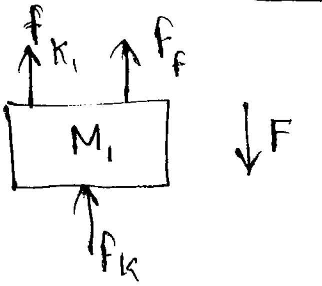
$$\frac{-A\cos\omega t}{\omega} = M_2 \frac{dy_2}{dt}$$

$$\frac{-A\sin\omega t}{\omega^2} = M_2 y_2$$

$$y_2 = \frac{-A\sin\omega t}{M_2 \omega^2}$$

(c) The condition for mass M_1 does not vibrate is algebraic sum of the forces must be less than M_1 x acceleration.

F.B.D

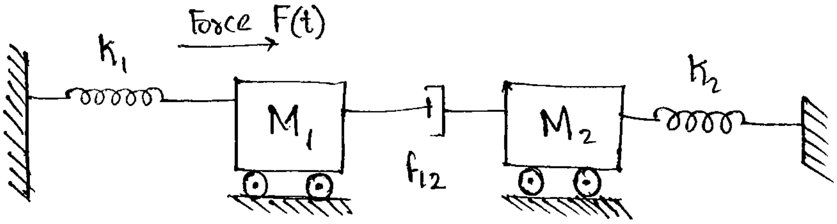


$$F - f_{K_1} - f_K - f_f < M_1 a_1$$

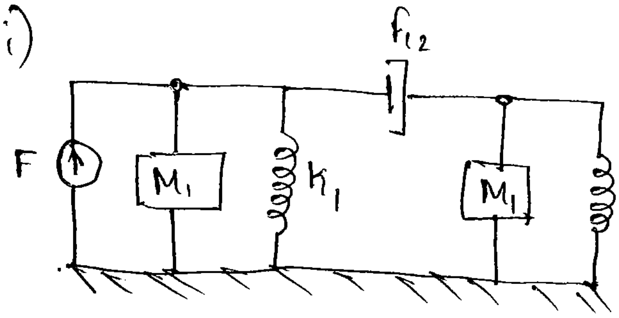
Q(i) For a mechanical system of figure shown below, obtain differential equation

(ii) Sketch the mechanical equivalent ckt.

(iii) Draw the electrical analogous ckt based on Force current Analogy.



(ii)



(i)

$$F = M_1 \frac{d^2x_1}{dt^2} + K_1 x_1 + f_{12} \frac{d}{dt}(x_1 - x_2)$$

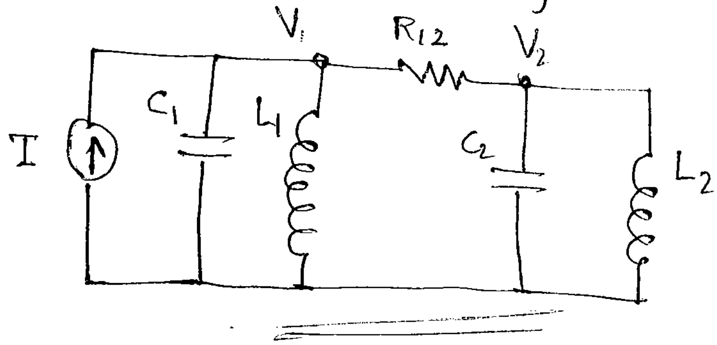
$$0 = M_2 \frac{d^2x_2}{dt^2} + K_2 x_2 + f_{12} \frac{d}{dt}(x_2 - x_1)$$

(iii)

$$I = C_1 \frac{dV_1}{dt} + \frac{1}{L_1}\int V_1\, dt + \cancel{R_2}\, \frac{1}{R_{12}} (V_1 - V_2)$$

MDK $\rightarrow$ $\frac{C}{RL}$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int V_2\, dt + \frac{1}{R_{12}} (V_2 - V_1)$$



GATE

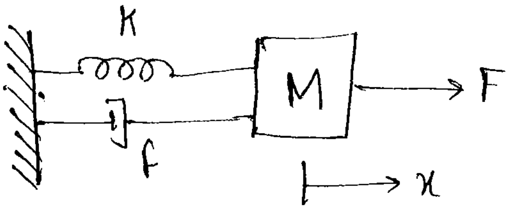
Q. ~~For~~ a mechanical system shown in figure, write the differential equation representing the system. Draw the integrator based electronic ckt to simulate this mechanical system, to study vibrations of x, for the different values of parameter.

OR

GATE

write the force equation for the mechanical system shown below and draw the state diagram.

$$\boxed{\text{Integrater Based Electronic ckt} \equiv \text{State diagram}}$$



According to the ~~law of~~ newton's Laws of motion,

$$\sum \text{forces} = Ma$$

$$F - f_K - f_F = Ma$$

$$F = Ma + f_F + f_K$$

$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx$$

$$F = M\ddot{x} + f\dot{x} + Kx.$$

To draw the state diagram, or integrator based diagram, the highest order of differential state variable bring to the to the left hand side.

$$M\ddot{x} = F - f\dot{x} + Kx$$

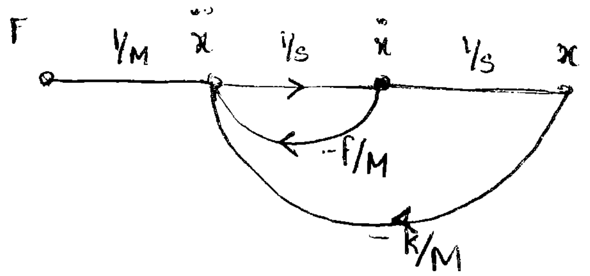
$$\ddot{x} = \frac{F}{M} - \frac{f}{M}\dot{x} - \frac{K}{M}x \longrightarrow (A)$$

STATE DIAGRAM OR INTEGRATOR BASED CKT

→ The first node should be input node.

→ The second node should highest power of differential state variable node.

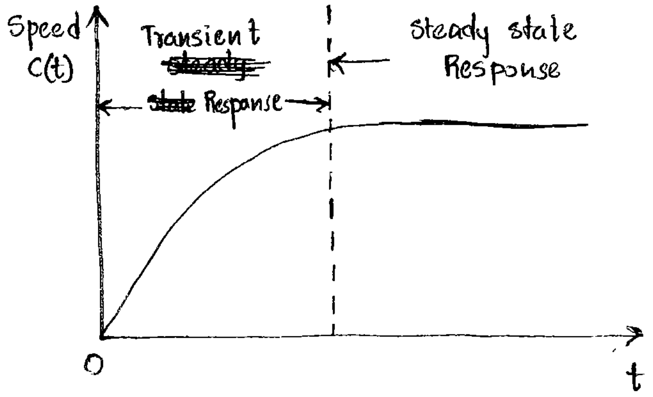
→ The next successive nodes are integrable nodes upto x_1



TIME DOMAIN ANALYSIS

Purpose : To evaluate the performance of the system w.r.t time

TIME RESPONSE



→ If the response of the system varies w.r.t time. Then it is called time Response.

→ The time Response is sum of transient and steady state Response.

$$\boxed{c(t) = C_{tr}(t) + C_{ss}(t)}$$

Q. Identify the transient and steady state terms in the given Response.

$$c(t) = \underbrace{10 + 2\sin 2t + 3\cos 3t}_{\text{Steady state term.}} + \underbrace{4te^{-4t} + 5e^{-5t}\sin 5t + 6t\,e^{-6t}\cos 6t}_{\text{Transient term}}$$

In transient terms, $\boxed{\lim_{t\to\infty} c_{tr}(t) = 0}$

(OR) Those with exponential decay are transient term.

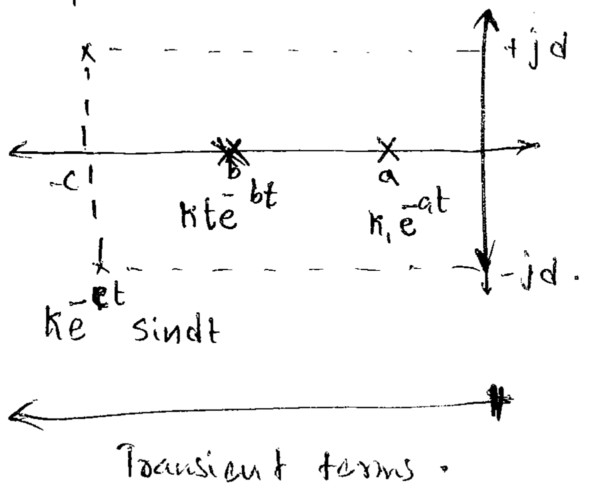
TRANSIENT TERM (Transient Response)

It is a part of time response that becomes Zero as time becomes very large. ie,

$$\boxed{\lim_{t\to\infty} C_{tr}(t) = 0}$$

→ The terms which consists of exponential decay is called the transient term.

→ The poles which lies in the left hand side gives the transient term.

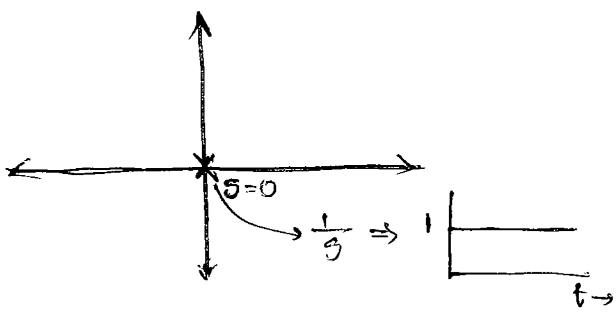


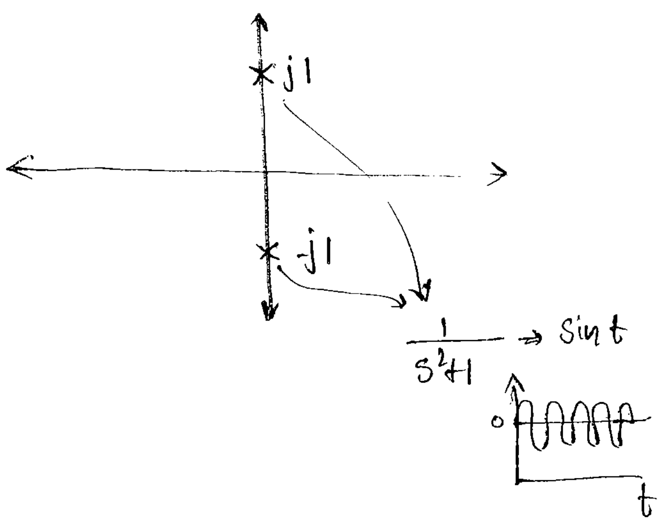
on the imaginary axis ~~and right side~~ are steady state terms.

STEADY STATE TERMS (steady state Response).

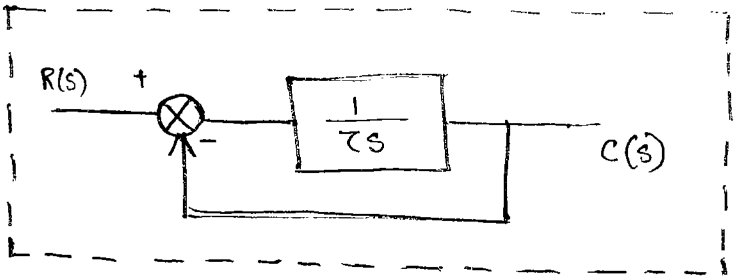
It is a part of the time Response that remains after the transients becomes zero

The poles lies on the imaginary response gives the steady state response.





TIME RESPONSE TO THE FIRST ORDER SYSTEMS.



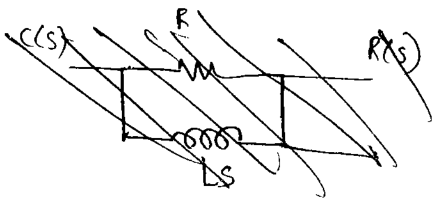
$$\frac{C(s)}{R(s)} = \frac{1}{\tau s+1}$$

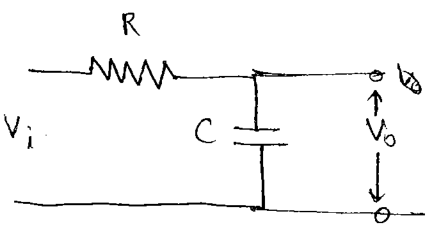
For type and order of a closed loop system required OPTF of unity feedback system. Above found is CLTF

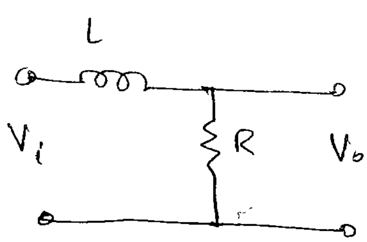
$$\frac{G(s)}{1+G(s)} = \frac{1}{\tau s+1}$$

$$G(s) = \frac{1}{\tau s} \qquad \text{Type 1 , Order 1}$$

The practical circuit to the first order system is RC or RL circuit of Low pass filter.







$$\frac{V_o}{V_i} = \frac{\frac{1}{Cs}}{R+\frac{1}{Cs}}$$

$$\boxed{\frac{V_o}{V_i} = \frac{1}{sCR+1}}$$

$$\frac{V_o}{V_i} = \frac{R}{R+sL}$$

$$\boxed{\frac{V_o}{V_i} = \frac{1}{s\,L/R+1}}$$

IMPULSE RESPONSE

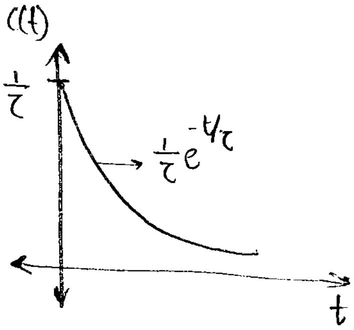
$r(t) = \delta(t)$

$R(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{1}{(\tau s+1)}$$

$$C(s) = \frac{1}{\tau s+1}$$

$$C(s) = \frac{1}{\tau(s+1/\tau)}$$



→ System is stable.

ILT

$$c(t) = \frac{1}{\tau}\boxed{e^{-t/\tau}}$$

Transient term.

→ Impulse response consists of only transient term.

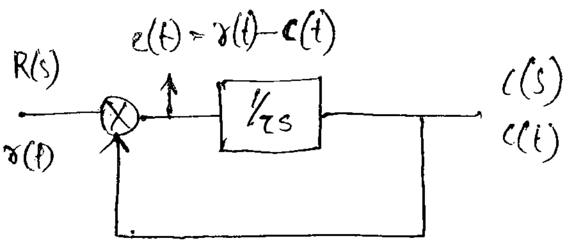
→ Always transient term represent system parameters.

→ Hence the impulse response is called System Response or Natural response, or Free Forced Response.

ERROR

~~Deviation~~ Error is the deviation ~~from~~ of the output from input

$$e(t) = r(t) - c(t)$$



STEADY STATE ERROR (e_{ss})

The error at $t \to \infty$

$$\boxed{e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} r(t) - c(t)}$$

→ Steady state error does not exist for Impulse Response. or not defined, because there is no input present at $t \to \infty$. Hence we cannot compare output with input, at $t \to \infty$.

UNIT STEP RESPONSE

$$r(t) = u(t)$$

$$R(s) = 1/s$$

$$C(s) = \frac{1}{s(s\tau+1)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s\tau+1}$$

$$K_1(s\tau+1) + K_2 s = 1$$

$$K_1 \times 1 = 1$$

$$c(t) = \frac{1}{s} - \frac{\tau}{s\tau+1} = \frac{1}{s} - \frac{\cancel{\tau}}{\cancel{\tau}(s+1/\tau)}$$

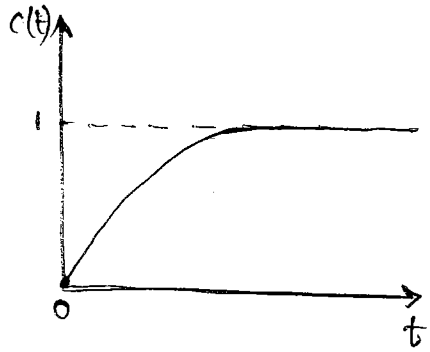
ILT

$$= \Big(\underbrace{1}_{\text{Steady state term.}} \underbrace{- e^{-t/\tau}}_{\text{Transient term.}}\Big)$$

→ In the response, the steady state term is because of the input and transient term is because of the system.

$$e_{ss} = \underset{t\to\infty}{Lt}\left[\cancel{1} - (\cancel{1} - e^{-t/\tau})\right]$$

$$= \underset{t\to\infty}{Lt}\, e^{-t/\tau} = \underline{\underline{0}}$$



UNIT RAMP RESPONSE

$$r(t) = t\,u(t)$$

$$R(s) = \frac{1}{s^2}$$

$$C(s) = \frac{1}{s^2(s\tau+1)} = \frac{K_1}{s} + \frac{K_2}{s^2} + \frac{K_3}{(s+1/\tau)}$$

$$C(s) = \frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{s\tau+1}$$

$$K_1 s(s+1/\tau) + K_2(s+1/\tau) + K_3 s^2 = 1$$

$$K_3\left(\frac{1}{\tau}\right)^2 = 1$$

$$K_3 = \tau^2.$$

$$K_1 s(s+1/\tau) + K_2(s+1/\tau) + K_3 s^2 = 0$$

$$c(s) = \frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau}{s+1/\tau}$$

ILT

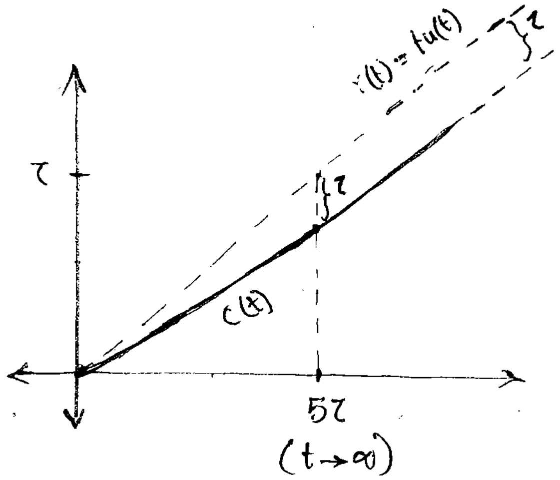
$$\boxed{c(t) = \underbrace{t-\tau}_{\text{Steadystate}} + \underbrace{\tau e^{-t/\tau}}_{\text{Transient}}}$$

$$e_{ss} = \lim_{t\to\infty}\left(r(t) - c(t)\right)$$

$$= \lim_{t\to\infty}\left(\cancel{t} - (\cancel{t} - \tau + \tau e^{-t/\tau})\right)$$

$$= \lim_{t\to\infty}\left(+\tau - \tau e^{-t/\tau}\right)$$

$$e_{ss} = \tau.$$



UNIT PARABOLIC ~~INPUT~~ RESPONSE

$$r(t) = \frac{t^2}{2}u(t)$$

$$R(s) = \frac{1}{s^3}$$

$$\frac{C(s)}{R(s)} = \frac{1}{(s\tau+1)}$$

$$C(s) = \frac{1}{s^3(s\tau+1)}$$

$$C(s) = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{D}{s\tau+1}$$

$$= \frac{A(s\tau+1) + Bs(s\tau+1) + Cs^2(s\tau+1) + Ds^3}{s^3(s\tau+1)}$$

$A = 1$

$A\tau + B = 0$

$B = -A\tau$

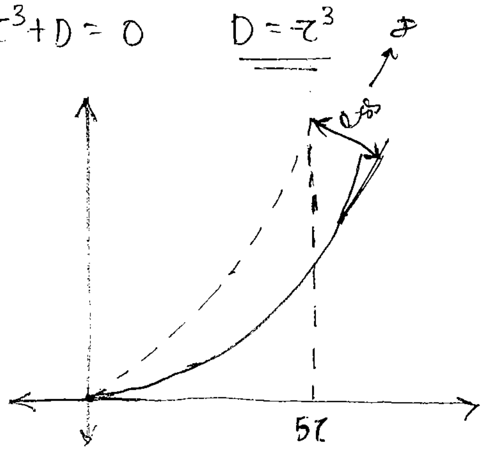
$B = \underline{\underline{-\tau}}$

$B\tau + C = -\tau^2 + C = 0$

$C = \tau^2$

$C\tau + D = 0$

$\tau^3 + D = 0 \qquad \underline{\underline{D = -\tau^3}}$



$$C(s) = \frac{1}{s^3} - \frac{\tau}{s^2} + \frac{\tau^2}{s} - \frac{\tau^3}{(s\tau+1)}$$

$$= \frac{t^2}{2} - \tau t + \tau^2 - \frac{\tau^{\cancel{3}2}}{\cancel{\tau}} e^{-t/\tau}$$

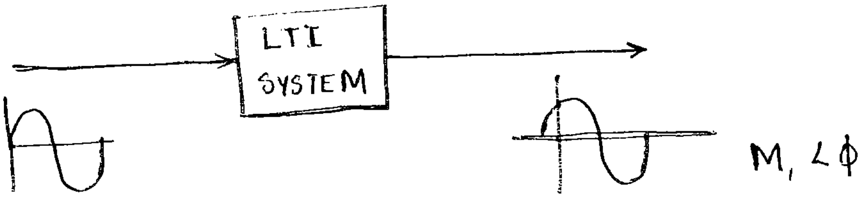
$$= \frac{t^2}{2} - \tau t + \tau^2 - \tau^2 e^{-t/\tau}$$

$$= \frac{t^2}{2} - \tau t + \tau^2\left(1 - e^{-t/\tau}\right)$$

$$e_{ss} = \lim_{t\to\infty}\left(r(t) - c(t)\right) = \lim_{t\to\infty} \cancel{\frac{t^2}{2}} - \cancel{\frac{t^2}{2}} + \tau t - \tau^2\left(1 - e^{-t/\tau}\right)$$

$$= \lim_{t\to\infty} \tau t - \tau^2 \times 1 \qquad = \underline{\underline{\infty}}$$

SINUSOIDAL RESPONSE



→ For any LTI System, if the input is Sinusoidal, then o/p will also be sinusoidal. But the difference in magnitude and phase.

The standard forms of input and outputs are as follows.

$$\boxed{\begin{aligned} r(t) &= A\sin(\omega t \pm \theta) \Longrightarrow c(t) = A \times M \sin(\omega t \pm \theta \pm \phi) \\ r(t) &= A\cos(\omega t \pm \theta) \Longrightarrow c(t) = A \times M \cos(\omega t \pm \theta \pm \phi) \end{aligned}}$$

Q. The closed transfer function of a linear time invarient system is given as $\frac{C(s)}{R(s)} = \frac{1}{s+1}$. For ~~inp~~ the input $r(t) = \sin t$, the steady state output is (OR) the Sinusoidal response is,

$$C(s) = \frac{1}{s+1} R(s)$$

$$= \cancel{\frac{1}{(s+1)(s^2+1)}}$$

$$= \cancel{\frac{1}{s+j}} \quad \cancel{\frac{A}{s+1} + \frac{B}{s+j} + \frac{C}{s-j}}$$

Note

Don't use above method. Just compare with the above std equn

$$c(t) = A \times M \cos(\omega t \pm \theta \pm \phi) \qquad M = \frac{1}{\sqrt{2}}, \quad \angle\phi = \frac{\angle 1}{\angle(j1+1)}$$

$$= -\tan^{-1}(1) = -45$$

$$\omega = 1$$

$$s = j\omega = j1$$

$$c(t) = 1 \times \frac{1}{\sqrt{2}} \sin(t - 45^\circ)$$

$$= \frac{1}{\sqrt{2}} \sin(t - 45^\circ)$$

$$\frac{C(s)}{R(s)} = \frac{1}{j+1}$$

Q, Repeat the above problem with $\frac{C(s)}{R(s)} = \frac{s+2}{s+1}$

and $r(t) = 10\cos(2t + 45^\circ)$

$$\frac{C(s)}{R(s)} = \frac{s+2}{s+1}$$

$$= \frac{j\omega+2}{j\omega+1}$$

$\omega = 2$

$$= \frac{j2+2}{(j2+1)} \times \frac{(j2-1)}{(j2-1)}$$

$$= \frac{-4-j2+4j-2}{4+1}$$

$$= \frac{-6+2j}{5}$$

$M = \sqrt{\frac{8}{5}}$, $\angle\phi = \tan^{-1}\left(\frac{2}{2}\right) - \tan^{-1}\left(\frac{2}{1}\right)$

$$= \tan^{-1}1 - \tan^{-1}2$$

$$= 45 - 63.43^\circ$$

$$= -18.43^\circ$$

$$c(t) = 10\times\sqrt{\frac{8}{5}}\cos(2t + 45^\circ - 18.43^\circ)$$

$$c(t) = 12.64\cos(2t + 26.57^\circ).$$

Q, A system $\frac{Y(s)}{X(s)} = \frac{s}{s+P}$

with ~~x(t)~~ o/p $y(t) = 1\cdot\cos(2t - \pi/3)$

i/p $x(t) = P\cos(2t - \pi/2)$

Then the system parameter P is ?

(i) $M = \sqrt{\frac{4}{4+p^2}} = \frac{2}{\sqrt{4+p^2}}$ $\qquad \angle\phi = \pi/2 - \tan^{-1}\frac{2}{p}$

$y(t) = 1\cdot\cos(2t - \pi/3)$

$\cancel{-\frac{\pi}{2}} \cancel{+\frac{\pi}{2}} - \tan^{-1}\frac{2}{p}$

$\cancel{\frac{1}{\sqrt{1+p^2}}}\ \frac{2p}{\sqrt{4+p^2}} = 1$

$\cancel{p = \sqrt{1+p^2}}$

$\cancel{p^2 = 1+p^2}$

1

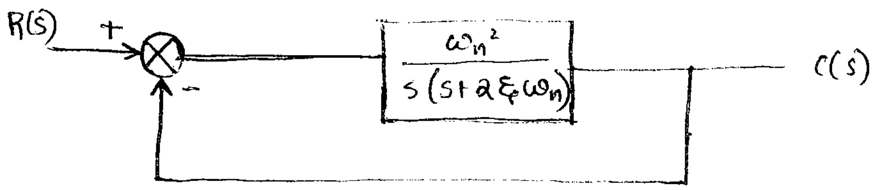
$4p^2 = 4 + p^2$

$p = 2/\sqrt{3}$

$\frac{2}{p} = \tan 60^\circ = \sqrt{3}$

$p = 2/\sqrt{3}$

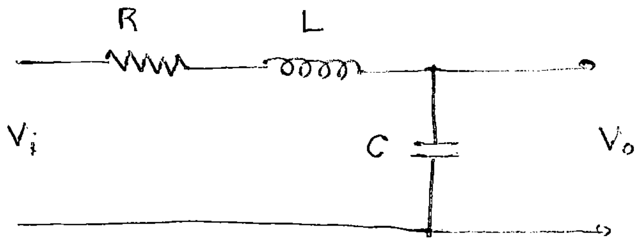
TIME RESPONSE TO THE SECOND ORDER SYSTEM



Type 1 Order 2.

$$\boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}}$$

→ The practical circuit to the second order system is R, L, C circuit.



$$\frac{V_o}{V_i} = \frac{1/Cs}{R + Ls + 1/Cs} = \frac{1}{RCs + LCs^2 + 1}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

comparing it with $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \frac{1}{LC}}$

$$\omega_n^2 = \frac{1}{LC} \qquad \boxed{\omega_n = \frac{1}{\sqrt{LC}}}$$

$\omega_n \rightarrow$ Natural Frequency of Oscillation of Undamped Oscillation.

$$2\xi\omega_n = \frac{R}{L}$$

$$2\xi \frac{1}{\sqrt{LC}} = \frac{R}{L}$$

$$\xi = \sqrt{LC}\,\frac{R}{2L}$$

$$\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

$\xi \rightarrow$ Damping Ratio.
(It gives the ratio of energy lost to energy stored.)

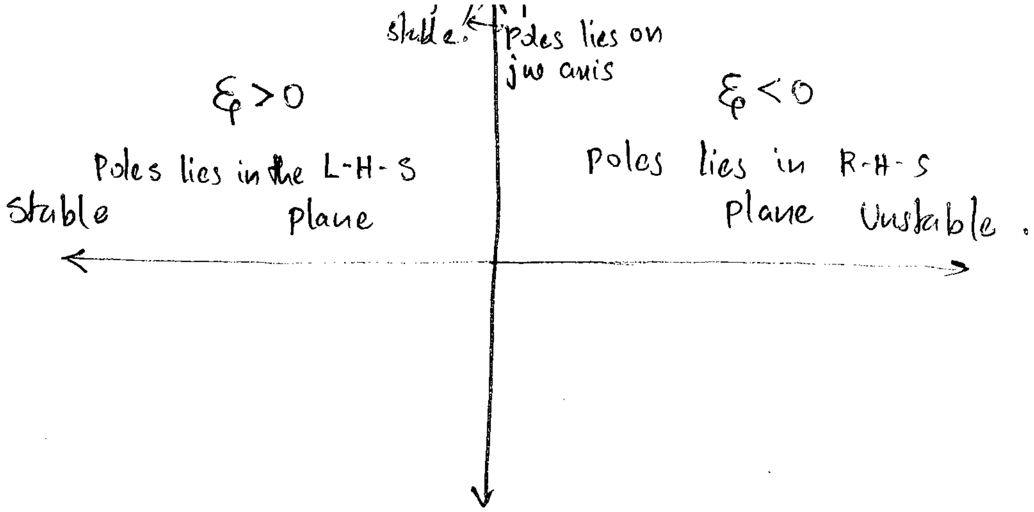
$\xi\omega_n \rightarrow$ Damping Factor or Actual Damping.

QUALITY FACTOR

$$\boxed{Q = \frac{1}{2\xi} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

→ The second order system completely depends only on ξ, the damping ratio.

→ The second order system response completely depends on ξ.
The second order system is stable for all the positive values of ξ. $0 < \xi < \infty$, because the poles lies in the left of s plane.



IMPULSE RESPONSE

$r(t) = \delta(t)$

$R(s) = 1$

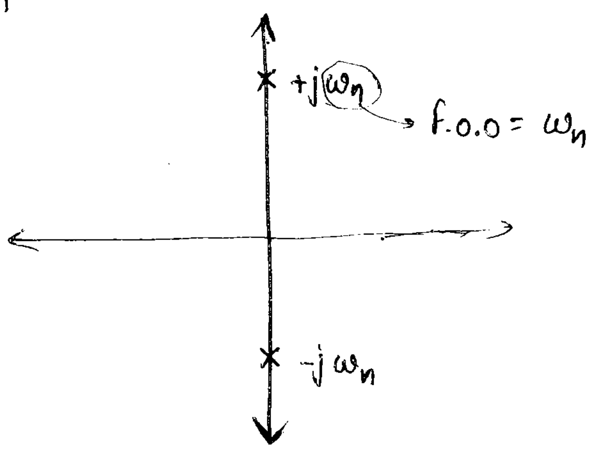
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\cancel{s = -2\xi\omega_n +}$

Case 1 $\xi = 0 \longrightarrow$ UNDAMPED SYSTEM

$$C(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}$$



$$\tau = \frac{-1}{Re(pole)}$$

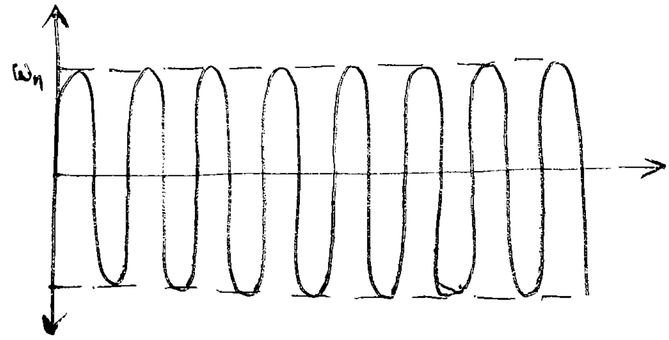
$$\tau = \frac{-1}{0} = \underline{\underline{\infty}}$$

frequency of operation = ω_n

Non repeated poles on $j\omega$-axis.

ILT

$c(t) = \omega_n \sin(\omega_n) t$



Constant Amplitude & Frequency of oscillation.

UNDAMPED OSCILLATION

Sustained oscillation with f.o.o ω_n.

→ when $\xi = 0$, the poles lies on imaginary axis which are non repeated. The system is marginal stable. The system response is constant amplite and frequency of oscillations, which are called Undamped oscillations.

→ Any system which produce Undamped oscillations, is called Undamped system.

→ Irrespective of the input amplied, if $\xi = 0$, we get Undamped system.

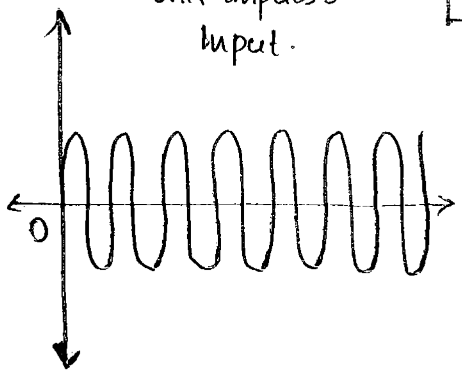
→ The second order system nature completely depends on ξ. For eg, when $\xi = 0$, the second order system nature is constant amplitude and frequency of oscillations around the input, which never be changed by changing the input signal. Hence when $\xi = 0$, the second order system is called Undamped system, irrespective of all the inputs.

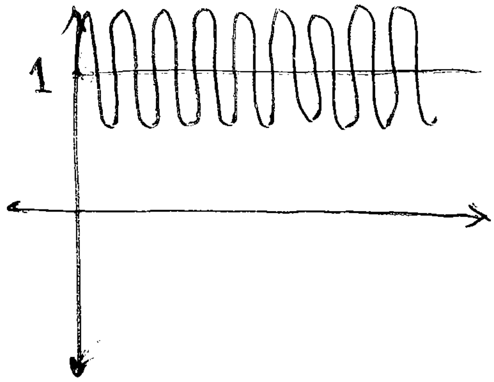
Similarly when $0 < \xi < 1$, the system is called Underdamped.

when $\xi = 1$, critically damped.

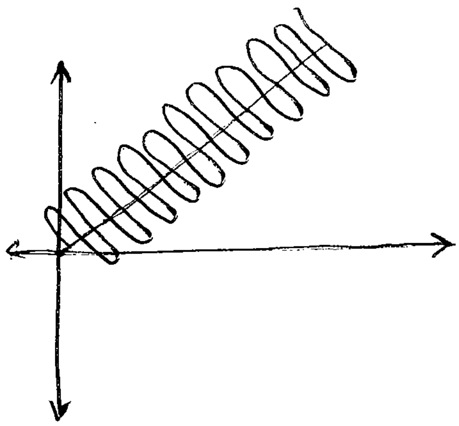
when $\xi > 1$ overdamped system.

Unit Impulse Input.

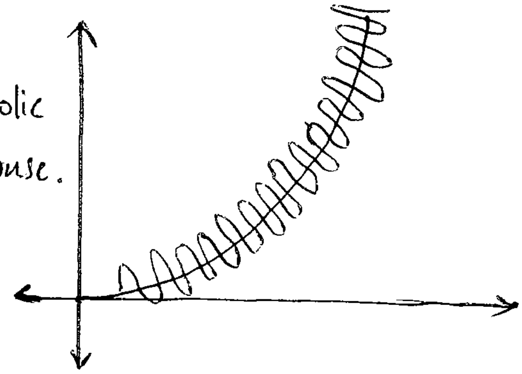




Unit ramp Response.



Unit parabolic Response.



→ when $\xi_p = 0$, we cannot find the steady state errors because the system is marginal stable.

→ Steady state errors are calculated to only stable systems.

Case II $0 < \xi_p < 1$ UNDER DAMPED SYSTEM

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n \xi_p s + \omega_n^2}$$

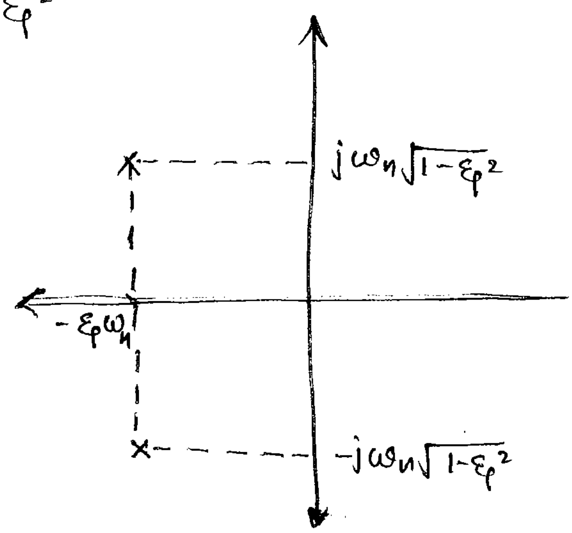
$$s = \frac{-2\omega_n \xi_r \pm \sqrt{4\omega_n^2 \xi_p^2 - 4\omega_n^2}}{2}$$

$$= -\omega_n \xi_r \pm \sqrt{\omega_n^2 \xi_p^2 - \omega_n^2}$$

$$s = -\omega_n \xi_r \pm \omega_n \sqrt{\xi_p^2 - 1}$$

This is valid only when $\xi_p > 1$ Here $0 < \xi_p < 1$

ie, $s_1, s_2 = -\varepsilon_p \omega_n \pm j\omega_n\sqrt{1-\varepsilon_p^2}$



Time Constant $= \dfrac{-1}{Re(pole)}$

$$= \frac{-1}{-\varepsilon_p \omega_n}$$

$$\boxed{\tau = \frac{1}{\varepsilon_p \omega_n}}$$

$$\boxed{\text{Frequency of } \cancel{\text{operation}} \text{ oscillation} = \omega_n\sqrt{1-\varepsilon_p^2}}$$

$$\boxed{f.o.o = \omega_n\sqrt{1-\varepsilon_p^2}}$$

$$c(s) = \frac{\omega_n^2}{\left(s+\varepsilon_p\omega_n + j\omega_n\sqrt{1-\varepsilon_p^2}\right)\left(s+\varepsilon_p\omega_n - j\omega_n\sqrt{1-\varepsilon_p^2}\right)}$$

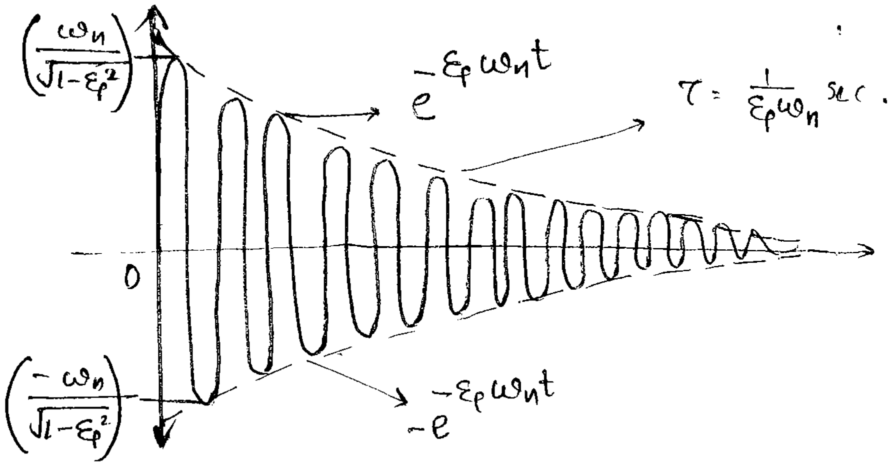
$$c(s) = \frac{\omega_n^2}{(s+\varepsilon_p\omega_n)^2 + \left(\omega_n\sqrt{1-\varepsilon_p^2}\right)^2}$$

ILT

$$c(t) = \frac{\omega_n^{\cancel{2}}\, e^{-\varepsilon_p\omega_n t}}{\cancel{\omega_n}\sqrt{1-\varepsilon_p^2}} \sin\left(\omega_n\sqrt{1-\varepsilon_p^2}\,t\right)$$

$$c(t) = \left(\frac{\omega_n}{\sqrt{1-\varepsilon_p^2}}\right) e^{-\varepsilon_p\omega_n t} \sin\left(\omega_n\sqrt{1-\varepsilon_p^2}\,t\right)$$

Response is Exponential ~~frequ~~ decay frequency of operations.



Damped Oscillations.

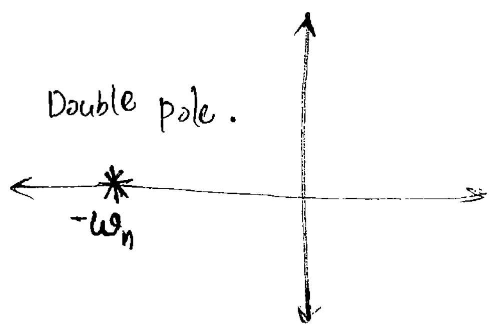
→ when $0<\varepsilon_p<1$, the poles lies in the left hand side which are complex conjugate, the system response is exponential decay, frequency of oscillations which are called damped oscillations.

→ Any system which produce damped oscillations is called Underdamped System.

Case III $\varepsilon_p = 1$ CRITICALLY DAMPED

$$C(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{(s+\omega_n)^2}$$



Time Constant $\tau = \frac{-1}{Re(pole)}$

$$= \frac{-1}{-\omega_n}$$

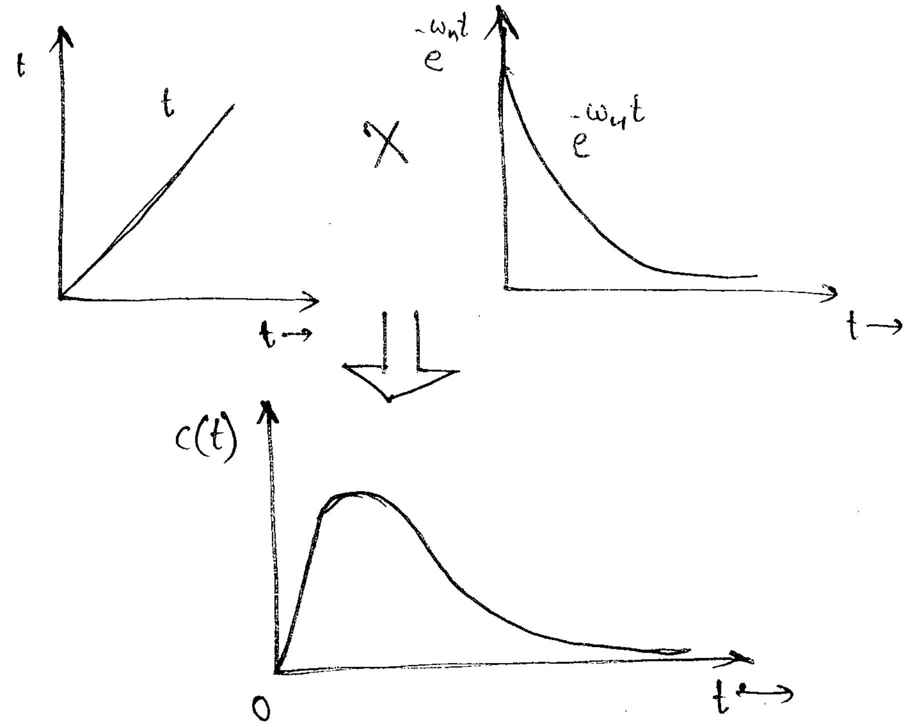
$$\boxed{\tau = \frac{1}{\omega_n}}$$

Frequency of oscillation = 0

$$C(s) = \frac{\omega_n^2}{(s+\omega_n)^2}$$

ILT

$$c(t) = \omega_n^2\left(t\, e^{-\omega_n t}\right)$$



→ when $\xi = 1$, Both the poles lies on the negative real axis at the same location. The system is stable. The system response is critical damped system because it generates critically one damped oscillation.

Case IV $\xi > 1$ OVERDAMPED SYSTEM.

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$$

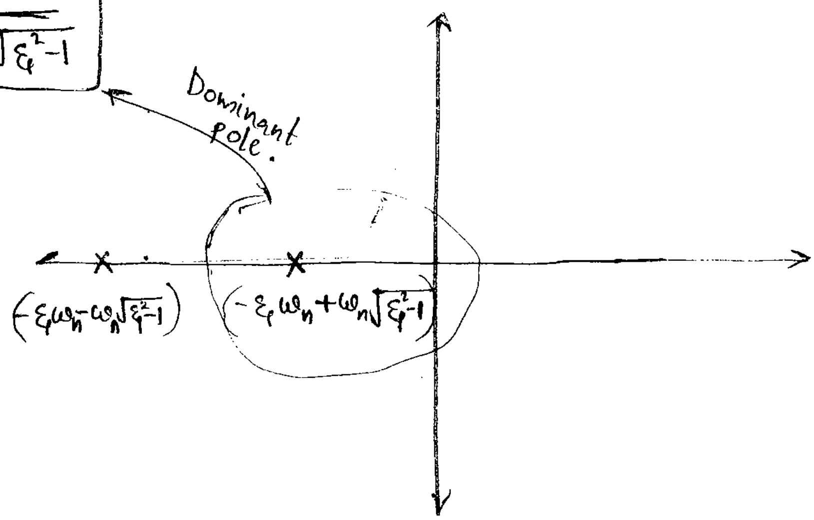
$$s_1, s_2 = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$s_1, s_2 = -\xi_p\omega_n \pm \omega_n\sqrt{\xi_p^2-1}$$

$$C(s) = \frac{\omega_n^2}{\left(s+\xi_p\omega_n \pm \omega_n\sqrt{\xi_p^2-1}\right)\left(s+\xi_p\omega_n \mp \omega_n\sqrt{\xi_p^2-1}\right)}$$

$$\boxed{\text{Time constant } \tau = \frac{1}{\xi_p\omega_n - \omega_n\sqrt{\xi_p^2-1}}}$$

$$\boxed{\text{F.O.O} = 0 \text{ r/sec}}$$



$$C(s) = \frac{K_1}{\left(s+\xi_p\omega_n+\omega_n\sqrt{\xi_p^2-1}\right)} + \frac{K_2}{\left(s+\xi_p\omega_n-\omega_n\sqrt{\xi_p^2-1}\right)}$$

$$C(s) = \frac{\omega_n}{2\sqrt{\xi_p^2-1}}\left[\frac{-1}{s+\xi_p\omega_n+\omega_n\sqrt{\xi_p^2-1}} + \frac{1}{s+\xi_p\omega_n-\omega_n\sqrt{\xi_p^2-1}}\right]$$

$$\cancel{-\xi_p\omega_n-\omega_n\sqrt{\xi^2-1}+\xi_p\omega_n-\omega_n\sqrt{\xi_p^2-1}}$$

$$K_1\left(0-2\omega_n\sqrt{\xi_p^2-1}\right) = \omega_n^2$$

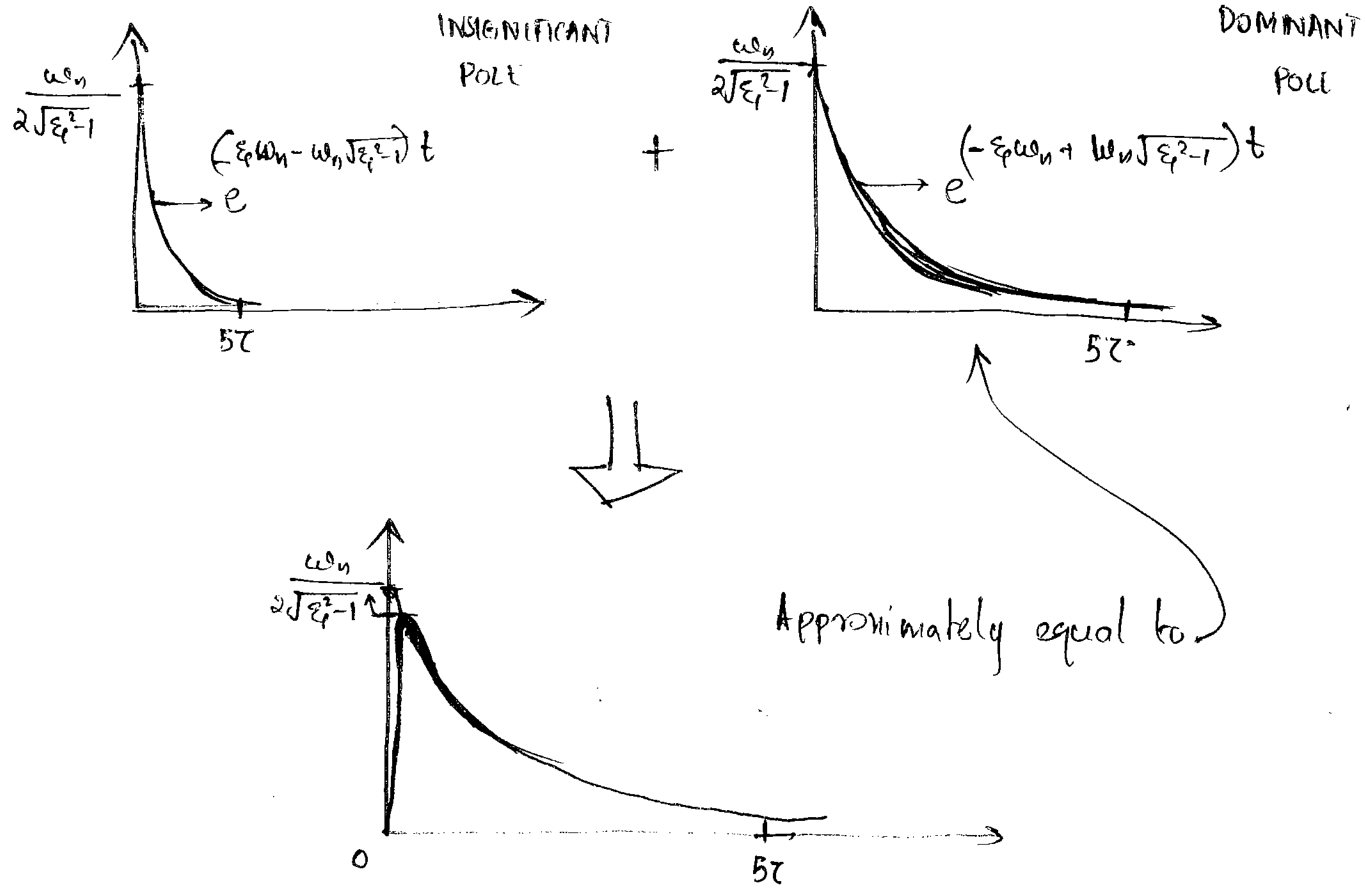
$$K_1 = \frac{-\omega_n}{2\sqrt{\xi_p^2-1}}$$

$$K_1 = -K_2$$

$$K_2 = \frac{\omega_n}{2\sqrt{\xi_p^2-1}}$$

ILT

$$\boxed{c(t) = \frac{\omega_n}{2\sqrt{\xi_p^2-1}}\left[-e^{\left(-\xi_p\omega_n-\omega_n\sqrt{\xi_p^2-1}\right)t} + e^{\left(-\xi_p\omega_n+\omega_n\sqrt{\xi_p^2-1}\right)t}\right]}$$

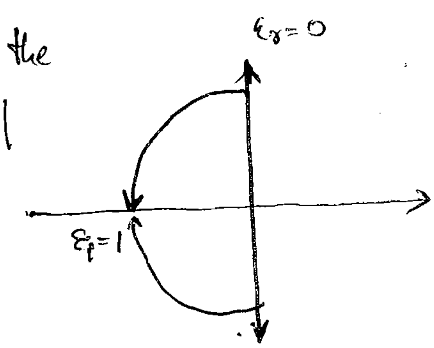


→ when $\xi_r > 1$, both the poles lies in the left hand side at different locations. The system is stable. The system response is called over damped system because the system eliminates or overcomes the damped oscillations.

CONCLUSIONS

1. when ξ_r increased from -1 to 1, then the second order systems poles path is circle. of radius ω_n

2. when ξ_r increases from 0 to 1, then both the poles moves towards the left and near to the real axis. In this case

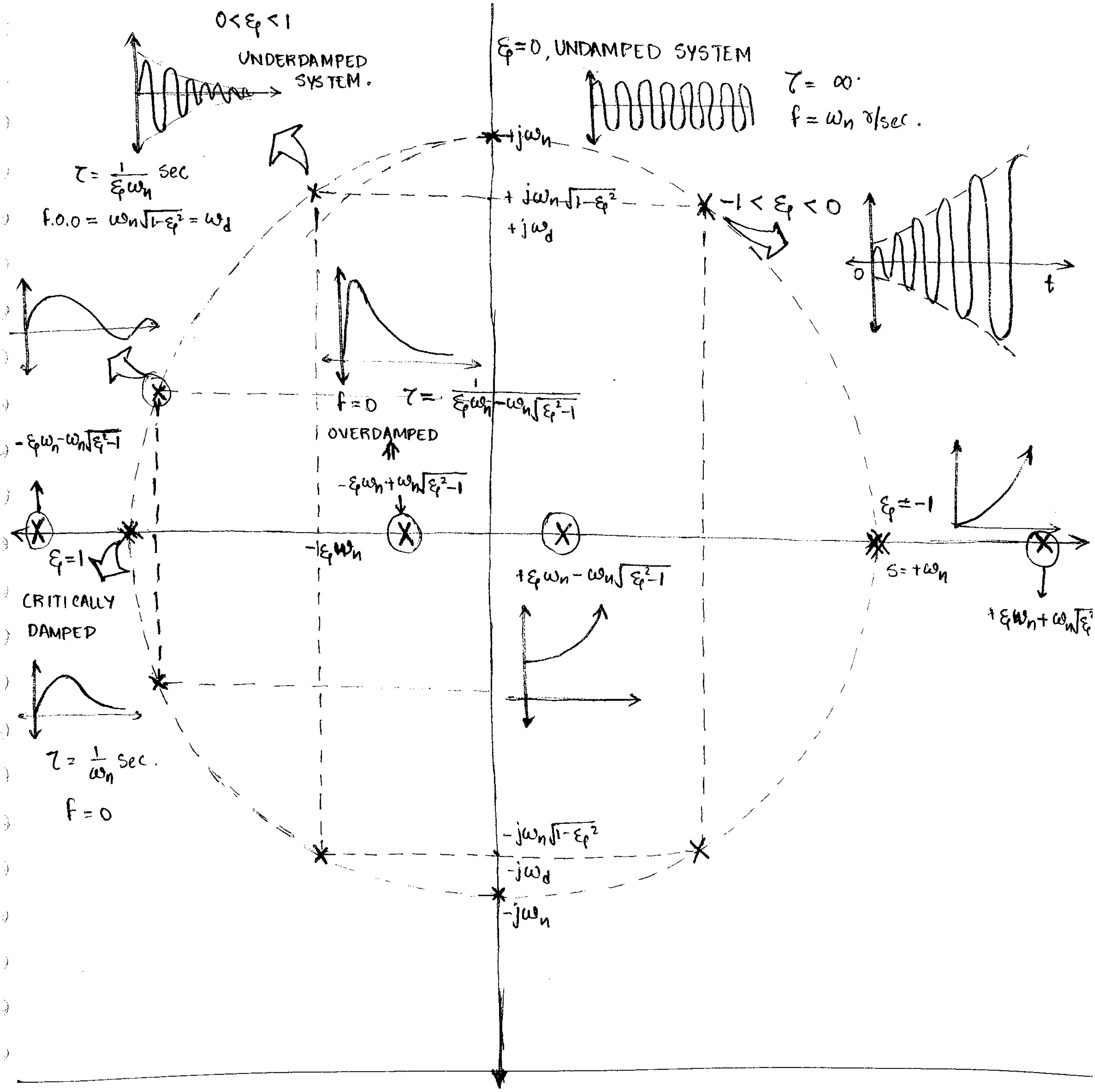


step 1 : Time constant decreases.

step 2 : Settling time decreases.

step 3 : Damped oscillation decreases.

Specifications delay time (td), rise time(tr)



step 5 : Relative stability increases.

3. when $\xi \geq 1$ and increases then one pole moves towards the origin on the real axis. In this case

step 1 : Time constant increases.

step 2 : Settling time increases.

step 3 : Damped oscillation becomes zero.

step 4 : Relative stability decreases.

4. ORDER OF TIME CONSTANTS

Largest sluggish Resp

Medium Resp

Smallest τ

$$\tau_{UNDAMPED} > \tau_{OVERDAMPED} > \tau_{UNDERDAMPED} > \tau_{CRITICAL\ DAMPER}$$

$$\underbrace{\tau_{UNDAMPED}}_{\text{MARGINALLY STABLE}} \quad \underbrace{\tau_{OVERDAMPED} > \tau_{UNDERDAMPED} > \tau_{CRITICAL\ DAMPER}}_{\text{STABLE}}$$

$$\infty > \frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}} > \frac{1}{\xi\omega_n} > \frac{1}{\omega_n}$$

UNIT STEP RESPONSE

$$r(t) = 1 \times u(t)$$

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{\omega_n^2}{s(s^2+2\omega_n\xi s+\omega_n^2)}$$

Case I $\quad \xi = 0 \quad$ UNDAMPED SYSTEM

$$C(s) = \frac{\omega_n^2}{s(s^2+\omega_n^2)}$$

~~$(s+j\omega_n)$~~ $(s$

$$= \frac{A}{s} + \frac{B}{s+j\omega_n} + \frac{C}{s-j\omega_n}$$

$$A(s+j\omega_n)(s-j\omega_n) + Bs(s-j\omega_n) + Cs(s-j\omega_n)$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+j\omega_n^2}$$

$$A + B = 0$$

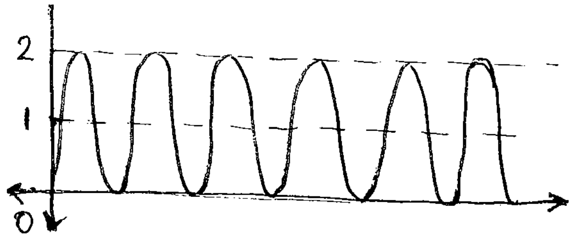
$$A(s^2+j\omega_n^2) + (Bs+C)s = \omega_n^2$$

$$= \frac{1}{s} + \frac{-s}{s^2+\omega_n^2}$$

$$\omega_n^2 A = \omega_n^2 \qquad A = 1$$

ILT

$$c(t) = \left(1 - \cos(\omega_n t)\right)$$



UNDAMPED SYSTEM

Marginally stable.

Case II $0 < \xi < 1$ UNDERDAMPED SYSTEM

$$C(s) = \frac{\omega_n^2}{s\left(s^2 + 2\xi\omega_n + \omega_n^2\right)}$$

$$\cancel{s_1 s_2 = -\xi\omega_n}$$

$$C(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\xi\omega_n + \omega_n^2}$$

$$c(s) = \frac{A\left(s^2 + 2\xi\omega_n s + \omega_n^2\right) + s\left(Bs + C\right)}{s^2 + 2\xi\omega_n + \omega_n^2}$$

$$A\,\omega_n^2 = \omega_n^2$$

$$A = 1$$

$$-2\xi\omega_n = C$$

$$A + B = 0 \qquad B = -A$$

$$B = -1$$

$$c(s) = \frac{1}{s} + \frac{-s - 2\xi\omega_n}{s^2 + 2\omega_n\xi s + \omega_n^2}$$

$$\cancel{c(s) = \frac{1}{s} +}$$

$$s_1\, s_2 = -\xi_\rho \omega_n \pm j\omega_n\sqrt{1-\xi_\rho^2}$$

$$c(s) = \frac{1}{s} + \frac{-(s + 2\xi_\rho\omega_n)}{(s+\xi_\rho\omega_n)^2 + (\omega_n\sqrt{1-\xi_\rho})^2}$$

$$\cancel{= \frac{1}{s} - \frac{s}{(s+\xi_\rho\omega_n)^2+(\omega_n\sqrt{1-\xi_\rho})^2} + \frac{2\xi_\rho\omega_n}{(s+\xi_\rho\omega_n)^2+(\omega_n\sqrt{1-\xi_\rho})^2}}$$

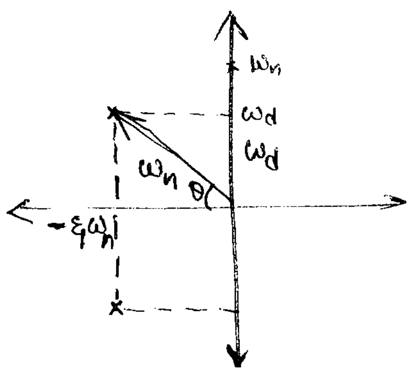
$$= \frac{1}{s} - \left[\frac{s+\xi_\rho\omega_n}{(s+\xi_\rho\omega_n)^2+(\omega_n\sqrt{1-\xi_\rho})^2} + \frac{\frac{\xi}{\sqrt{1-\xi_\rho^2}}(\omega_n\sqrt{1-\xi_\rho^2})}{(s+\xi_\rho\omega_n)^2+(\omega_n\sqrt{1-\xi_\rho})^2}\right]$$

$$= 1 - e^{-\xi_\rho\omega_n t}\cos(\omega_n\sqrt{1-\xi_\rho})t + \frac{\xi_\rho}{\sqrt{1-\xi_\rho^2}} e^{-\xi_\rho\omega_n t}\sin(\omega_n\sqrt{1-\xi_\rho})t$$

$$= 1 - \frac{e^{-\xi_\rho\omega_n t}}{\sqrt{1-\xi_\rho^2}}\left[\sqrt{1-\xi_\rho^2}\cos(\omega_n\sqrt{1-\xi_\rho})t + \xi_\rho\sin(\omega_n\sqrt{1-\xi_\rho})t\right]$$

$$= 1 - \frac{e^{-\xi_\rho\omega_n t}}{\sqrt{1-\xi_\rho^2}}\left[\sqrt{1-\xi_\rho^2}\cos(\omega_d)t + \xi_\rho\sin(\omega_d)t\right]$$

consider



$$\cos\theta = \frac{\xi_\rho\omega_n}{\omega_n} = \xi_\rho$$

$$\sin\theta = \frac{\omega_d}{\omega_n} = \frac{\cancel{\omega_n}\sqrt{1-\xi_\rho^2}}{\cancel{\omega_n}} = \sqrt{1-\xi_\rho^2}$$

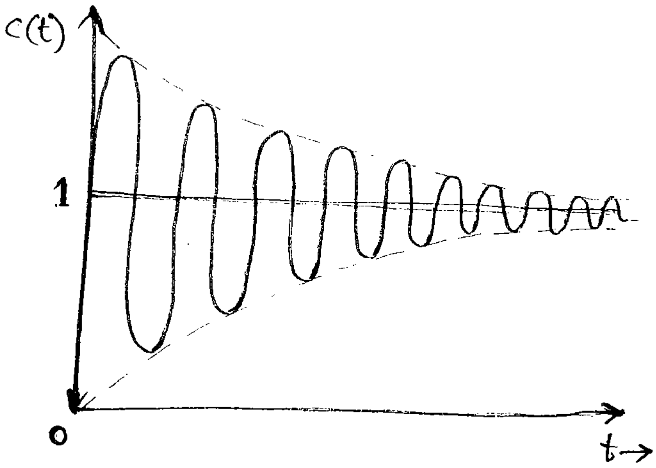
$$\tan\theta = \frac{\sqrt{1-\xi_\rho^2}}{\xi_\rho}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\theta \cos\omega_d t + \cos\theta \sin\omega_d t\right]$$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)}$$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n\sqrt{1-\xi^2}\, t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right)}$$

where $\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$



Case III $\xi = 1$

$$\frac{C(s)}{\cancel{R(s)}} = \frac{\omega_n^2}{s(s^2+2\omega_n s+\omega_n^2)}$$

$$C(s) = \frac{\omega_n^2}{(s+\omega_n)^2} \times \frac{1}{s}$$

$$\frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{(s+\omega_n)}$$

$$\omega_n^2 = A(s+\omega_n)^2 + Bs + Cs(s+\omega_n)$$

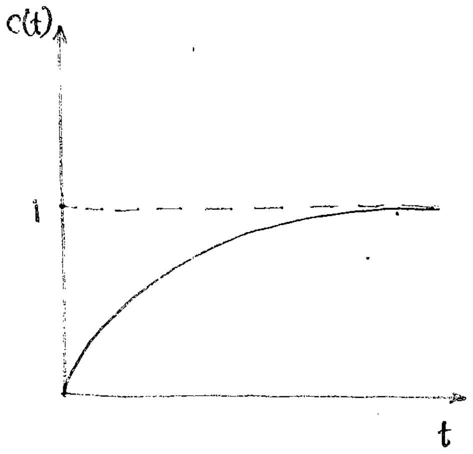
$s=0$ $\quad \omega_n^2 = A\cdot\omega_n^2 \quad \therefore \underline{A=1}$

$0 = A+C \quad \therefore \underline{C=-1} \quad \omega_n^2 = B_n \cdot -\omega_n \quad \therefore \underline{B = -\omega_n}$

$$C(s) = \frac{1}{s} + \frac{-\omega_n}{(s+\omega_n)^2} + \frac{-1}{(s+\omega_n)}$$

$$c(t) = 1 - \omega_n t\, e^{-\omega_n t} + -e^{-\omega_n t}$$

$$= 1 - e^{-\omega_n t}\left[1+\omega_n t\right]$$



Case IV OVERDAMPED SYSTEM $(\xi > 1)$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \times \frac{1}{s}$$

$$s = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\times 1\times\omega_n^2}}{2} = \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2-1}}{2}$$

$$s_1 s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$C(s) = \frac{\omega_n^2}{s\left(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)\left(s+\xi\omega_n+\omega_n\sqrt{\xi^2-1}\right)} = \frac{\omega_n^2}{s\left(s^2+2\xi\omega_n s+\omega_n^2\right)}$$

$$\cancel{= \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+}}$$

$$C(s) = \frac{A}{s} + \frac{B}{\left(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)} + \frac{C}{\left(s+\xi\omega_n+\omega_n\sqrt{\xi^2-1}\right)}$$

$$= \frac{A\left(s^2+2\xi\omega_n s+\omega_n^2\right) + Bs\left(s+\xi\omega_n+\omega_n\sqrt{\xi^2-1}\right) + Cs\left(s+\xi\omega_n-\omega_n\sqrt{\xi^2-1}\right)}{s\left(s^2+2\xi\omega_n s+\omega_n^2\right)}$$

$s=0$ $\quad \omega_n^2 = A\omega_n^2 \quad \therefore A=1$

$0 = A+B+C \quad ; \quad B+C=-1$

$s' \Rightarrow 0 = A \cdot 2\xi_1\omega_n + B\left(\xi_1\omega_n + \omega_n\sqrt{\xi_1^2-1}\right) + C\left(\xi_1\omega_n - \omega_n\sqrt{\xi_1^2-1}\right)$

$$0 = 2\xi_1\omega_n + \xi_1\omega_n(B+C) + \omega_n\sqrt{\xi_1^2-1}\,(B-C)$$

$$\therefore\ B-C = \frac{-\xi_1}{\sqrt{\xi_1^2-1}}$$

$$B+C = -1$$

$$2B = -\left[1+\frac{\xi_1}{\sqrt{\xi_1^2-1}}\right] \qquad \therefore\ B = -\frac{1}{2}\left[\frac{\sqrt{\xi_1^2-1}+\xi_1}{\sqrt{\xi_1^2-1}}\right]$$

$$C = -1-B = -1+\frac{1}{2}\left[\frac{\sqrt{\xi_1^2-1}+\xi_1}{\sqrt{\xi_1^2-1}}\right]$$

$$= \frac{-2\sqrt{\xi_1^2-1}+\sqrt{\xi_1^2-1}+\xi_1}{\sqrt{\xi_1^2-1}} = \underline{\underline{\frac{-\sqrt{\xi_1^2-1}+\xi_1}{2\cdot\sqrt{\xi_1^2-1}}}}$$

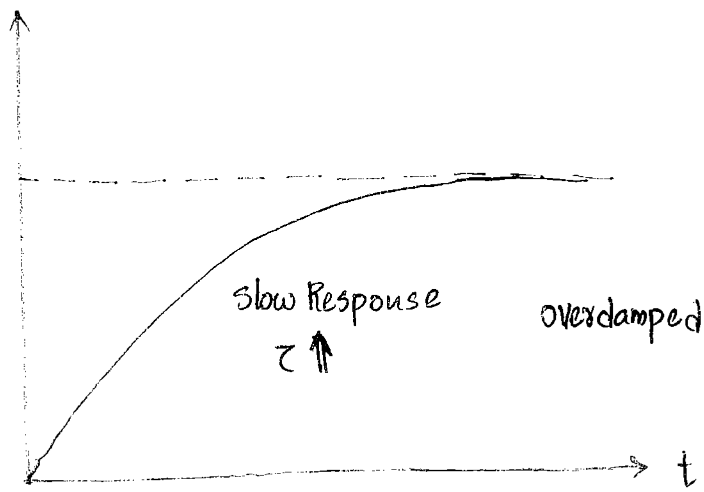
$$B = -\frac{1}{2}\left[1+\frac{\xi_1}{\sqrt{\xi_1^2-1}}\right] \qquad C = \frac{1}{2}\left[\frac{\xi_1}{\sqrt{\xi_1^2-1}}-1\right]$$

$$C(s) = \frac{1}{s} + \frac{\frac{1}{2}\left[1+\frac{\xi_1}{\sqrt{\xi_1^2-1}}\right]}{\left(s+\xi_1\omega_n-\omega_n\sqrt{\xi_1^2-1}\right)} + \frac{\frac{1}{2}\left[\frac{\xi_1}{\sqrt{\xi_1^2-1}}-1\right]}{\left(s+\xi_1\omega_n+\omega_n\sqrt{\xi_1^2-1}\right)}$$

$$\boxed{c(t) = 1-\frac{1}{2}\left[1+\frac{\xi_1}{\sqrt{\xi_1^2-1}}\right]e^{-\left(\xi_1\omega_n-\omega_n\sqrt{\xi_1^2-1}\right)t} + \frac{1}{2}\left[-1+\frac{\xi_1}{\sqrt{\xi_1^2-1}}\right]e^{-\left(\xi_1\omega_n+\omega_n\sqrt{\xi_1^2-1}\right)t}}$$

At $t=0$ $\quad c(t) = 1-\frac{1}{2}\left[1+\frac{\xi_1}{\sqrt{\xi_1^2-1}}\right]+\frac{1}{2}\left[-1+\frac{\xi_1}{\sqrt{\xi_1^2-1}}\right] = 0$

At $t=\infty$ $\quad c(t) = 1$



Slow Response

$\tau \Uparrow$

overdamped s/m.

TIME DOMAIN SPECIFICATIONS

For time domain specifications, select underdamped system and unit step response.

	T.R	S.S	STABILITY
Impulse	✓	✗	✓
step	✓	✓	✓
Ramp	✓	✓	✗
Parabolic	✓	✓	✗

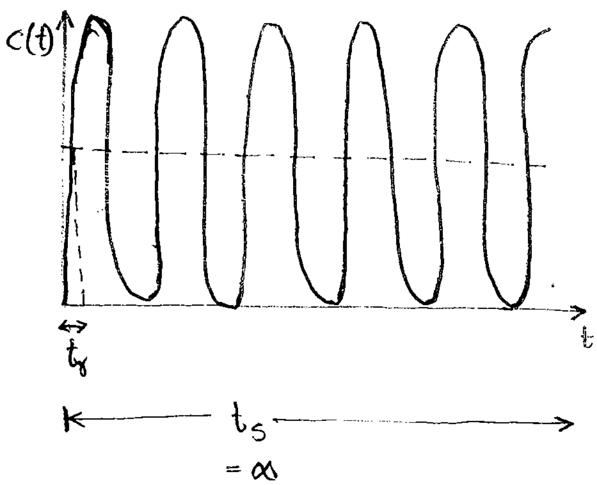
Impulse ⟹ Practically does not exist.

step ⟹ Practically exist & widely used. Provides complete analysis of the s/m.

Ramp, Parabolic: Unbounded i/p.

~~$\xi = 0$ un~~

$\xi = 0$ UNDAMPED

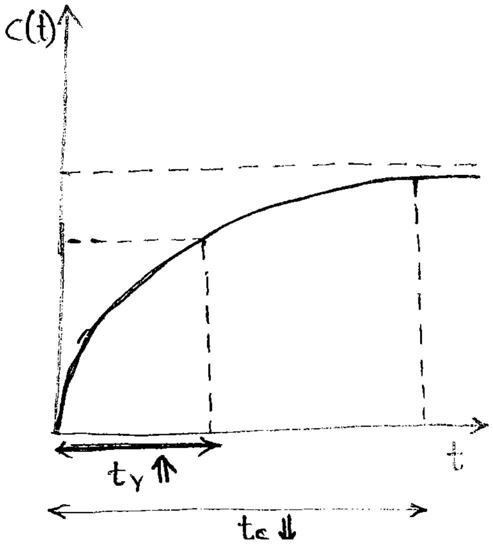


LOW t_r

INFINITE t_s

Not preffered.

$\xi \geq 1$ CRITICAL | OVERDAMPED

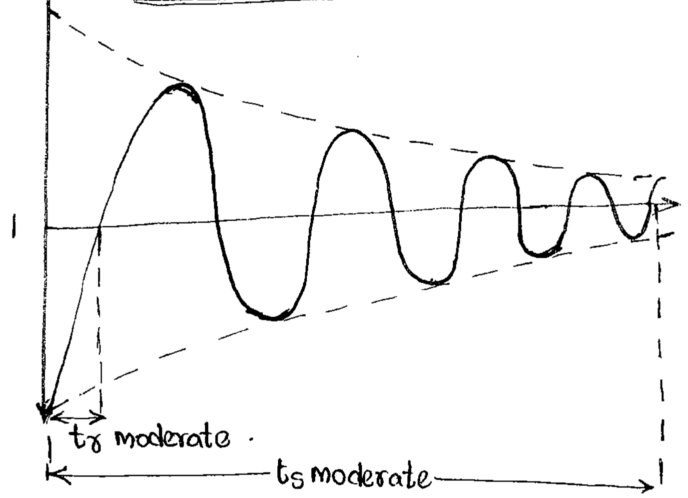


HIGH t_r

LOW t_s

Not preferred.

UNDERDAMPED



PREFERRED t_s and t_r

Optimum Value of ξ ; $\boxed{0.4 < \xi < 0.7}$

⇒ Underdamped systems are selected for Time domain Systems, because if we select the undamped system, the rise time is very small, but settling time is ∞, whereas if we select critical and overdamped system, the rise time is very large, but settling time is very small

⇒ Practically for any s/m, requires Smallest rise time and Smallest settling time.

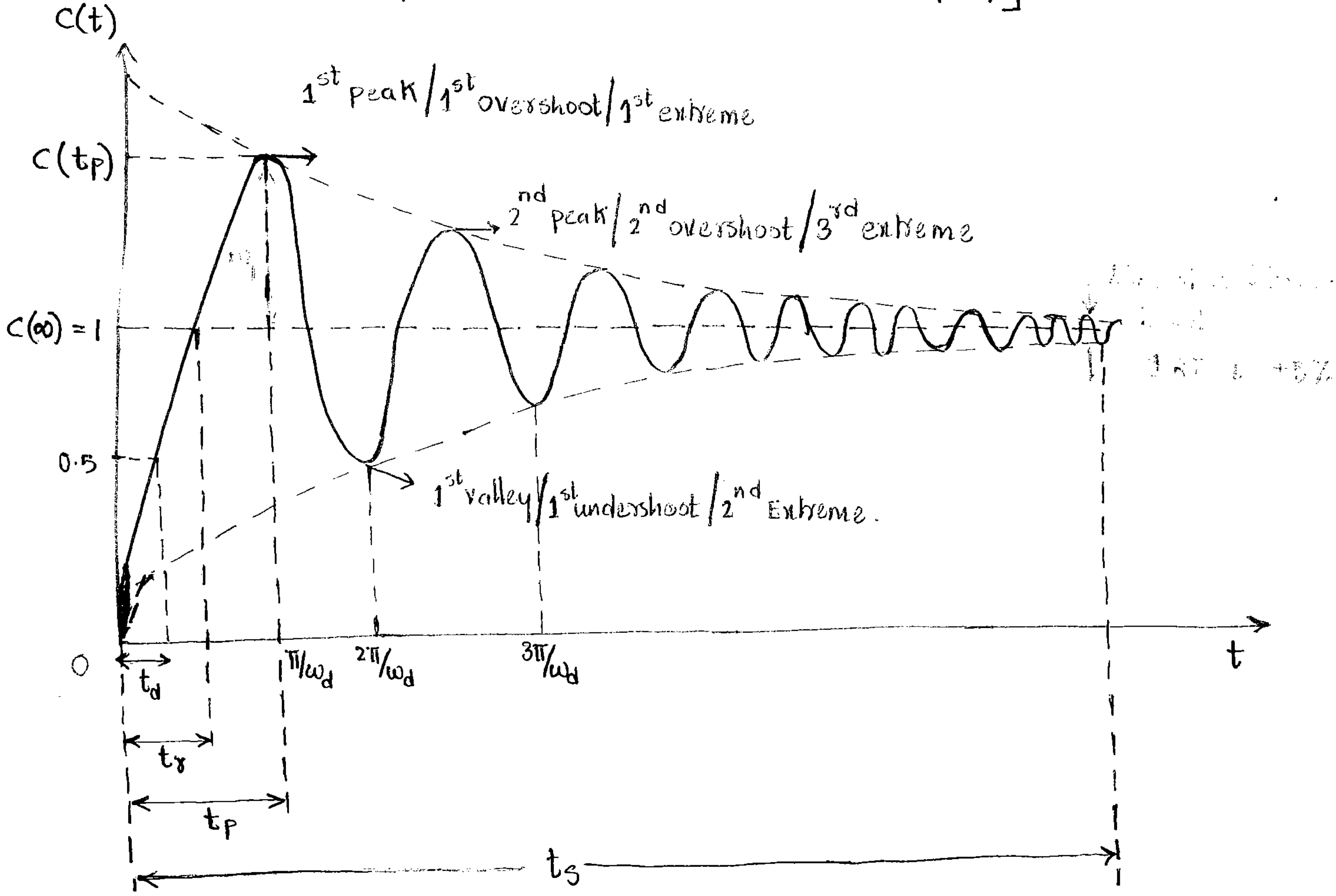
⇒ In underdamped system, we can get the moderate values of rise time and settling time.

⇒ To Implement any second order system, the ξ values selected should be b/w 0.4 to 0.7

For $0<\xi_\rho<1$, the unit step response of a second order system is

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$= 1 - \frac{e^{-\xi_\rho\omega_n t}}{\sqrt{1-\xi_\rho^2}} \sin\left[\left(\omega_n\sqrt{1-\xi_\rho^2}\right)t + \tan^{-1}\left(\frac{\sqrt{1-\xi_\rho^2}}{\xi_\rho}\right)\right]$$



DELAY TIME (t_d)

It is the time taken by the response to rise from 0 to 50 % of the final value.

$$\boxed{t_d = \frac{1+0.7\,\xi_\rho}{\omega_n}} \text{ s}$$

RISE TIME (t_r)

It is the time taken by the response to rise from

0 to 100% for underdamped s/m.

5 to 95% for critical damped s/m.

10 to 90% for overdamped s/m.

$$\boxed{t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_d}} \qquad \boxed{t_r = \frac{\pi - \cos^{-1}(\xi)}{\omega_d}}$$

PEAK TIME (t_P)

It is the time required for the response to rise from 0 to peaks of the time response.

$$\boxed{t_p = \frac{n\pi}{\omega_d}} \text{ s}$$ $n=1$ by default. (1st peak)

$$\boxed{t_p = \frac{\pi}{\omega_d}} \text{ s}$$

→ 2nd peak, $t_p = \frac{3\pi}{\omega_d}$ s

→ 1st valley, $t_p = \frac{2\pi}{\omega_d}$ s

PEAK OVERSHOOT (M_P)

It is the difference b/w time response peak to steady state value.

$$\boxed{M_p = c(t_P) - c(\infty)}$$

% OF PEAK OVERSHOOT

It gives the normalized difference b/w time response peak to steady state value.

$$\boxed{\% M_p = \frac{c(t_P) - c(\infty)}{c(\infty)} \times 100\%}$$

$$\% M_p = \left[\frac{c(t_p)-1}{c(\infty)}\right] \times 100$$

$$\% M_p = e^{-\left(n\pi \xi/\sqrt{1-\xi^2}\right)} \times 100\%$$ n value similar to peak time.

$$\boxed{\% M_p = e^{\left(-\xi\pi/\sqrt{1-\xi^2}\right)} \times 100\%}$$

The undershoot to the 1st valley point is

$$\boxed{\% M_p = e^{\left(-2\xi\pi/\sqrt{1-\xi^2}\right)} \times 100\%}$$

SETTLING TIME (t_s)

It is the time required for the response to rise from 0 to specified tolerance band. Usually $\pm 2\%$ ~~to~~ or $\pm 5\%$

$\pm 5\%$	$t_s = 3\tau = \frac{3}{\xi\omega_n}$ sec.	
$\pm 2\%$	$t_s = 4\tau = \frac{4}{\xi\omega_n}$ sec.	
0%	$t_s = 5\tau = \frac{5}{\xi\omega_n}$ sec.	

TIME PERIOD OF OSCILLATIONS

$$T_{osc} = \frac{2\pi}{\omega_d} = 2 \times \frac{\pi}{\omega_d} = 2 \cdot t_p$$

$$\boxed{T_{osc} = 2t_p}$$

Number of oscillations before reaching steady state.

$$N = \frac{t_s(\pm 2\% \text{ or } \pm 5\%)}{T_{osc}} = \frac{t_s}{2\pi/\omega_d} = \frac{t_s}{2t_p}$$

$$\boxed{N = \frac{t_s}{2t_p}}$$

(i) Rise Time.

$$c(t) = 1 - \frac{e^{-\xi_p \omega_n t}}{\sqrt{1-\xi_p^2}} \sin(\omega_d t + \theta)$$

$t = t_r$, $c(t_r) = 1$

$$c(t) = 1 - \frac{e^{-\xi_p \omega_n t}}{\sqrt{1-\xi_p^2}} \sin\left(\omega_d t + \tan^{-1}\left(\frac{\sqrt{1-\xi_p^2}}{\xi_p}\right)\right)$$

std form of unit step respons for underdamped s/m.

$$\cancel{1} = \cancel{1} - \frac{e^{-\xi_p \omega_n t_r}}{\sqrt{1-\xi_p^2}} \sin\left[\omega_d t_r + \tan^{-1}\left(\frac{\sqrt{1-\xi_p^2}}{\xi_p}\right)\right]$$

$$0 = \frac{-e^{-\xi_p \omega_n t_r}}{\sqrt{1-\xi_p^2}} \sin\left[\omega_d t_r + \tan^{-1}\left(\frac{\sqrt{1-\xi_p^2}}{\xi_p}\right)\right]$$

$$\sin\left[\omega_d t_r + \tan^{-1}\left(\frac{\sqrt{1-\xi_p^2}}{\xi_p}\right)\right] = 0 = \sin\pi$$

$$\omega_d t_r + \tan^{-1}\left(\frac{\sqrt{1-\xi_p^2}}{\xi_p}\right) = \pi$$

π ?

$$\therefore \boxed{t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\xi_p^2}}{\xi_p}\right)}{\omega_d}}$$

(ii) Peak Time.

At $t = t_p$, the response $c(t)$ should be max.

$$c(t) = 1 - \frac{e^{-\xi_p \omega_n t}}{\sqrt{1-\xi_p^2}} \sin(\omega_d t + \theta)$$

$$\frac{dc(t)}{dt} = \frac{-1}{\sqrt{1-\xi_p^2}}\left[e^{-\xi_p \omega_n t_p} \cos(\omega_d t_p + \theta)\,\omega_d + \sin(\omega_d t_p + \theta)\, e^{-\xi_p \omega_n t_p}\, (-\xi_p \omega_n)\right] = 0$$

$$\cos(\omega_d t_p + \theta)\,\omega_d = \sin(\omega_d t_p + \theta)\,\xi_p \omega_n$$

$$\frac{\omega_d}{\omega_n \xi_p} = \tan(\omega_d t_p + \theta)$$

$$\frac{\sqrt{1-\xi_p^2}}{\xi_p} = \tan(\omega_d t_p + \theta) \Rightarrow \tan(\omega_d t_p + \theta) = \tan\theta$$

$\because \tan\theta = \frac{\sqrt{1-\xi}}{\xi_p}$

From trignometric identity

$$\tan(\omega_d t_p + \theta) = \tan(n\pi + \theta)$$

$$\omega_d t_p = n\pi$$

$$\therefore \boxed{t_p = \frac{n\pi}{\omega_d}}$$

(iii) <u>Overshoot</u>

$$M_p = c(t_p) - c(\infty)$$

$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\,\%$$

$$c(t_p) = 1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$$

$$\% M_p = \frac{\cancel{1} - \frac{e^{-\xi_p \omega_n t_p}}{\sqrt{1-\xi_p^2}} \sin(\omega_d t_p + \theta) \cancel{-1}}{1} \times 100\%$$

$$= -\frac{e^{-\xi_p \omega_n t_p}}{\sqrt{1-\xi_p^2}} \sin(\omega_d t_p + \theta) \times 100\%$$

$$= -\frac{e^{-\frac{\xi_p \cancel{\omega_n} n\pi}{\cancel{\omega_n}\sqrt{1-\xi_p^2}}}}{\sqrt{1-\xi_p^2}} \sin\left(\frac{\cancel{\omega_n \sqrt{1-\xi_p^2}} \times n\pi}{\cancel{\omega_n \sqrt{1-\xi_p^2}}} + \tan^{-1}\left(\frac{\sqrt{1-\xi_p^2}}{\xi_p}\right)\right) \times 100\%$$

$$= -\frac{e^{-(n\xi_p\pi/\sqrt{1-\xi_p^2})}}{\sqrt{1-\xi_p^2}} \sin\left[n\pi + \theta\right] \times 100\% = -\frac{e^{-\frac{n\xi_p\pi}{\sqrt{1-\xi_p^2}}}}{\sqrt{1-\xi_p^2}} \sin(-\theta) \times 100\,\%$$

$$= \frac{e^{-n\xi_p\pi/\sqrt{1-\xi_p^2}}}{\sqrt{1-\xi_p^2}} \times \sin\theta. = \frac{e^{-n\xi_p\pi/\sqrt{1-\xi_p^2}}}{\cancel{\sqrt{1-\xi_p^2}}} \times \cancel{\sqrt{1-\xi_p^2}} \times 100\%$$

$$\boxed{\% M_p = e^{-n\xi_p\pi/\sqrt{1-\xi_p^2}} \times 100\%}$$

(iv) Settling time:

$\pm 2\% \Longrightarrow$ ie At $t = t_s$, $c(t) = 0.98$

At $t = t_s$, the oscillations completely becomes zero. Hence we require to consider only exponential terms in the response.

ie, $c(t) = 1 - e^{-\xi \omega_n t_s} = \left[1 - e^{-t_s/\tau}\right]$?

At $t = t_s$, $c(t) = 0.98$, $0.98 = 1 - e^{-t_s/\tau}$ with $\boxed{\tau = \frac{1}{\xi \omega_n}}$

ie, $0.02 = e^{-t_s/\tau}$

$$-\frac{t_s}{\tau} = \ln(0.02)$$

$$-t_s = -3.91\tau$$

$\boxed{t_s = 3.91\tau} \approx \boxed{t_s \simeq 4\tau}$ $\boxed{t_s = \frac{4}{\xi \omega_n}}$

$\pm 5\% \Longrightarrow$ ie At $t = t_s$, $c(t) = 0.95$

$$c(t) = 1 - e^{-t_s/\tau} = 1 - e^{-\xi \omega_n t_s}$$

$$0.95 = 1 - e^{-\xi \omega_n t_s}$$

$$0.05 = e^{-\xi \omega_n t_s}$$

$$-\frac{t_s}{\tau} = \ln(0.05) = -3$$

$\boxed{t_s = 3\tau}$ $\boxed{t_s = \frac{3}{\xi \omega_n}}$

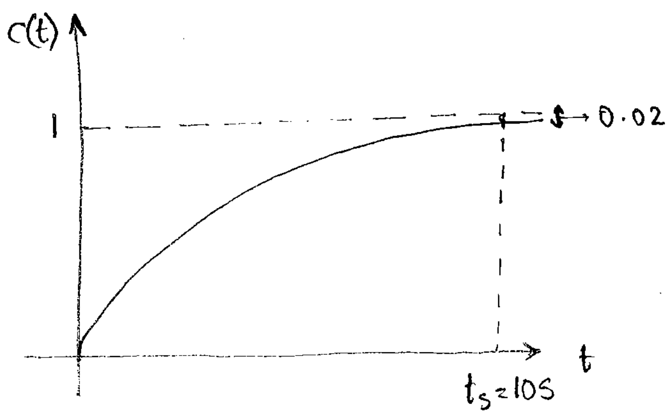
Q) The unit step response of the s/m is shown in figure. Find the following factors.

i) τ

ii) Delay time.

iii) t_r

iv) t_p and M_p



(i) Given tolerance = 0.02.

$\therefore t_s = 4\tau$, $\tau = \frac{t_s}{4} = \frac{10}{4} = \underline{2.5\ s}$

(ii) At $t = t_d$, $c(t) = 0.5$

The standard form of unit step response to the first order system or overdamped system is

$$\boxed{c(t) = K\left(1 - e^{-t/\tau}\right)}$$

$K = 1$ (Steady state Value)

$$0.5 = 1 - e^{-t/\tau}$$

$$0.5 = e^{-t/\tau}$$

$$\frac{-t_d}{\tau} = \ln(0.5) = -0.693$$

$$\therefore t_d = 0.693\tau = \underline{1.732\ s}$$

(iii) Rise time

For rise time, consider the time durations from 10% to 90% of the final value.

10%, $c(t) = 0.1 \Longrightarrow t_{r1}$

90%, $c(t) = 0.9 \Longrightarrow t_{r2}$

$$t_r = t_{r2} - t_{r1}$$

$$c(t) = 1 - e^{-t_{r1}/\tau}$$

$$0.1 = 1 - e^{-t_{r1}/2.5} \Longrightarrow \frac{t_{r1}}{2.5} = 0.105 \Longrightarrow t_{r1} = \underline{0.263\ s}$$

$$0.9 = 1 - e^{-t_{r1}/2.5} \Longrightarrow \frac{t_{r2}}{2.5} = 2.3 \Longrightarrow t_{r2} = \underline{5.75\ s}$$

$t_r = t_{r2} - t_{r1} = \underline{5.487\ s}.$

$t_r = t_{r2} - t_{r1} = \underline{2.2\tau}$

iv) No peaks exists $\therefore$ No peak time and peak overshoot.

Q) The Impulse response of the system is $c(t) = e^{-3t} \sin 4t$

Find the following factors to the unit step response.

(i) t_p

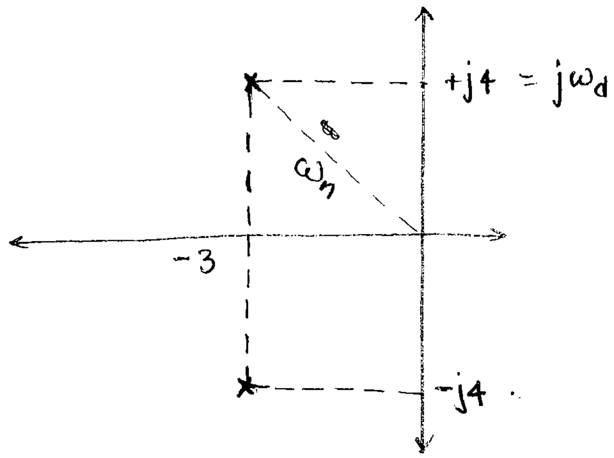
(ii) t_s

(iii) Damping Ratio

(iv) ω_n

(v) t_d, t_r and M_p

$$c(t) = e^{-3t} \sin 4t.$$



$$\tau = \frac{-1}{-3} = \frac{1}{3}\ s.$$

(i) $t_p = \frac{\pi}{\omega_d} = \underline{\frac{\pi}{4}\ s}$

(ii) $t_s = 4\tau$

$$t_s = \frac{4}{3} = \underline{1.33\ s}$$

(iii) ξ $\omega_n \rightarrow$ Distance from origin to pole.

$\therefore \omega_n = 5$ rad/s.

$$-\xi_p \omega_n = -3$$

$$\therefore \xi_p = \frac{3}{\omega_n} = \frac{3}{5} = \underline{\underline{0.6}}$$

(iv) $\omega_n = 5 \text{rad/s}.$

(v) $$t_d = \frac{1 + 0.7\xi}{\omega_n}$$

$$= \frac{1+0.7 \times 0.6}{5} = \underline{\underline{0.284 s}}$$

$$t_r = \frac{\pi - \cos^{-1}(\xi_p)}{\omega_d} = \frac{\pi - \cos^{-1}(0.6)}{5\sqrt{1-0.6^2}}$$

$$= \frac{\pi - 0.927}{5 \times 0.8} = \frac{\pi - 0.927}{4}$$

$$= \underline{\underline{0.5536 s}}$$

$$\% M_p = e^{-\xi_p \pi / \sqrt{1-\xi_p^2}} \times 100\%$$

$$= e^{-0.6\pi/\sqrt{1-0.6^2}} \times 100\%$$

$$= \underline{\underline{9.47\%}}$$

Q Find the percentage of peak overshoot to the given transfer function.

(i) $\dfrac{C(s)}{R(s)} = \dfrac{25}{s^2+25}$

$\xi = 0$ undamped system.

$\% M_p = e^{-\pi \times 0/\sqrt{1-0^2}} \times 100\% = 100\%$

(ii) $\dfrac{C(s)}{R(s)} = \dfrac{100}{s^2+20s+100}$

$\omega_n = 10 r/sec.$

$\xi = 1$

$\therefore$ Critical damped system.

$\Rightarrow \% M_p = e^{-\pi \times 1/\sqrt{1-1}} \times 100 = e^{-\infty} \times 100 = \underline{\underline{0\ \%}}$

Note:

→ As the damping ratio ξ increases from 0 to 1, the percentage of peak overshoot ($\% M_p$) decreases from 100% to 0%.

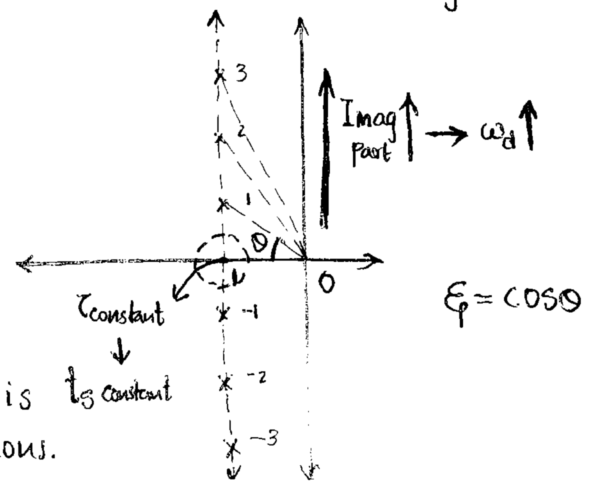
→ when $\xi \geq 1$ and increases, the percentage of peak overshoot is ZERO, because the system response does not consists any oscillations.

Q Find the variations in time domain specifications to the given poles path in s plane.

~~For such qns~~

~~For such qns~~

(For such qns first consider Real part and then Imaginary part.)



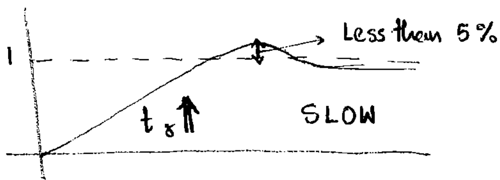
As the real part of poles is constant, the time constant is constant. Hence settling time is same for all pole locations.

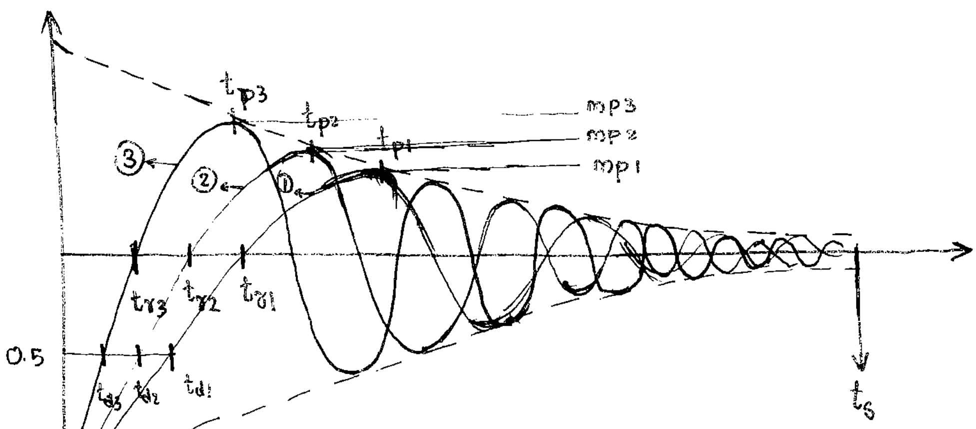
※ As Imaginary part increases, the damped oscillations ω_d increases. As ω_d increases, the time specifications t_d, t_r, t_p must be decreasing. As the inclination of the poles θ increases, the damping ratio ξ decreases. As ξ decreases, the percentage overshoot increases. The large peak overshoot makes the system more oscillatory and less relative stable. ~~The~~

→ The optimum range of percentage peak overshoot is (5 – 40 %)

→ If peak overshoot is more than 40%, the system becomes ~~more osci~~ less relative stable.

→ If peak overshoot is less than 5 %, then the system response become slow or sluggy.





$t_{r3} < t_{r2} < t_{r1}$

$t_{p3} < t_{p2} < t_{p1}$

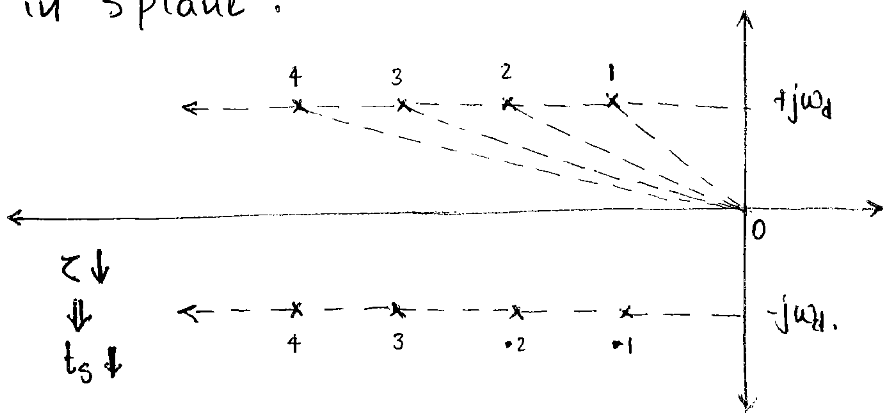
$t_{d3} < t_{d2} < t_{d1}$

$t_p \downarrow$

$t_r \downarrow$

$t_d \downarrow$

Q Find the variations in time domain specifications to given poles path in s plane.

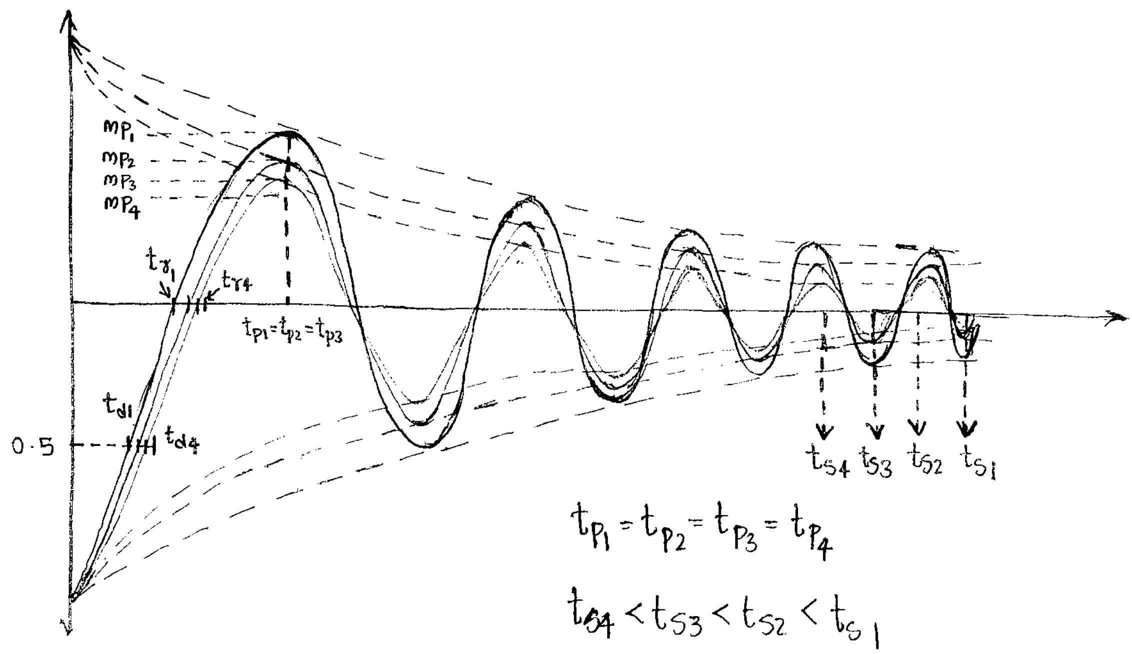


→ poles are moving towards left. so the time constant and the settling time decreases.

→ As imaginary part is constant, the damped oscillations ω_d constant. Hence the peak time is constant.

$$t_p = \text{const} + \frac{\pi}{\omega_d} \longrightarrow \text{Constant}.$$

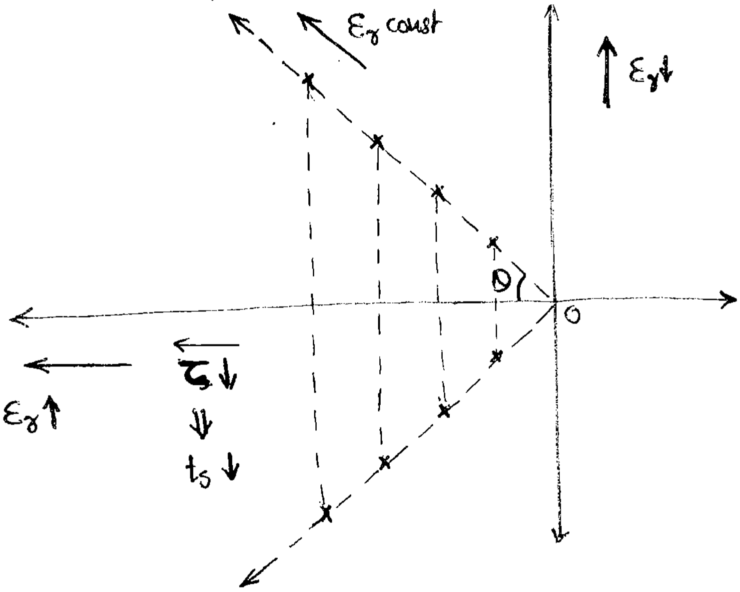
→ But there exist a slight variations in delay and rise time. As the inclination of the poles θ decreases, the damping ratio ξ ~~decrease~~ increases. Hence the percentage peak overshoot decreases, The system become more stable and generates less number of oscillation.



$$t_{p_1} = t_{p_2} = t_{p_3} = t_{p_4}$$

$$t_{s4} < t_{s3} < t_{s2} < t_{s1}$$

Q Find the variation in time domain specifications for given poles path in s plane.



$\theta \rightarrow$ Constant.
$\xi \rightarrow$ constant.
$\% M_p \rightarrow$ constant.

→ As the inclination of poles, θ constant, the damping ratio ξ constant. Hence percentage of peak overshoot constant. So there is no effect on system stability.

→ As the poles moves towards the left, time constant decreases. Hence settling time also decreases.

→ As the imaginary part increases, ω_d increases, t_p, t_r, t_d decreases.

Conclusions: Settling Time depends Only on Real part of Poles.
Peak Time depends only on Imaginary part of poles.
$\% M_p$ depends only on damping ratio inturn inclination θ

Q Find the time domain specifications to the given unity feedback system.

$$G(s) = \frac{25}{s(s+4)}, \quad H(s) = 1$$

$$CLTF = \frac{G(s)}{\quad} = \frac{25}{s(s+4)+25}$$

$$= \frac{25}{s^2+4s+25}.$$

$$\omega_n = 5$$

$$\cancel{\xi = \frac{1}{2}} \qquad 2\omega_n \xi = 4$$

$$\omega_n \xi = 0.5$$

$$\xi = \cancel{\frac{0.5}{5}}\ \underline{0.4}$$

$$t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7\times 0.4}{5} = \frac{1+0.28}{5} = \frac{1.28}{5} = \underline{0.256 \text{ se}}$$

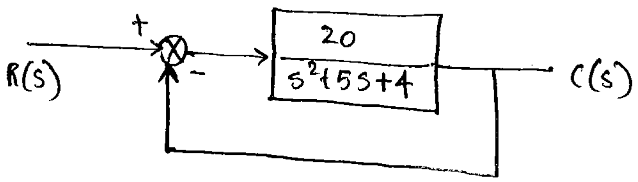
$$t_p = \frac{n\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-0.16}} = \underline{0.685 \text{ sec}}$$

$$t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_d} = 0.44 \text{ sec}$$

$$\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100\% = \underline{25.34\%}$$

$$\pm 2\% \quad t_s = \underline{2 \text{ sec.}}$$

Q. Repeat the above problem.



$$\frac{C(s)}{R(s)} = \frac{20}{s^2+5s+4+20} = \frac{20}{s^2+5s+24} = \frac{20}{24}\left(\frac{24}{s^2+5s+24}\right)$$

Effect of steady state value but not any T.D specifications.

$$\omega_n = \underline{4.899}$$

$$2\xi\omega_n = 5 \qquad \xi = \underline{0.51}$$

$$t_d = \frac{1+0.7\xi}{\omega_n} = \underline{0.277 \text{ sec.}}$$

$$t_p = \frac{n\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi_1^2}} = 0.7455 \text{ sec.}$$

$$t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\xi_1^2}}{\xi_1^2}\right)}{\omega_d} = \underline{0.442 \text{ sec}}$$

$t_r = 0.501$ sec

x

$$\% \, m_p = e^{-\pi\xi_1/\sqrt{1-\xi_1^2}} \times 100\% = \underline{15.52\%}$$

$$\pm 2\% \; t_s = \underline{1.6 \text{ sec}}$$

Q The unit step response to the above system is

$$\boxed{c(t) = 1 - \frac{e^{-\xi_1\omega_n t}}{\sqrt{1-\xi_1^2}}\left[\sin(\omega_d t) + \cos^{-1}(\xi_1)\right]}$$

$$c(t) = \frac{20}{24}\left[1 - \frac{e^{-\xi_1\omega_n t}}{\sqrt{1-\xi_1^2}}\left(\sin(4.2t) + \cos^{-1}(0.51)\right)\right]$$

$$c(t) = \frac{20}{24}\left[1 - \frac{e^{-2.5t}}{0.917}\left(\sin(4.2t + 1.036)\right)\right]$$

$$c(t) = \frac{20}{24}\left[1 - \frac{e^{-2.5t}}{0.917}\sin(4.2t + 59.336^\circ)\right]$$

VARIATION IN TIME DOMAIN SPECIFICATIONS W.R.T ξ

1. As ξ increases, the poles moves towards the left and near to the real axis.

In this case (i) time constant τ decreases

(ii) Settling time t_s decreases.

(iii) Damped oscillation ω_d decreases.

(iv) As ω_d decreases, the time specification t_d, t_r, t_p increases.

(v) % M_p decreases.

(vi) The system becomes more relatively stable.

H.W

Q Find the time domain specifications to the given system.

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 8y = 8x$$

where y is o/p, x is i/p

$$\frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8}$$

Comparing it with $\dfrac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2}$

$\omega_n^2 = 8$ $\quad \omega_n = 2\sqrt{2} = \underline{2.828}$ rad/s.

$2\xi\omega_n = 4$

$\xi = \dfrac{2}{\omega_n} = \dfrac{\cancel{2}}{\cancel{2}\sqrt{2}} = \underline{0.707}$

$$t_d = \frac{1 + 0.7\,\xi}{\omega_n} = \underline{\underline{0.529\,sec}}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 2\sqrt{2}\sqrt{1-\frac{1}{2}}$$

$$= 2\sqrt{2}\sqrt{1/2}$$

$$= \underline{\underline{2}}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{2}\,s$$

$$= \underline{\underline{1.571\,s}}$$

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d} = \underline{\underline{1.178\,s}} \quad \left(\frac{3}{8}\pi\right)$$

$$\%\,M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\,\%$$

$$\frac{-\pi \frac{1}{\cancel{\sqrt{2}}} \times \cancel{\sqrt{2}}}{\cancel{\sqrt{3}}}$$

$$= e^{-\pi} \times 100\%$$

$$= \underline{\underline{4.321\,\%}}$$

$$\pm 2\,\%\; t_s = 4\tau$$

$$= \frac{4}{\xi\omega_n} = \frac{4}{\frac{1}{\cancel{\sqrt{2}}} \times 2\cancel{\sqrt{2}}} = \underline{\underline{2\,sec}}$$

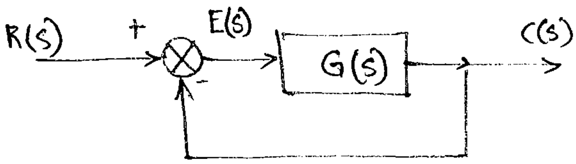
$$\therefore \pm 2\,\%\; t_s = \underline{\underline{2\,sec}}$$

STEADY STATE ERRORS

The error is deviation of the output from the input.

<u>steady state error</u> : The error at $t \to \infty$
(e_{ss})

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t)} \qquad \boxed{e_{ss} = \lim_{s \to 0} sE(s)}$$



~~Steady state~~

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$

$$E(s) = \frac{R(s)}{1+G(s)}$$

ie, $$\boxed{e_{ss} = \lim_{s \to 0} s\left(\frac{R(s)}{1+G(s)}\right)}$$

→ The steady state errors depends on two factors.

(i) Type of input. → (R(s))

(ii) Type of system. → (G(s)) (H(s) = 1)

→ Steady state errors are calculated to only closed loop stable systems.

→ steady state errors are valid for unity feedback systems.

If given non unity feedback, we require to convert it into unity feedback system.

TYPE OF INPUT

(i) Step Input

$$r(t) = A\,u(t)$$

$$R(s) = A/s$$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{s\,R(s)}{1+G(s)} = \underset{s\to 0}{Lt} \frac{s \times \frac{A}{s}}{1+G(s)}$$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{A}{1+G(s)}$$

$$= \frac{A}{1+\underset{s\to 0}{Lt}\, G(s)}$$

Let $K_p \Rightarrow$ position error constant $= \boxed{\underset{s\to 0}{Lt}\, G(s) = K_p}$

$$\therefore \boxed{e_{ss} = \frac{A}{1+K_p}}$$

(ii) Ramp Input

$$r(t) = At\,u(t)$$

$$R(s) = A/s^2$$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{s\,R(s)}{1+G(s)} = \underset{s\to 0}{Lt} \frac{\cancel{s} \times A/s^{\cancel{2}}}{1+G(s)}$$

$$= \underset{s\to 0}{Lt} \frac{A}{s+sG(s)}$$

$$= \frac{A}{\underset{s\to 0}{Lt}\left[\underset{0}{s} + sG(s)\right]}$$

$$= \frac{A}{\underset{s\to 0}{Lt}\left[s\,G(s)\right]} \qquad \boxed{e_{ss} = \frac{A}{K_v}}$$

$\boxed{K_v = Lt\, sG(s)}$

(III) PARABOLIC Input.

$$r(t) = At^2/2 \, u(t)$$

$$R(s) = A/s^3$$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{s \times A/s^3}{1+G(s)} = \underset{s\to 0}{Lt} \frac{A}{s^2 + s^2 G(s)} = \frac{A}{\underset{s\to 0}{Lt} \; s^2 G(s)}$$

Let K_a = Acceleration error constant.

$$\boxed{K_a = \underset{s\to 0}{Lt} \; s^2 G(s)}$$

$$\boxed{e_{ss} = \frac{A}{K_a}}$$

TYPE OF SYSTEMS

The standard form of s/m is represented as

$$\boxed{G(s)H(s) = \frac{K(1+s\tau_1)(1+s\tau_2)\ldots\ldots}{s^n(1+s\tau_a)(1+s\tau_b)\ldots\ldots}}$$

$\quad \parallel$

$\quad 1$

Type n systems.

I

Consider the step input and different types of systems.

we have for step i/p, $e_{ss} = \frac{A}{1+K_p}$

✡ TYPE - 0 - SYSTEM

$$K_p = \underset{s\to 0}{Lt} \; G(s) = \underset{s\to 0}{Lt} \frac{K(1+s\tau_1)(1+s\tau_2)\ldots\ldots}{s^0(1+s\tau_a)(1+s\tau_b)\ldots\ldots}$$

$$K_p = K$$

$$\boxed{e_{ss} = \frac{A}{1+K} = \text{constant}}$$

✡ TYPE - 1 - SYSTEM

$$K_p = \underset{s\to 0}{Lt}\, G(s) = \underset{s\to 0}{Lt} \frac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^1(1+s\tau_a)(1+s\tau_b)\cdots} = \infty$$

$$\boxed{e_{ss} = \frac{A}{1+\infty} = 0}$$

✡ TYPE - 2 - SYSTEM

$$K_p = \underset{s\to 0}{Lt}\, G(s) = \underset{s\to 0}{Lt} \frac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^1(1+s\tau_a)(1+s\tau_b)\cdots} = \infty$$

$$\boxed{e_{ss} = \frac{A}{1+\infty} = 0}$$

Conclusion

	e_{ss}
Type = Input t power ⟹	constant
Type > Input t power ⟹	0
Type < Input t power ⟹	∞

→ The steady state errors required to calculate only in 3 cases

(i) step input and Type 0 system

(ii) Ramp input and Type 1 system

(iii) parabolic Input and Type 2 system

Remaining all the ~~et~~ cases, the steady state error either become ZERO or INFINITY

II Consider Ramp Input,

$$\cancel{e_{ss} = \frac{A}{sK_v}}$$

$$e_{ss} = \frac{A}{K_v}$$

where $K_v = \underset{s\to 0}{Lt}\ sG(s)$

✡ TYPE - 0 - SYSTEM

$$K_v = \underset{s\to 0}{Lt}\ sG(s) = \underset{s\to 0}{Lt}\ \frac{sK(1+s\tau_1)(1+s\tau_2)\ldots}{s^0(1+s\tau_a)(1+s\tau_b)\ldots}$$

$$= \underline{\underline{0}}$$

$$\boxed{e_{ss} = \frac{A}{0} = \infty}$$

✡ TYPE - 1 - SYSTEM

$$K_v = \underset{s\to 0}{Lt}\ sG(s) = \underset{s\to 0}{Lt}\ \frac{\cancel{s}K(1+s\tau_1)(1+s\tau_2)\ldots}{\cancel{s^1}(1+s\tau_a)(1+s\tau_b)\ldots}$$

$$K_v = K$$

$$\boxed{e_{ss} = \frac{A}{K} = \text{constant.}}$$

✡ TYPE - 2 - SYSTEM

$$K_v = \underset{s\to 0}{Lt}\ sG(s) = \underset{s\to 0}{Lt}\ \frac{\cancel{s}K(1+s\tau_1)(1+s\tau_2)\ldots}{s^2(1+s\tau_a)(1+s\tau_b)\ldots}$$

$$K_v = \infty$$

$$\boxed{e_{ss} = \frac{A}{\infty} = 0}$$

III consider parabolic input.

$$e_{ss} = \frac{A}{K_a}$$

$$K_a = \lim_{s \to 0} s^2 G(s)$$

✡ TYPE - 0 - SYSTEM

$$K_a = \lim_{s \to 0} s^2 G(s) = \frac{s^2 K (1+s\tau_1)(1+s\tau_2)\ldots}{s^0 (1+s\tau_a)(1+s\tau_b)\ldots}$$

$$K_a = 0$$

$$\boxed{e_{ss} = \frac{A}{0} = \infty}$$

✡ TYPE - 1 - SYSTEM

$$K_a = \lim_{s \to 0} s^2 G(s) = 0$$

$$\boxed{e_{ss} = \frac{A}{0} = \infty}$$

TYPE - 2 - SYSTEM

$$K_a = \lim_{s \to 0} s^2 G(s) = \lim_{s \to 0} \frac{\cancel{s^2} \times K \times (1+s\tau_1)(1+s\tau_2)\ldots}{\cancel{s^2}(1+s\tau_a)(1+s\tau_b)\ldots\ldots} = K$$

$$\boxed{e_{ss} = \frac{A}{K} = \text{constant.}}$$

Q Find the steady state error to the given unity feed back system

$$G(s) = \frac{10(s+1)}{s^2(s+2)(s+10)} \quad , H(s)$$

$$r(t) = (10 + 5t + t^2)\,u(t)$$

Method 1 Essay Type.

$$\cancel{G(s) = \frac{10(s+1}{20\ } \frac{10(1+s)}{s^2(1+s/2)(1+s/10)}}$$

$$e_{ss} = \lim_{s\to 0} s\left[\frac{R(s)}{1+G(s)}\right]$$

$$= \lim_{s\to 0} \frac{s\left[\frac{10}{s} + \frac{5}{s^2} + \frac{2}{s^3}\right]}{1 + \frac{10(s+1)}{s^2(s+2)(s+10)}}$$

$$= \lim_{s\to 0} \frac{10 + \frac{5}{s} + \frac{2}{s^2}}{\frac{s^2(s+2)(s+10) + 10(s+1)}{s^2(s+2)(s+10)}}$$

$$= \lim_{s\to 0} \frac{(s+2)(s+10)(10s^2 + 5s + 2)}{s^2(s+2)(s+10) + 10(s+1)}$$

$$= \frac{2 \times \cancel{10} \times 2}{\cancel{10}}$$

$$= \underline{\underline{4}}$$

Method II

Type		Input	ess
2	>	0	0
2	>	1	0
2	=	2	$\frac{A}{K}$

For A taking std form of parabolic $At^2/2$

Here t^2 Hence $A = 2$

$K = 1/2$

$$e_{ss} = \frac{A}{K} = \underline{\underline{4}}$$

Q. Find the steady state errors to the given unity feedback system to the following inputs.

$$G(s) = \frac{10}{s(s+5)}, \quad H(s) = 1$$

i) $10\,u(t)$

ii) $10t\,u(t)$

iii) $10t^2\,u(t)$

iv) $(1+t)\,u(t)$

v) $(1+t+t^2)\,u(t)$

i) $10\,u(t)$

$$G(s) = \frac{10}{s(s+5)} \longrightarrow \text{Type } 1$$

$$e_{ss} = 0$$

ii) $10t\,u(t)$

$$e_{ss} = \frac{A}{K} = \frac{10}{2} = \underline{\underline{5}}$$

iii) $10t^2\,u(t)$

$$e_{ss} = \infty$$

iv) $1+t$

$$\cancel{e_{ss} = \lim_{s\to 0} \frac{s\left(\frac{1}{s}+\frac{2}{s^3}\right)}{1+}}$$

$$e_{ss} = 0 + \frac{1}{10/5} = 0.5$$

v) $(1+t+t^2)\,u(t) \Rightarrow 0 + 0.5 + \infty = \underline{\underline{\infty}}$

Q, $G(s) = \dfrac{s+1}{s^2(s+2)(s+4)}$, $H(s) = 1$

Given system is closed loop system, so finding CLTF

$$CLTF = \frac{G}{1+G} = \frac{s+1}{s^4+6s^3+8s^2+1}$$

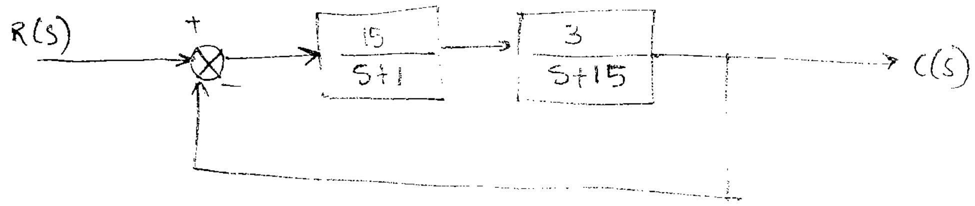
Here s^1 terms are missing. Hence directly can be write, the system is unstable.

Steady state errors are calculated only for closed loop stable systems. Hence no need to calculate for each.

Note:

while finding the steady state error, observe the options. If the one of the option is none (or) not defined, then verify the closed loop system stability by using RH criteria.

Q For the system shown in figure, the steady state error for the unit step i/p is



$$G(s)\ \cancel{\left(\frac{C(s)}{R(s)}\right)} = \frac{45}{(s+1)(s+15)}, \quad H(s) = 1$$

$$\cancel{CLTF = \frac{45}{(s+1)(s+15)}}$$

Type 0

$$e_{ss} = \lim_{s\to 0} \frac{A}{1+K_p} = \frac{1}{1+\frac{45}{15}} = \frac{1}{4} = 0.25$$

To calculate the steady state error required openloop transfer function of unity feedback system.

Q The openloop transfer function of a unity feed back system

$G(s) = \dfrac{k}{s(s+1)(s+2)}$, $H(s) = 1$. The K value to get the steady state error equal to 0.1 to the unit ramp i/p is

$$e_{ss} = \frac{A}{K} =$$

$$0.1 = \frac{1}{K/2}$$

$$\underline{\underline{K = 20}}$$

Q The open loop transfer function of a unity feedback system is $G(s)$. The steady state error is 0 for

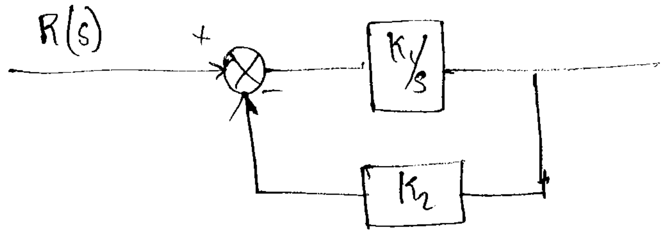
(a) step i/p and Type 1 system. $e_{ss} = 0$

(b) step i/p and Type 0 system. $\frac{A}{1+K}$ = const

(c) Ramp i/p and Type 1 System. $\frac{A}{K}$ const

(d) Ramp i/p and Type 0 system. $e_{ss} = \infty$

Q For the system shown in fig, the steady state gain equal to 2 with time Constant of 0.4 s. The values of K_1 and K_2 are



$$CLTF = \frac{K_1/s}{1+K_1K_2} = \frac{K_1}{s+K_1K_2} = \frac{\cancel{K_1}}{\cancel{K_1}K_2\left(\frac{s}{K_1K_2}+1\right)}$$

(5) we have 1^{st} order system

is $\frac{C(s)}{R(s)} = \frac{K}{s\tau + 1}$ where $K \to$ Steady state gain, or system gain.

If not given, take $K = 1$

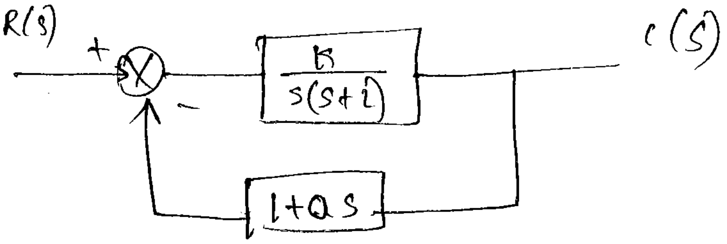
Given $K = 2$

ie, $\frac{1}{K_2} = 2$

$K_2 = 0.5$

$\frac{1}{K_1 K_2} = 0.4$, $\Rightarrow$ $\frac{2}{K_1} = 0.4$ $K_1 = \frac{2}{0.4} = \underline{\underline{5}}$

Q, For the system shown if figure, the undamped frequency of oscillation are 4 rad/s. and damping ratio is 0.7, the values of K and a is



$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\frac{\frac{K}{s(s+2)}}{1 + (1+as)\left(\frac{K}{s(s+2)}\right)} = \frac{K}{s(s+2) + K + Kas}$$

$$= \frac{K}{s^2 + 2s + K + Kas}$$

$$= \frac{K}{s^2 + (2 + Ka)s + K}$$

$\omega_n = 4$ rad/s.

$\underline{K = 16 \text{ rad/s}}$

$(2 + Ka) = 2\xi\omega_n$

$2 + 16 \times a = 2 \times 0.7 \times 4$

$a = 0.225$ //

Q. A control system described by following diff equs,

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 10\left(1-e^{-2t}\right)$$. The response at $t\to\infty$ is

$$\boxed{\text{we need to find } \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)}$$

$$s^2y + 2sy + 5y = L\left[10 - 10e^{-2t}\right]$$

$$s^2y + 2sy + 5y = \frac{10}{s} - \frac{10}{(s+2)}$$

$$Y(s) = \frac{\frac{10}{s} - \frac{10}{s+2}}{s^2+2s+5}$$

$$Y(s) = \frac{\frac{10(s+2) - 10s}{s(s+2)}}{s^2+2s+5}$$

$$Y(s) = \frac{20}{s(s+2)(s^2+2s+5)}$$

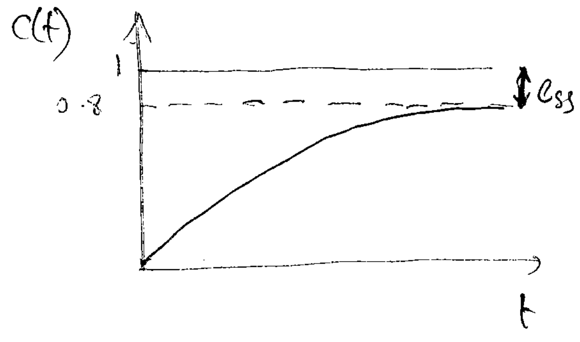
$$\lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{\cancel{s}\;20}{\cancel{s}(s+2)(s^2+2s+5)}$$

$$= \frac{20}{2\times 5}$$

$$= \underline{\underline{2}}$$

Q. The unit step response of the system is given as

The system gain is



$$c(t) = 0.8\left(1 - e^{-t/\tau}\right)$$

$$C(s) = \frac{0.8}{s} - \frac{0.8}{s+\tau}$$

~~Hence~~ Here e_{ss} given as 0.2

$$e_{ss} = \frac{A}{1+K} = \frac{1}{1+K} = 0.2$$

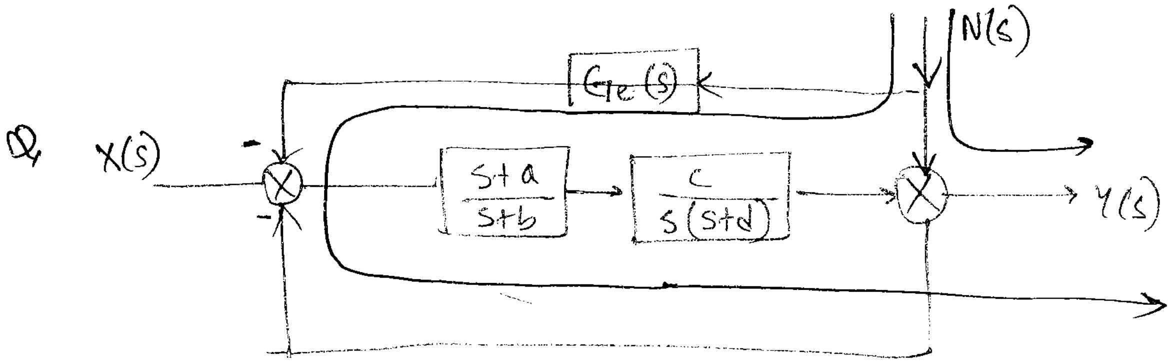
$$1 = 0.2 + 0.2K$$

$$0.2K = 0.8$$

$$\underline{\underline{K = 4}}$$

The error constants are ~~not~~ equal to the SYSTEM GAIN.

Q



Q In the above figure,

To nullify the effect of noise, the gain of feed forward path is

The feed forward path is the additional forward path which is used to reduce the effect of noise and disturbances in the system.

we need to find $G_e(s)$

such that noise effect $\doteq 0$

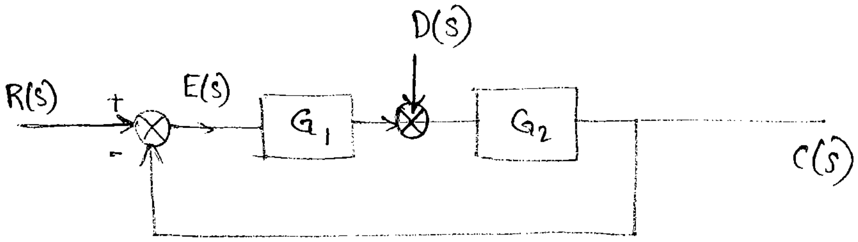
ie, find $\frac{Y(s)}{N(s)} = 0$

$$\frac{Y(s)}{N(s)} = \frac{1 - G_e(s)\dfrac{c(s+a)}{s(s+b)(s+d)}}{1 + \dfrac{c(s+a)}{s(s+b)(s+d)}} = 0$$

$$G_e(s)\, c(s+a) = s(s+b)(s+d)$$

$$G_e(s) = \frac{s(s+b)(s+d)}{c(s+a)}$$

STEADY STATE ERRORS, TO DISTURBANCE ~~ER~~ INPUT



Study procedure and equation here.

E(S) is always produced with respect to reference input only not w.r.t D(S).

e_{ss} due to R(s)

$$\frac{E(s)}{R(s)} = \frac{1}{1+G_1G_2}$$

$$E(s) = \frac{R(s)}{1+G_1G_2}$$

$$e_{ss} = \lim_{s\to 0} s\,E(s)$$

$$e_{ss} = \lim_{s\to 0} s\left(\frac{R(s)}{1+G_1G_2}\right)$$

e_{ss} due to D(s)

$$\frac{E(s)}{D(s)} = \frac{-G_2}{1+G_1G_2}$$

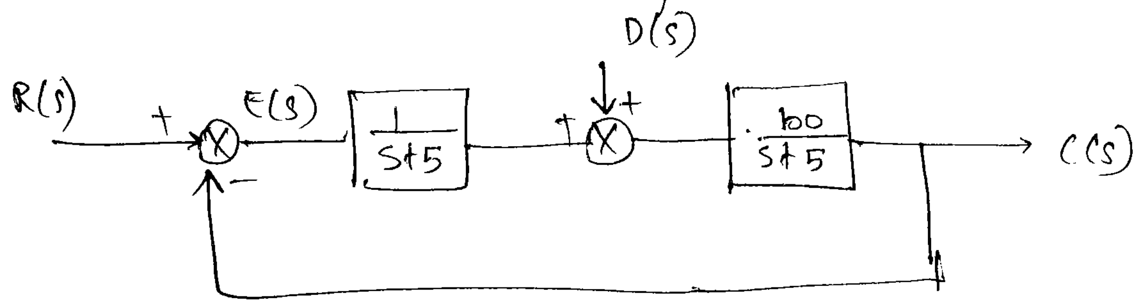
$$E(s) = \frac{-G_2 D(s)}{1+G_1G_2}$$

$$e_{ss} = \lim_{s\to 0} s\,E(s)$$

$$= \lim_{s\to 0} s\left(\frac{-G_2 D(s)}{1+G_1G_2}\right)$$

Above is not a formula, take it as a procedure because N(s) or D(s) can be given anywhere.

Q Find the steady state error due to step i/p and step disturbance to the following system.



① e_{ss} due to R(s)

$$e_{ss} = \lim_{s \to 0} \frac{s(R(s))}{1 + G_1 G_2}$$

$$= \lim_{s \to 0} \frac{\cancel{s} \times \frac{1}{\cancel{s}}}{1 + \frac{100}{(s+5)(s+5)}}$$

$$= \lim_{s \to 0} \frac{1}{1 + \frac{100}{(s+5)^2}}$$

$$= \frac{1}{1 + \frac{100}{25}} = \frac{1}{1+4} = \frac{1}{5} = 0.2$$

e_{ss} due to D(s)

$$e_{ss} = \lim_{s \to 0} \frac{s(-G_2 D(s))}{1 + G_1 G_2}$$

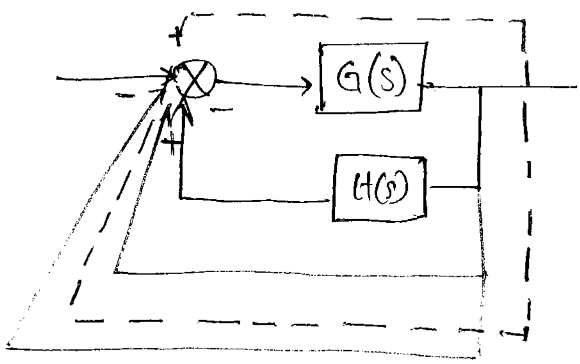
$$= \lim_{s \to 0} \frac{\cancel{s} \times -\frac{100}{(s+5)} \times \frac{1}{\cancel{s}}}{1 + \frac{100}{(s+5)^2}}$$

$$= \frac{-\frac{100}{5}}{1 + \frac{100}{25}} = \frac{-\frac{100}{5}}{1+4} = \frac{-100}{25} = \underline{\underline{-4}}$$

$$\text{Total } e_{ss} = 0.2 - 4 = \underline{\underline{-3.8}}$$

STEADY STATE ERRORS TO THE NON-UNITY FEEDBACK SYSTEM.

The steady state errors to calculated to only unity feed back systems. If non unity feed back system is given, it is required to convert to unity feedback, as follows.

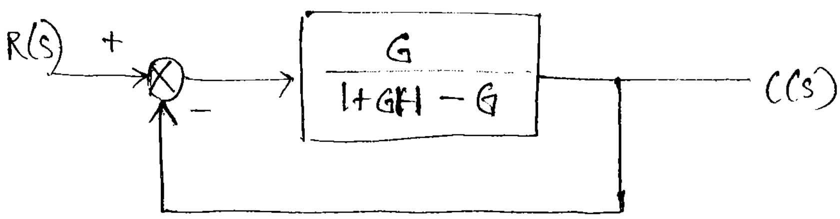


NON UNITY ⟷ UNITY

Adding one unity positive feed back and one unity negative feedback

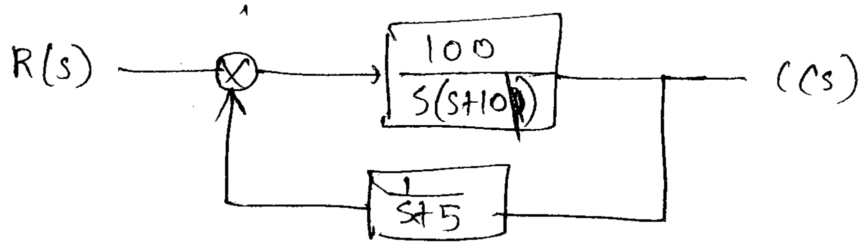
$G_{NUF} \rightarrow$ Equivalent OLTF to the Non-Unity Feedback

⇓

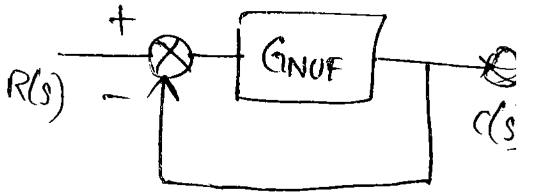


$$\boxed{G_{NUF} = \frac{G_{UF}}{1 + G_{UF}H - G_{UF}}}$$

Q. Find the steady state error to the given non unity feedback system to the unity step i/p.



$$G_{NUF} = \frac{\dfrac{100}{s(s+10)}}{1 + \dfrac{100}{s(s+5)(s+10)} - \dfrac{100}{s(s+10)}}$$



$$\frac{E(s)}{R(s)} = \frac{1}{1+G_{NUF}}$$

$$E(s) = \frac{R(s)}{1+G_{NUF}}$$

$$e_{ss} = \underset{s\to 0}{Lt}\; s\,E(s) = \underset{s\to 0}{Lt} \frac{s\,R(s)}{1+G_{NUF}}$$

$$= \underset{s\to 0}{Lt} \frac{\cancel{s}\times\frac{1}{\cancel{s}}}{1+G_{NUF}}$$

$$K_p = \underset{s\to 0}{Lt} \frac{\dfrac{100}{\cancel{s(s+10)}}}{\dfrac{s(s+5)(s+10)+100-100(s+5)}{\cancel{s}(s+5)\cancel{(s+10)}}}$$

$$K_p = \underset{s\to 0}{Lt} \frac{100(s+5)}{s(s+5)(s+10)+100-100(s+5)} = \frac{500}{100-500}$$

$$K_p = \frac{-5}{4}$$

$$G_{NUF}(s) = \frac{100(s+5)}{s^3+15s^2-50s-400}$$

$$e_{ss} = \frac{1}{1-5/4} = -4$$

Type 0 Order 3

For the type and order of a non-unity feedback system required openloop transfer function of a unity feed back system. $G_{NUF}(s)$, $H(s) = 1$

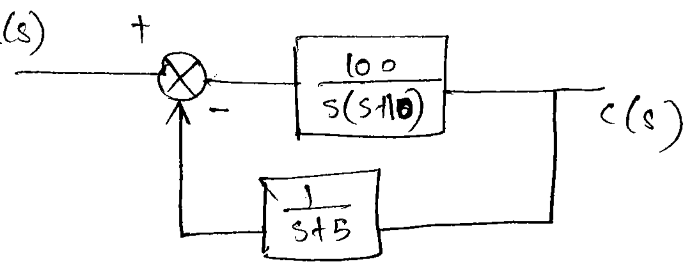
Method II

If CLTF is given (or) Non Unity FB system or

General Equation for all cases,

$$\boxed{e_{ss} = \lim_{s \to 0} s\left[R(s) - C(s)\right]}$$

consider the above case.



$$\frac{C(s)}{R(s)} = \frac{100}{s(s+10) + 100\left(\frac{1}{s+5}\right)}$$

$$C(s) = \frac{100(s+5)\,R(s)}{s(s+10)(s+5) + 100}$$

$$e_{ss} = \lim_{s \to 0} s\left[R(s) - C(s)\right]$$

$$e_{ss} = \lim_{s \to 0} s\left[R(s) - \frac{100(s+5)\,R(s)}{s(s+10)(s+5)+100}\right]$$

$$e_{ss} = \underset{s\to 0}{Lt}\ s \times \frac{1}{s}\left[1 - \frac{100(s+5)}{s^3+15s^2+50s+100}\right]$$

$$= 1 - \frac{500}{100} = \underline{\underline{-4}}$$

If in case instead in s domain, if it is given as c(t) is given

$$\boxed{e_{ss} = \underset{t\to\infty}{Lt}\left[r(t) - c(t)\right]}$$

IES

Q. A closed loop transfer function of a unity feedback system is

$$\frac{C(s)}{R(s)} = \frac{20s^2}{(s+1)(s+3)(s+5)}$$

Determine the response of the system, when excitation is $r(t) = 1+2t+3t^2/2$

$$R(s) = \frac{1}{s} + \frac{2}{s^2} + \frac{3}{s^3}$$

$$C(s) = \frac{20s^2 \times \left(\frac{1}{s}+\frac{2}{s^2}+\frac{3}{s^3}\right)}{(s+1)(s+3)(s+5)}$$

$$= \frac{20\left(s+2+\frac{3}{s}\right)}{(s+1)(s+3)(s+5)} = \frac{20(s^2+2s+3)}{s(s+1)(s+3)(s+5)}$$

$$C(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3} + \frac{D}{s+5}$$

$A \times 12 = 20s^2 + 40s + 60$ $\quad A = 5,$

$B(-1\times 2\times 4) = 20-40+60$

$-8B = 40 \quad \underline{B = -5}$

$C(-3 \times -2 \times 2) = 20 \times 9 - 120 + 60$

$+12 \, C = 120$

~~C = 20~~ $\quad \underline{C = +10}$

$D(-5 \times -4 \times -2) = 20 \times 25 - 200 + 60$

~~D = 9/5~~ $\quad D = -9$

$$C(s) = \frac{4}{s} - \frac{5}{s+1} + \frac{10}{s+3} - \frac{9}{s+5}$$

ILT $\quad c(t) = 4 - 5e^{-t} + 10e^{-3t} - 9e^{-5t}$

Q, ~~Determ~~ For the s/m shown below, determine the characteristic eqn, hence the following, when the excitation is unit step.

(i) Undamped fr.o.o

(ii) Damping ratio

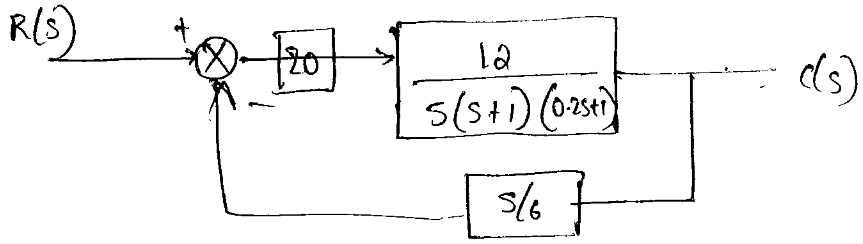
(iii) Damping factor

(iv) Damped frequency of oscillation

(v) Max over shoot.

(vi) settling time

(vii) No: of cycles completed before the o/p is settled within ± 5% of the final value.



characteristic eqn.

$$1 + GH = 0$$

$$1 + \frac{240 \times s/6}{s(s+1)(0.2s+1)} = 0$$

$$1 + \frac{40\cancel{s}}{\cancel{s}(s+1)(0.2s+1)} = 0$$

$$(s+1)(0.2s+1) + 40 = 0$$

$$0.2s^2 + s + 0.2s + 41 = 0$$

$$0.2s^2 + (1.2)s + 41 = 0$$

$$s^2 + 6s + 205 = 0$$

(i) $\omega_n^2 = 205$

$\omega_n = \underline{14.318}$ rad/s.

Undamped f.o.o = $\underline{14.318}$ rad/s.

(ii) $2\xi\omega_n = 6$

$$\xi = \frac{6}{2\omega_n}$$

$\xi = \underline{0.2095}$

(ii) Damping factor $= \xi \omega_n$

$= \underline{\underline{3}}$

(iii) $\omega_d = \omega_n \sqrt{1-\xi^2}$

$= \underline{\underline{13.992 \text{ rad/s}}}$

(iv) $M_p = e^{-\pi \xi / \sqrt{1-\xi^2}}$

$= 0.51012$

$= \underline{\underline{51.012}}$

(v) $\pm 2\% \, t_s = 4\tau$

$t_s = 4 \times \dfrac{1}{\xi \omega_n}$

$= 1.33$.

$\pm 5\% \, t_s = 3\tau$

$= \underline{\underline{1}}$

(vi) $N = \dfrac{t_s}{2 t_p} = \dfrac{t_s}{T_{osc}}$ $\cancel{t_p = \pi}$

$= \dfrac{1}{2\pi/\omega_d} = \dfrac{13.992}{2\pi}$

$= \underline{\underline{2.2268}}$

$\approx$ 3 cycles.

Maxima - $t_p = \frac{\pi}{\omega_d} = 0.2245\,s$

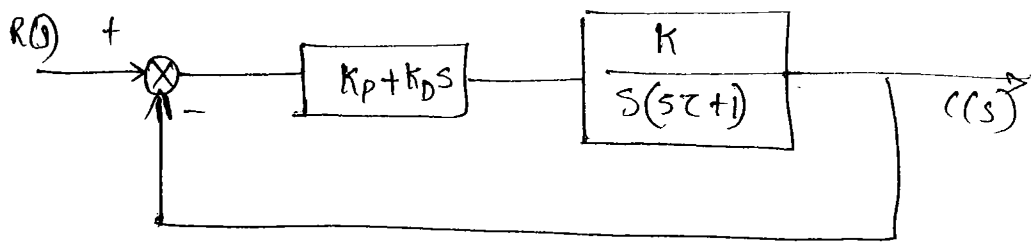
Minima $\rightarrow$ $t_p = \frac{2\pi}{\omega_d} = 0.4491\,s$

Q. For a second order control system with PD controller, is shown in figure. Derive the expression for its

(i) steady state error to the velocity i/p.

(ii) Natural frequency of oscillation.

(iii) Damping ratio in terms of system parameters.



(i) $e_{ss} = \frac{A}{K_v}$

$$K_v = \lim_{s \to 0} \frac{\cancel{s}\,(K_p + K_D s)\,K}{\cancel{s}\,(s\tau+1)}$$

$$K_v = \frac{K_p \times K}{1} = \underline{K K_p}$$

$$K_v = \underline{K K_p}$$

$$e_{ss} = \frac{A}{K K_p}$$

(ii) Finding characteristic eqn.

$$1 + GH = 0$$

$$1 + \frac{(K_p + K_D s) K}{s(s\tau+1)} = 0$$

$$s(s\tau+1) + (K_p + K_D s) K = 0$$

$$s^2\tau + s + KK_D s + KK_p = 0$$

$$s^2\tau + (1+KK_D) s + KK_p = 0$$

$$s^2 + \left(\frac{1+KK_D}{\tau}\right) s + \frac{KK_p}{\tau} = 0$$

$$\omega_n^2 = \frac{KK_p}{\tau}$$

$$\boxed{\omega_n = \sqrt{\frac{KK_p}{\tau}}}$$

(iii)

$$2\xi_p \omega_n = \left(\frac{1+KK_D}{\tau}\right)$$

$$\xi_p = \left(\frac{1+KK_D}{\tau}\right) \frac{1\sqrt{\tau}}{2 \times \sqrt{KK_p}}$$

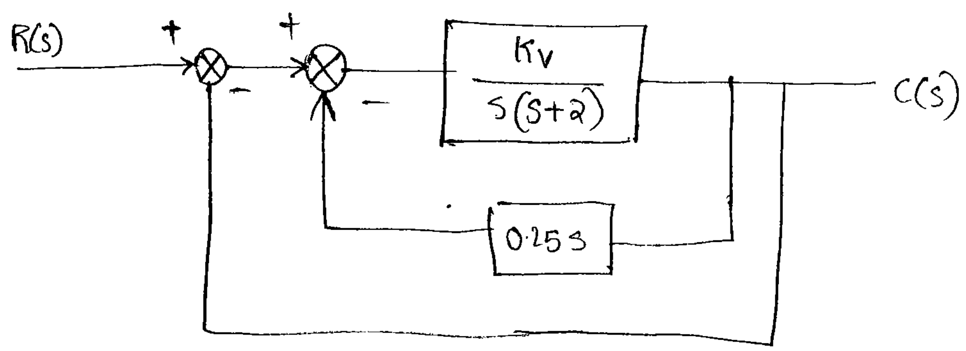
$$\boxed{\xi_p = \frac{1}{2} \frac{1+KK_D}{\sqrt{KK_p \tau}}}$$

Q A control system is represented by a block diagram of the figure (i) Find the characteristic eqn by using block diagram reduction technique.

(ii) calculate its damping ratio and damping factor, undamped frequency of oscillation, when $K_V = 10$.

(iii) what should be the value of K_V for the critical damped system.

(iv) For $K_V = 10$, Find the expression of $C(t)$ for unit step, and obtain time at which first overshoot occurs. and also find ~~P~~ peak overshoot.



(i)

$$R(s) \rightarrow \boxed{\frac{K_V}{s(s+2) + 0.25K_V s + K_V}} \rightarrow C(s)$$

(ii) $$\frac{K_V}{s^2 + 2s + 0.25K_V s + K_V}$$

$$\frac{K_V}{s^2 + (2 + 0.25K_V)s + K_V}$$

$\omega_n^2 = K_V$ $\omega_n = \sqrt{K_V}$

$\omega_n = 3.1622$

~~2ξ~~ $2\xi\omega_n = 2 + 0.25K_V$

$\xi = \frac{2 + 0.25K_V}{2\omega_n}$

$\xi_p = \underline{0.7115}$ Damping factor $= \underline{2.25}$

(iii) For critical damped system, $\xi_p = 1$

$$2 + 0.25K_v = 2\xi_p\omega_n$$

$$2 + 0.25K_v = 2\sqrt{K_v}$$

$$(2 + 0.25K_v)^2 = 4K_v$$

$$0.25^2K_v^2 + 4 + K_v = 4K_v$$

$$0.0625K_v^2 - 3K_v + 4 = 0$$

$$\underline{K_v = 46.627, \quad K_v = 1.3725}$$

(iv) $K_v = 10$

$$C(s) = \frac{10}{s^2 + 4.5s + 10}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3.1622 \times \sqrt{1 - \xi_p^2}}$$

$$= \underline{1.412}$$

$$M_p = e^{-\pi\xi_p/\sqrt{1-\xi_p^2}} = \underline{0.0415}$$

$$\% M_p = e^{-\pi\xi_p/\sqrt{1-\xi_p^2}} \times 100 = 4.15\%$$

For unit step. $R(s) = 1/s$

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + 4.5s + 10}$$

$$C(s) = \frac{10}{s(s^2+4.5s+10)}$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+4.5s+10}$$

$$A(s^2+4.5s+10) + (Bs+C)s = 10$$

$A \times 10 = 10$

$A = 1$

$\cancel{4.5A + B + D = 10}$ $\quad A \cdot 4.5 + C = 0$

$\cancel{B + D = 5.5}$ $\quad C = -4.5$

$A + B = 0 \quad B = -1$

$$C(s) = \frac{1}{s} + \frac{-s - 4.5}{s^2+4.5s+10}$$

$$= \frac{1}{s} - \frac{s+4.5}{s^2+4.5s+10}$$

$$= \frac{1}{s} - \frac{s+4.5}{s^2+4.5s + \frac{81}{16} - \frac{81}{16} + 10}$$

$1 - \frac{81}{16}$

$\frac{16-81}{16}$

$$= \frac{1}{s} - \frac{s+4.5}{(s+9/4)^2 + \frac{79}{16}}$$

$$= 1 - \left(\frac{s + 9/4}{(s+9/4)^2 + \frac{79}{16}} + \frac{4.5 - 9/4}{(s+9/4)^2 + \frac{79}{16}} \right)$$

ILT

$$= 1 - e^{-9/4 t} \cos 2.222t + \cancel{2.25\times}\ 1.012 \times \sin 2.22t\, e^{-9/4 t}$$

$$= 1 - \left[e^{-2.25t} \cos(2.22t) + 1.012 \times e^{-2.25t} \sin(2.222t) \right]$$

$$c(t) = 1 - e^{-2.25t} \left[\cos 2.25t + \overset{1.013}{\sin}(2.22t) \right]$$

finding t_p and M_p from $c(t)$.

Taking 1.013 ~~and~~.

$$\cancel{c(t) = 1 - \frac{e^{-2.25t}}{0.987} \left[0.987 \cos(2.22t) + \sin(2.22t) \right]}$$

~~sin A cos B + cos A sin B~~

Approximate $1.013 \approx 1$

$$c(t) = 1 - e^{-2.25t} \sqrt{2} \left[\frac{1}{\sqrt{2}} \cos(2.25t) + \frac{1}{\sqrt{2}} \sin(2.22t) \right]$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B)$$

$$c(t) = 1 - e^{-2.25t} \left[\sin(2.25t + 45^\circ) \right]$$

$$c(t) = 1 - \sqrt{2}\ e^{-2.25t}\left[\sin\left(2.22t + 45^\circ\right)\right]$$

std form is

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$\sqrt{1-\xi^2} = \frac{1}{\sqrt{2}} \qquad \omega_d = 2.22$$

$$1-\xi^2 = \frac{1}{2}$$

$$\xi^2 = 1/2$$

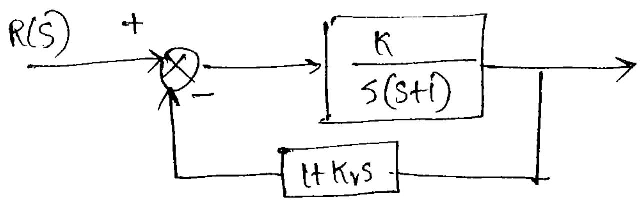
$$\xi = 1/2$$

$$t_p = \frac{\pi}{\omega_d} = 1.415 \text{ sec}$$

$$M_p = 4.15\ \%$$

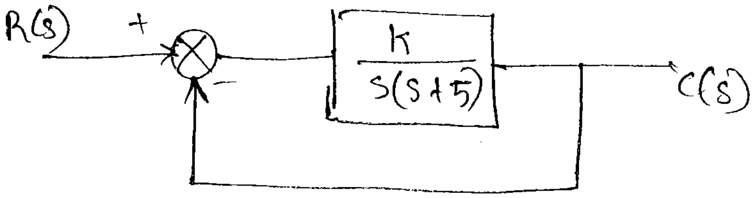
H.W

1, Determine the value of K and velocity feedback constant K_v, so that the max overshoot to the unit step response is 0.2 and peak time is 1s. With these values of K_v and K, obtain the rise time and settling time.



2, Find the value of gain K for the feedback control system shown in figure such that the system will be underdamped and will respond with 16 % overshoot. Then calculate the followin

1) undamped natural frequency of oscillations
2) Damping ratio.
3) Time required to reach first maxima.
4) $\pm 2\%$ tolerance band t_s?
5) peak time.

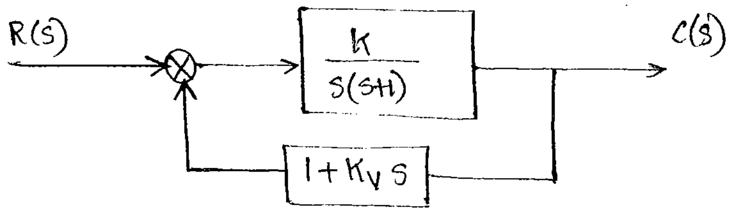


Q. A circuit has the following transfer function

$$\frac{C(s)}{R(s)} = \frac{s^2+3s+4}{s^2+4s+4}$$

Find $c(t)$ when $r(t)$ is unit step. and state the circuit nature.

Q1.



$t_p = 1s$

$M_p = 0.2$

$t_s = ?$

$t_r = ?$

$K, K_v = ?$

$$t_p = \frac{\pi}{\omega_d}$$

$$= \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \longrightarrow (1)$$

characteristic equ

$$1 + GH = 0$$

$$1 + \frac{K(1+K_v s)}{s(s+1)} = 0$$

$$s^2+s+K+KK_v s=0$$

$$s^2+(1+KK_v)s+K=0$$

$$\omega_n=\sqrt{K} \qquad 2\xi_1\omega_n = 1+KK_v$$

$$\xi_1 = \frac{1+KK_v}{2\sqrt{K}}$$

① ⇒ $$t_p = \frac{\pi}{\sqrt{K}\sqrt{1-\left(\frac{1+KK_v}{2\sqrt{K}}\right)^2}}$$ $t_p = \cdot 1s$

$$\cancel{t_p = \frac{\pi}{\sqrt{K}}}$$

$$\sqrt{K}\sqrt{1-\left(\frac{1+KK_v}{2\sqrt{K}}\right)^2} = \pi \implies \cancel{\sqrt{K}\times 1.952\times(1+KK_v)}$$

$$K\times\left(1-\frac{(1+KK_v)^2}{4K}\right)=\pi^2$$

$$K-\frac{(1+KK_v)^2}{4}=\pi^2$$

$$M_p = e^{-\pi\xi_1/\sqrt{1-\xi_1^2}} \cancel{100\%}$$

$$0.2 = e^{-\pi\xi_1/\sqrt{1-\xi_1^2}}$$

$$\frac{\pi\xi_1}{\sqrt{1-\xi_1^2}} = 1.6094$$

$$\frac{\sqrt{1-\xi_1^2}}{\xi_1} = 1.952$$

$$1-\xi_1^2 = 3.81\,\xi_1^2$$

$$1 = 4.81\,\xi_1^2$$

$$\xi_1 = \underline{0.4559}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi_1^2}}$$

$$\omega_n\sqrt{1-\xi_1^2} = \pi$$

$$\omega_n = 3.5299 \text{ rad/s.}$$

$$\omega_n = \sqrt{K}$$

$$K = \omega_n^2$$

$$K = 12.4602$$

$$\xi_1 = \frac{1+KK_v}{2\omega_n} = \frac{1+KK_v}{2\sqrt{K}} = 0.4559$$

$$KK_v = 2.2186$$

$$K_v = 0.1781$$

$$K = 12.4602 \qquad K_v = 0.1781$$

Rise time $t_r = \dfrac{\pi - \cos^{-1}\xi_1}{\omega_d}$

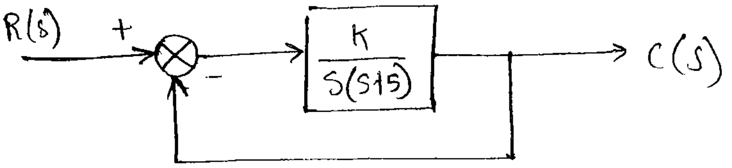
$$= 0.6507 \text{ s}$$

Settling time ± 2 $t_s = 4\tau = \dfrac{4}{\xi_1\omega_n} = 2.4856$ sec.

$K = 12.4602$ $\qquad t_r = 0.6507$ sec

$K_v = 0.1781$ $\qquad t_s = 2.4856$ sec

2



Underdamped with 16 % M_p.

So that $0<\xi<1$, and we can use the eqn $c(t) = 1 - \dfrac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$

Hence

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$16 = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$\frac{\pi\xi}{\sqrt{1-\xi^2}} = 1.8326$$

$$\sqrt{1-\xi^2} = 1.714\,\xi$$

$$1-\xi^2 = 2.9388\,\xi^2$$

$$\Rightarrow \xi = \underline{0.5039} \approx \underline{0.5}$$

characteristic eqn $1+GH = 0$

$$1 + \frac{K}{s(s+5)} = 0$$

$$s(s+5) + K = 0$$

$$s^2 + 5s + K = 0$$

$$2\xi\omega_n = 5$$

$$\omega_n = \frac{5}{2\xi} = \underline{4.9613} \approx \underline{5} \text{ rad/s}$$

(i) $\omega_n = 5$ rad/s. (4.9613)

(ii) $\xi = 0.5$ (0.5039)

(iii) $t_p = \dfrac{\pi}{\omega_d} = \underline{0.7255 \text{ sec}}$

(iv) $\pm 2\%$ tolerance band t_s.

$$\pm 2\% \; t_s = 4\tau = \frac{4}{\xi \omega_n} = \frac{4}{5/2} = \frac{8}{5} = 1.6$$

(v) $t_p = 0.7255$ sec

3Q $$\frac{C(s)}{R(s)} = \frac{s^2+3s+4}{s^2+4s+4}$$

Find $c(t)$ when $r(t)$ is unit step.

$$C(s) = \frac{s^2+3s+4}{s^2+4s+4} \times R(s)$$

$$= \cancel{\frac{\frac{1}{s}(s^2+3s+4)}{s^2+4s+4}} \qquad \frac{s^2+3s+4}{s(s^2+4s+4)}$$

$$= \cancel{s+3+\frac{4}{s}} \qquad = \frac{s^2+4s+4-s}{s(s^2+4s+4)}$$

$$C(s) = \frac{A}{s} + \frac{Bs+C}{s^2+4s+4} \qquad = \frac{1}{s}\left[1 - \frac{s}{(s+2)^2}\right]$$

$$A(s^2+4s+4) + s(Bs+C) = s^2+3s+4 \qquad = \frac{1}{s} - \frac{1}{(s+2)^2}$$

$$4A = 4 \quad A = 1 \qquad c(t) = 1 - te^{-2t}$$

$A+B=1 \quad B=-1$ critically damped.

$C + 4A = 3$

$C + 4 = 3$

$C = -1$

s^2+4s+4

$$C(s) = \frac{1}{s} - \frac{s+1}{s^2+4s+4}$$

$$= \frac{1}{s} - \frac{s+1}{(s+2)^2} = \frac{1}{s} - \left[\frac{s+2}{(s+2)^2} - \frac{1}{(s+2)^2}\right]$$

$$c(s) = \frac{1}{s} - \left[\frac{1}{(s+2)} - \frac{1}{(s+2)^2}\right]$$

ILT.

$$c(t) = 1 - \left[e^{-2t} - t\,e^{-2t}\right]$$

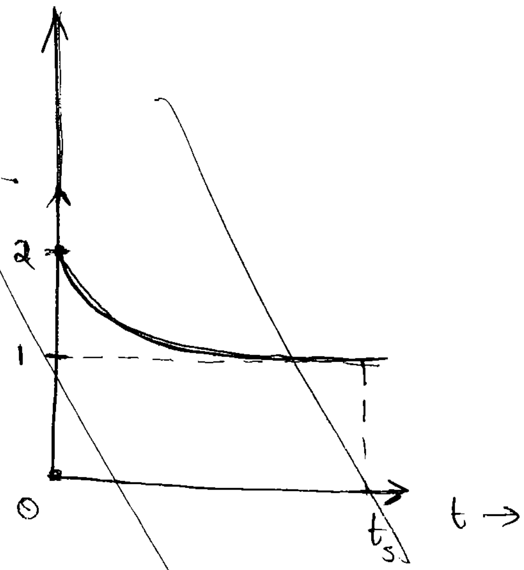
$$c(t) = 1 - e^{-2t} + t\,e^{-2t}$$

$$c(t) = 1 - e^{-2t}(-1+t)$$

Almost similar to critical damped unit step response.

$$c(t) = 1 - e^{-\omega_n t}(1+\omega_n t)$$

Not sure. system is stable.



~~Q, Find the value of k for the feedback control system shown in figure~~

Q Find the steady state error in the o/p of any linear control system with unity feedback when input ~~r(t)~~

~~r(t) = r_0 + r_1(t)~~

$r(t) = r_0 + r_1 t + r_2 t^2$.

Given input is combination of 3 inputs. ie, step, ramp and parabol.

Let $r(t) = r_1(t) + r_2(t) + r_3(t)$

(i) consider step i/p first,

$r_1(t) = r_0$

$R_1(s) = r_0/s$

$$e_{ss} = \underset{s \to 0}{Lt}\ s\left(\frac{R(s)}{1+G(s)}\right) = \underset{s \to 0}{Lt}\ \frac{\cancel{s} \times \frac{r_0}{\cancel{s}}}{(1+G(s))}$$

$$e_{ss} = \underset{s \to 0}{Lt}\ \frac{r_0}{1+G(s)} = \frac{r_0}{1 + \underset{s \to 0}{Lt}\ G(s)}$$

Let K_p = position error constant.

ie, $K_p = \underset{s \to 0}{Lt}\ G(s)$

Then $e_{ss} = \dfrac{r_0}{1+K_p}$

(ii) consider ramp i/p

$$r_2(t) = \cancel{r_1(t)}\ r_1 t$$

$$e_{ss} = \underset{s \to 0}{Lt} \left(\frac{s\,R(s)}{1+G(s)} \right)$$

$$= \underset{s \to 0}{Lt} \frac{\cancel{s} \times \frac{r_1}{s^{\cancel{2}}}}{1+G(s)}$$

$$= \underset{s \to 0}{lt} \frac{r_1}{s + s\,G(s)}$$

Let $K_v \rightarrow$ Velocity error constant.

$$K_v = \underset{s \to 0}{Lt}\ s\,G(s)$$

$$\therefore e_{ss} = \frac{r_1}{K_v}$$

(iii) consider parabolic i/p

$$r_3(t) = r_2 t^2$$

$$= 2r_2 \times \frac{t^2}{2}$$

$$e_{ss} = \underset{s \to 0}{lt} \frac{s\,R(s)}{1+G(s)}$$

$$= \underset{s \to 0}{lt} \frac{\cancel{s} \times \frac{2r_2}{s^{\cancel{3}2}}}{1+G(s)}$$

$$= \frac{2r_2}{s^2 + s^2 G(s)}$$

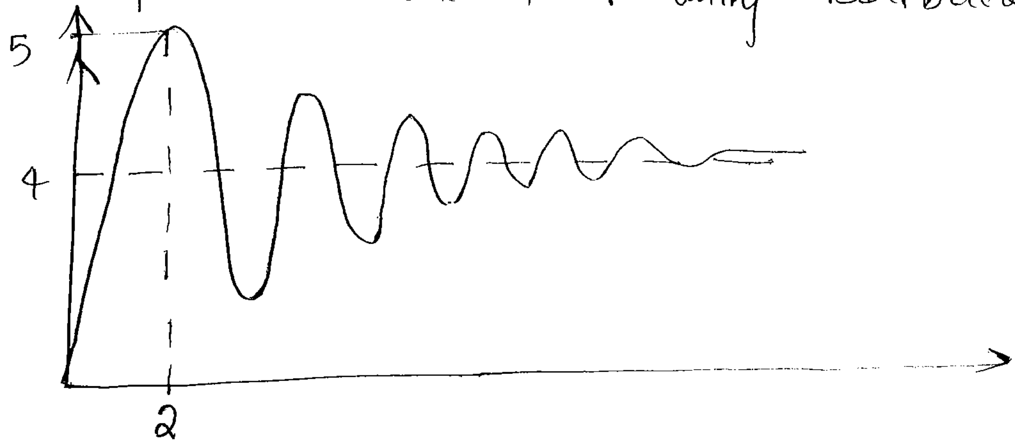
$$e_{ss} = \frac{2r_2}{K_a}$$

$K_a \rightarrow$ Acceleration ~~const~~ error constant

$$K_a = \underset{s \to 0}{Lt}\ s^2 G(s)$$

$$\text{Total error} = \frac{r_0}{1+K_p} + \frac{r_1}{K_v} + \frac{2r_2}{K_a}$$

Q. The step response of a second order system is shown in figure of an input of $4\,u(t)$. Determine open loop and closed loop Tranfer function of unity feedback system.



$t_p = 2$

$\% M_p = 25\%$

$$\frac{\pi}{\omega_d} = 2$$

$$\omega_d = \frac{\pi}{2} = 1.5708 \text{ rad/s.}$$

$$e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 25$$

$$\frac{\pi \xi}{\sqrt{1-\xi^2}} = 1.386$$

$$1.92(1-\xi^2) = 9.869 \times \xi^2$$

$$11.789\,\xi^2 = 1.92$$

$$\xi = 0.4036$$

~~$\omega_d = \omega_n\sqrt{1-\xi^2}$~~

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 1.5708$$

$$\omega_n = \frac{1.5708}{\sqrt{1-\xi^2}}$$

$$\omega_n = \underline{1.7168 \text{ rad/s}}$$

$$\cancel{1+G = s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\cancel{= s^2 + 1.3858}$$

$$\text{OLTF} = G = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

$$= \frac{2.947}{s(s+1.3858)}$$

$$\text{CLTF} = \frac{1}{1+G} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$= \frac{2.947}{s^2+1.3858s+2.947}$$

Q. Consider the unity feedback ~~system~~ control system of OLTF $G(s) = \dfrac{0.4s+1}{s(s+0.6)}$, Obtain the response to unit step i/p and also determine ξ, ω_n, t_r, t_p, $\% M_p$, t_s for $\pm 2\%$

$$G(s) = \frac{0.4s+1}{s^2+0.6s}$$

$$CLTF = \frac{0.4s+1}{s^2+0.6s+0.4s+1}$$

$$= \frac{0.4s+1}{s^2+s+1}$$

$$C(s) = \frac{(0.4s+1)}{s^2+s+1}\cdot\frac{1}{s}$$

$$\cancel{= \frac{0.4+\frac{1}{s}}{s^2+s+1}}$$

$$\cancel{= \frac{0.4}{s^2+s^2+1} + \frac{1}{s(s^2+s+1)}}$$

$$= \left(\frac{0.4}{s^2+}\right. = \frac{0.4}{s^2+s^2+1} + \frac{1}{s(s^2+s+1)}$$

$$\cancel{C(s) = \frac{A}{s} = \frac{A}{s} + \frac{Bs+C}{s^2+s+C}}$$

$$\cancel{A(s^2+s+1)}$$

$$\cancel{0 = }$$

$$c(s) = \frac{0.4}{\left(s+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)\left(s+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)} + \frac{1}{s(s^2+s+1)}$$

$$= c_1(t) + c_2(t)$$

$$C_1(t) = \cancel{\frac{0.4}{s+\frac{1}{2}-j\frac{\sqrt{3}}{2}}} \quad \frac{0.4}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$C_1(t) = L\left[\frac{0.4}{\frac{\sqrt{3}}{2}} \times \frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right]$$

$$= \frac{0.8}{\sqrt{3}} \times e^{-\frac{1}{2}t} \sin\frac{\sqrt{3}}{2}t$$

$$= 0.462\, e^{-\frac{1}{2}t} \sin\frac{\sqrt{3}}{2}t.$$

$$C_2(s) = \frac{A}{s} + \frac{Bs+C}{s^2+s+1}$$

$$A(s^2+s+1) + s(Bs+C) = 1$$

$$A = 1, \quad A+B=0 \quad B=-1$$

$$A+C=0$$

$$C=-1$$

$$C_2(s) = \frac{1}{s} - \frac{s+1}{s^2+s+1} = \frac{1}{s} - \frac{s+1}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$C_2(s) = \frac{1}{s} - \frac{(s+1/2)}{(s+1/2)^2+(\sqrt{3}/2)^2} - \frac{1/2}{(s+1/2)^2+(\sqrt{3}/2)^2}$$

$$= 1 - e^{-1/2 t}\cos\frac{\sqrt{3}}{2}t - 0.577\, e^{-1/2 t}\sin\frac{\sqrt{3}}{2}t$$

$$C_2(s) = 1 - e^{-t/2}\left[\cos\frac{\sqrt{3}}{2}t - 0.577\sin\frac{\sqrt{3}}{2}t\right]$$

$$C(t) = 0.46\, e^{-t/2}\sin\frac{\sqrt{3}}{2}t + 1 - e^{-t/2}\left[\cos\frac{\sqrt{3}}{2}t - 0.577\sin\frac{\sqrt{3}}{2}t\right]$$

$$c(t) = 1 - e^{-1/2 t}\cos\frac{\sqrt{3}}{2}t - 0.1\, e^{-t/2}\sin\frac{\sqrt{3}}{2}t.$$

To the above system, C.E is, $1+GH=0$

$$s^2+s+1=0$$

$$\omega_n = 1$$

$$\xi = 0.5$$

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d} = 2.418\ s$$

$$t_p = \frac{\pi}{\omega_d} = 3.628\ s$$

$$\%\, M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 16.30\,\%$$

Q. For a second order underdamped system, subjected to a unit step input, the step response shows that the first peak equal to 4 times of the second over shoot. Determine the damping ratio of the system and expected overshoot.

$$e^{-\pi\xi/\sqrt{1-\xi^2}} = 4\, e^{-2\pi\xi/\sqrt{1-\xi^2}}$$

$$\frac{-\pi\xi}{\sqrt{1-\xi^2}} = 1.38 - \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\frac{2\pi\xi}{\sqrt{1-\xi^2}} = 1.386$$

$$9.8696\xi^2 = 1.921(1-\xi^2)$$

$$39.4784\ \cancel{11.7906}\xi^2 = 1.921$$

$$\cancel{\xi = 0.4036}$$

$$\xi = 0.2205$$

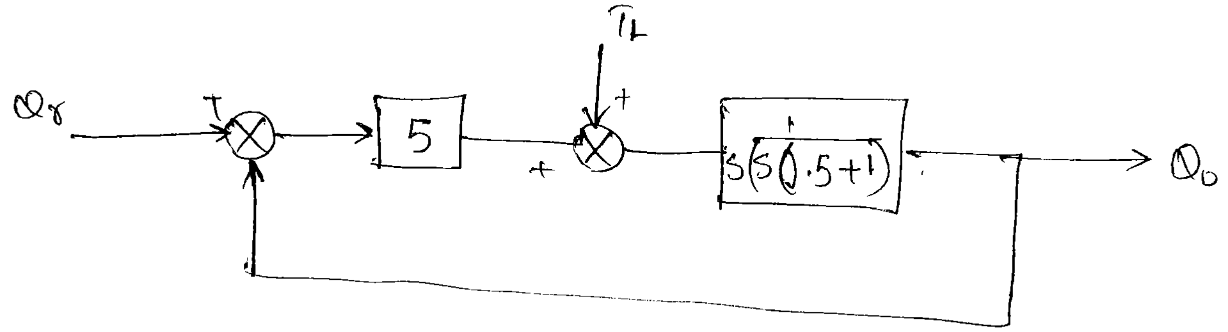
% overshoot = 0.491 x100

= 49.1 % 50 %

Q. For the system represented by the block diagram shown in figure (i) Evaluate closed loop transfer function.

(ii) Calculate ξ and ω_d

(iii) e_{ss} if the i/p is unit ramp, at θ_r



(i)

$$OLTF = \frac{5}{s(0.5s+1)}$$

$$CLTF = \frac{5}{0.5s^2+s+5}$$

$$CLTF = \frac{10}{s^2+2s+10}$$

(ii)

$$\omega_n = \sqrt{10}$$

$$2\xi\omega_n = 2$$

$$\xi\omega_n = 1$$

$$\xi = \frac{1}{\omega_n}$$

$$\xi = \frac{1}{\sqrt{10}} = \underline{0.3162}$$

$$\omega_d = \omega_n\sqrt{1-0.3162}$$

$$\omega_d = \underline{3}$$

(iii)

$$e_{ss} = \lim_{s\to 0} \frac{s \times R(s)}{1+GH}$$

$$= \frac{\cancel{s} \times \frac{1}{s^{\cancel{2}}}}{1+GH}$$

$$= \cancel{\frac{1}{s+s}}$$

$$K_v = \lim_{s\to 0} sG(s) = \frac{\cancel{s} \times 5}{\cancel{s}(0.5s+1)}$$

$$= \frac{5}{1} = \underline{5}$$

$$e_{ss} = \frac{1}{5} = \underline{0.2}$$

Q, Find the steady state error to the unit ramp i/p to the following system $G(s) = \frac{1}{s^2(s+2)}$, $H(s) = \frac{5(s+1)}{(s+5)}$

$$e_{ss} = \lim_{s \to 0} s\left[R(s) - C(s)\right]$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

$$= \frac{1}{s^2(s+2) + \frac{5(s+1)}{s+5}}$$

$$= \frac{s+5}{(s+5)s^2(s+2) + 5(s+1)}$$

$$\text{CLTF} = \frac{s+5}{s^2(s+5)(s+2) + 5(s+1)}$$

$$\cancel{\text{OLTF} = \frac{s+5}{s^2(s+5)(s+2) + 4(s+1)}}$$

~~Type 0~~ ~~Type 1~~ $$\frac{s+5}{s^2(s^2 + 7s + 10) + 5s + 5}$$

$$\text{OLTF} = \frac{s+5}{s^2(s+2)(s+5) + 5(s+1) - (s+5)}$$

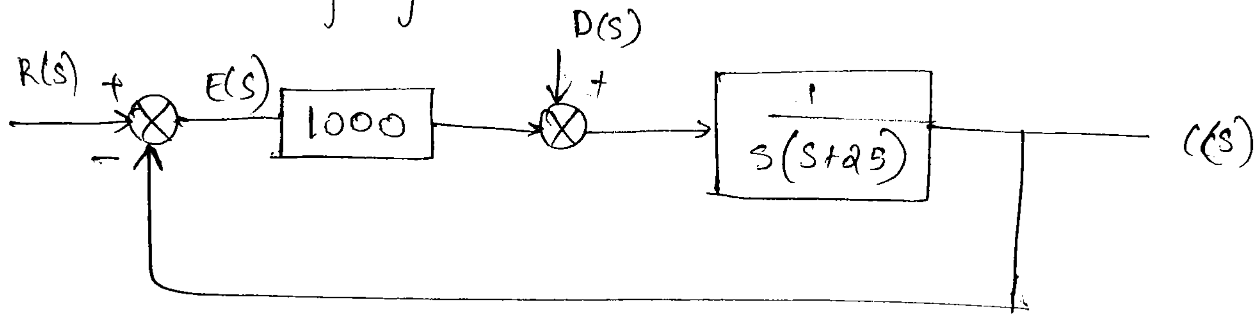
$$= \frac{s+5}{s^4 + 7s^3 + 10s^2 + 5s + \cancel{5} - \cancel{5}}$$

$$G_{NUF} = \frac{(s+5)}{s^{0}(s^3+\cancel{6}s^2+10s+4)}$$

Type ~~0~~ -1, order 4

$$e_{ss} = \frac{A}{K_v} = \frac{1}{5/4} = 4/5 = 0.8$$

Q, Find the steady state error due to the disturbance step i/p in the following system.



$$\frac{E(s)}{D(s)} = \frac{\frac{-1}{s(s+25)}}{1+\frac{1000}{s(s+25)}}$$

$$\frac{E(s)}{D(s)} = \frac{-1}{s(s+25)+1000}$$

$$E(s) = \frac{-1/s}{s(s+25)+1000}$$

$$\underset{s\to 0}{Lt}\ sE(s) = \underset{s\to 0}{Lt}\ \frac{\cancel{s}\times -\frac{1}{\cancel{s}}}{s^2+25s+1000}$$

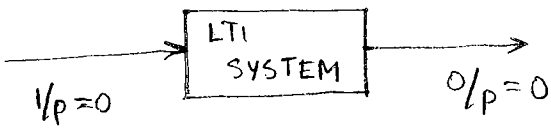
$$= \frac{-1}{1000} = -0.001$$

STABILITY

→ For any LTI system, if the input is bounded, the output also bounded.

→ If the input of the system is Zero, the o/p must be zero, irrespective of all the initial conditions.

→ If the above two conditions are satisfied, the system is STABLE.



stabilities is classified into two ways based on operating condition

→ ABSOLUTELY STABLE SYSTEM

Here the system is stable for all the values of system parameters, like K from 0 to ∞

$$0 < K < \infty$$

→ CONDITIONAL STABLE SYSTEM

Here the system is stable for certain range of system parameters, like K from 0 to 100

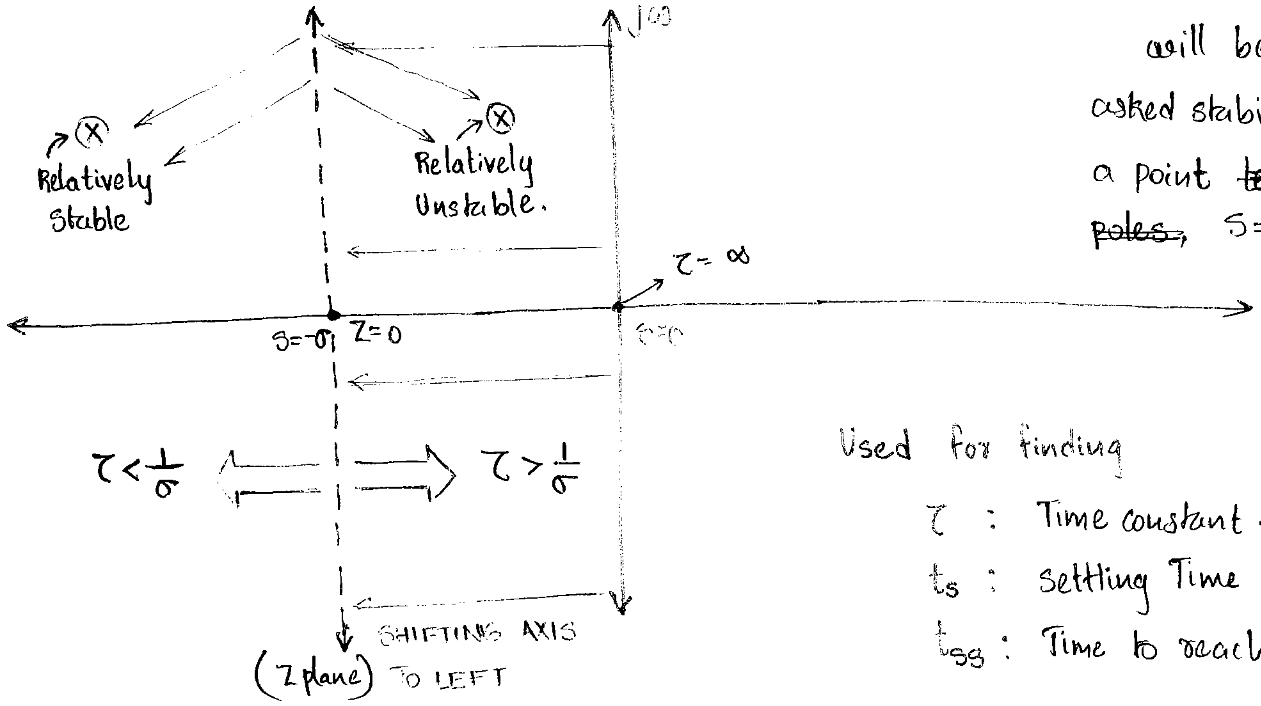
MARGINAL CRITICAL LIMITEDLY STABLE SYSTEM

A linear time invariant system is said to be maginally stable, if for the bounded i/p, the o/p maintain the ~~st~~ constant amplitude and frequency of oscillations.

The non-repeated poles on the imaginary axis, then it is marginally stable.

RELATIVE STABILITY

The relative stability concept applied for only closed loop stable systems.



will be ~~given~~ asked stability w.r.t a point ~~left of the poles,~~ $s=-\sigma$

Used for finding

τ : Time constant.

t_s : Settling Time

t_{ss} : Time to reach steady state

By using relative stability concept we can find system time constant, settling time and time required to reach steady state.

ROUTH HORWITZ CRITERIA (R.H CRITERIA)

(Main purpose: To find right pole)

PURPOSE

- To find the closed loop system stability
- To find the number of poles in the left, right, on imaginary axis of the s plane.
- To find the range of K value for system stability
- To find the K-Value to become the system marginal stable or undamped system.
- To find the natural frequency (undamped frequency) of oscillation.
- To find the relative stability, by using relative stability concept, we can find system time constant, settling time and time required to reach steady state.

CLOSED LOOP STABILITY

- RH criteria — Requires characteristic equation.
- Root Locus, Bode plot, Nyquist plot — Requires OLTF of a ~~no~~ unity feedback system

→ To find the closed loop stability using RH criteria required characteristic eqn where as remaining all other stability techniques requires open Loop Transfer function "OLTF"

→ The n^{th} order general form of characteristic equation is

$$CE \Rightarrow \boxed{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n s^0 = 0}$$

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & \cdots \\
s^{n-1} & a_1 & a_3 & a_5 & \cdots \\
s^{n-2} & \dfrac{a_1 a_2 - a_0 a_3}{a_1} & \dfrac{a_1 a_4 - a_0 a_5}{a_1} & \cdots & \\
\vdots & & & & \\
s^1 & & & & \\
s^0 & a_n & & &
\end{array}$$

→ The conditions for system stabilities are

(i) All the coefficients in the first column should have the same sign, and no coefficient is zero in the first column.

(ii) The number of sign changes in the first column equal to number of poles in the right hand side and the

Q. Find the stability to the following characteristic equation.

(1) $s+10=0$

(2) $s^2+25=0$

(3) $s^2+10s+10=0$

(4) $s^3+7s^2+5s+10=0$

(5) $s^3+6s^2+4s+100=0$

(6) $s^3+8s^2+4s+32=0$

1)
$$\begin{array}{c|c} s^1 & 1 \\ s^0 & 10 \end{array}$$
system stable.

First order system
$$\left.\begin{array}{c|c} s & a \\ s^0 & b \end{array}\right\}$$
$a, b > 0$, stable.

2)
$$\begin{array}{c|cc} s^2 & 1 & 25 \\ s^1 & 0 & 0 \\ s^0 & 250 & \end{array}$$
System unstable.

For second order system:
$$\left.\begin{array}{c|c} s^2 & a \\ s^1 & b \\ s^0 & c \end{array}\right\}$$
$a, c > 0$, $b=0$ marginally ~~marginally~~ stable.

3)
$$\begin{array}{c|cc} s^2 & 1 & 10 \\ s^1 & 10 & 0 \\ s^0 & 10 & 0 \end{array}$$
system stable

$$\left.\begin{array}{c|c} s^2 & a \\ s^1 & b \\ s^0 & c \end{array}\right\}$$
$a, b, c > 0$

4)
$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 7 & 10 \\ s^1 & \frac{25}{7} & 0 \\ s^0 & \frac{\frac{250}{7}}{\frac{25}{7}} = 10 & \end{array}$$
System stable.

5) $6 \times 4 < 100$

$24 < 100$

Hence system unstable.

6) $32 = 32$

Hence marginally stable

For third order system

Find Internal product − External product

if > 0 stable

if < 0 Unstable.

or

Internal prodt > External product

stable

$35 - 10 = 25$

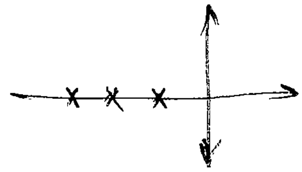
stable.

Consider third order system,

$$as^3 + bs^2 + cs + d = 0$$

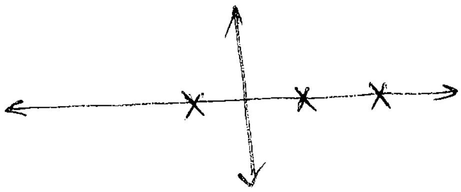
$$\begin{array}{c|cc}
s^3 & a & c \\
s^2 & b & d \\
s^1 & \frac{bc-ad}{b} & \\
s^0 & d &
\end{array}$$

~~If~~ Hence conditions, $d>0$, $a>0$, $b>0$



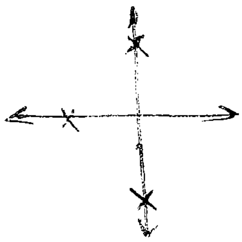
$\Rightarrow$ If ~~Also~~ $\frac{bc-ad}{b} > 0$, ie, $bc > ad \Rightarrow$ Stable,
All poles in left.

$\Rightarrow$ If $\frac{bc-ad}{b} < 0 \Rightarrow$ Unstable.,
$bc < ad$, and if $d > 0$
Then 2 sign changes, Hence two poles in right.



$\Rightarrow$ If $bc = ad$

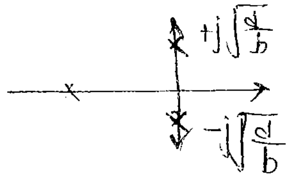
Marginally stable.



ie, only even powers of s terms.

$as^3+bs^2+cs+d=0$ changes to $bs^2+d=0$

$$s = \pm j\sqrt{\frac{d}{b}}$$



Q Find the number of poles in the right of s ~~splat~~ plane

$$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$$

s^4	1	3	5
s^3	2	4	
s^2	1	5	
s^1	-6		
s^0	5		

2 sign changes. Hence 2 poles in the right side of s plane. System Unstable.

2 poles $\longrightarrow$ right

2 poles $\longrightarrow$ Left

Q $s^4 + 2s^3 + 3s^2 + 2s + 1 = 0$

s^4	1	3	1
s^3	2	2	
s^2	2	1	
s^1	1		
s^0	1		

Zero sign changes. Hence no poles in Right side of ~~poles~~ s plane.

Hence stable. 4 pole is left of s plane.

Q_1 $\quad s^4 + 2s^3 + 3s^2 + s + 2 = 0$

$$
\begin{array}{c|ccc}
s^4 & 1 & 3 & 2 \\
s^3 & 2 & 1 & \\
s^2 & 5/2 & 2 & \\
s^1 & -3/5 & & \\
s^0 & 2 & &
\end{array}
$$

sign ch

sign change

$$\frac{2.5 - 4}{5/2}$$

$$\frac{-3/2}{5/2}$$

2 sign changes, 2 poles in right
2 poles in left.

System unstable.

Q_1 $\quad s^4 + 2s^3 + 2s^2 + 4s + 8 = 0$

$$
\begin{array}{c|ccc}
s^4 & 1 & 2 & 8 \\
s^3 & 2 & 4 & \\
s^2 & 0 \to \varepsilon & 8 & \\
s^1 & \frac{4\varepsilon - 16}{\varepsilon} & & \\
s^0 & 8 & &
\end{array}
$$

sign change

sign change

-16

$\cancel{4\varepsilon - 16}$

$$\frac{4\varepsilon - 16}{\varepsilon} \Longrightarrow 4 - \frac{16}{1\varepsilon} \Rightarrow -\infty$$

2 sign changes hence 2 roots of RHS.

System unstable.

⇒ If any one element is zero in first column, replace by smallest positive constant ε and continue the routh stability. Finally substitute $\varepsilon = 0$ and check the number of sign changes.

Q, $s^5 + s^4 + 2s^2 + 3s + 15 = 0$

s^5	1	2	3
s^4	1	2	15
s^3	$0 \rightarrow \varepsilon_1$	-12	
s^2	$\frac{2\varepsilon_1 + 12}{\varepsilon_1}$	15	
s^1	-		
s^0	15		

Sign change

Sign change.

-12×15

$\cancel{2\varepsilon_1 + 12}$

$2 + \frac{12}{\varepsilon_1} \rightarrow +\infty$

$$\frac{-12\left(\frac{2\varepsilon_1 + 12}{\varepsilon_1}\right) - 15\varepsilon_1}{\frac{2\varepsilon_1 + 12}{\varepsilon_1}}$$

$= -\infty$

2 sign change, unstable.

2 root on right hand side.

Q $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2$

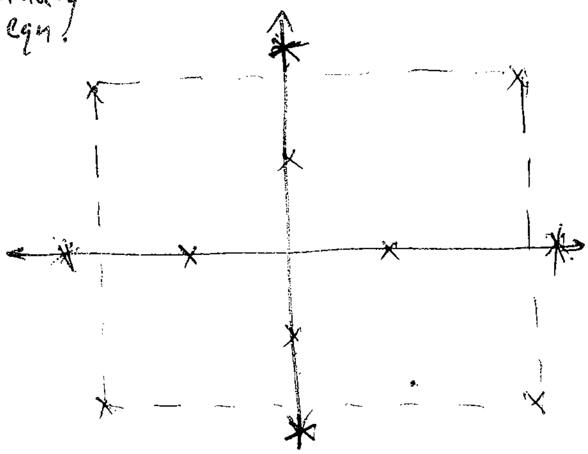
s^5	1	3	2
s^4	1	3	2
s^3	0	0	0
s^2	$3/2$	2	
s^1	$2/3$		
s^0	2		

Auxillary eqn!

One time row of zero indicates NON REPEATED POLES

$s^4 + 3s^2 + 2 = 0$

Diff $4s^3 + 6s = 0$



DIFFICULTY - 2

whenever the row of zeros occurs in the routh tabular form, then we require to form the auxillary eqn using the above the row of zero coefficients and differentiate the auxillary equation and replace the zeros by the coefficients of differential auxilary equation.

⇒ If Row of zero occur in RH tabular form, when the poles are lying symmetrically about the origin (which includes double pole of imaginary axis.

⇒ The auxillary equation must contain even power of s only.

⇒ The roots of auxillary equation must be symmetrical about the origin.

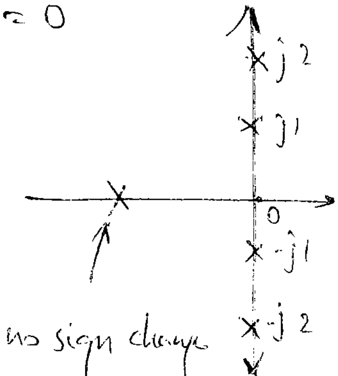
⇒ The row of zero must be occur only in odd power of s power of rows.

⇒ The roots of auxillary equation is ~~the~~ actually the closed

In last q^n,

the auxillary equ is given by $(s^2+1)(s^2+2) = 0$

$s = \pm j1, \pm j2$



Here no sign change so the fifth root will be in left side only. (not exact position).

~~ie, if all the elements in first~~

Note :

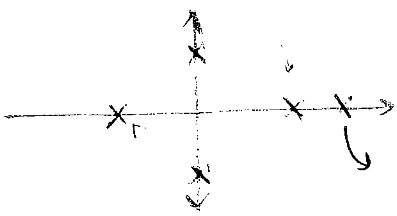
whenever only once the row of zero occurs and all the coefficients in the first column are having same sign then the system is marginal stable. because the poles must lies on imaginary axis which are non repeated.

Q. consider the case of

s^5	$-$	No symm pole LH to the pole placed in RH
s^4	$+$	→ A.E (4th order − 4 poles).
s^3	0 0 0	1ROZ
s^2	$+$	
s^1	$+$	Symm pole in LH to the pole placed in RH
s^0	$-$	

Then first consider the auxillary equ. A.E is an 4th order equ. Hence will have 4 roots or poles symmetric to origin.

Now consider the number of sign change below the auxillary equ. All those below will contribute to A.E



sign change above A.E.

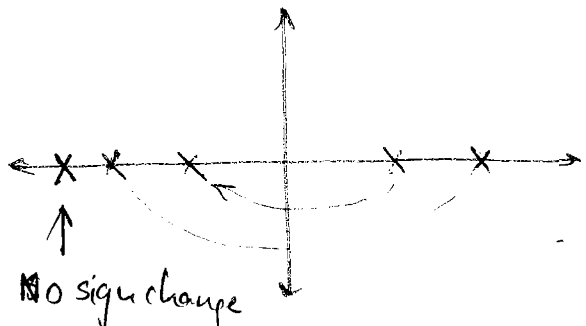
Note:

The sign changes occurs below auxillary equation there must be a symmetrical pole, in the left to the pole placed in the right hand side.

The sign changes ~~ab~~ occurs above the auxillary equation, there is no symmetrical pole in the left to the pole placed in the right of s plane.

Q.

s^5	+			
s^4	+			⟶ Auxillary eqⁿ 4^{th} order → 4 poles.
s^3	0	0	0	⟶ IRO2 → Non repeated poles but symm about.
s^2	+			
s^1	−			
s^0	+			



Q. $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$

~~s^6~~	~~1~~	~~4~~	~~5~~	~~2~~
s^5	3	6	3	
~~s^4~~	~~4~~ 2	~~3~~	~~2~~	
s^3	$3/2$	−1		
s^2		2		
s^1				

$\frac{3}{2} x$

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	$0 \to 8$	$0 \to 8$		
s^2	2	2		
s^1	$0 \to 4$			
s^0	2			

s^4 row → (AE) ; $A \cdot E \Rightarrow 2s^4 + 4s^2 + 2$

s^3 row → 1 ROZ ; $\frac{dA \cdot E}{ds} = 8s^3 + 8s$

s^2 row → $(A \cdot E)_1$; $2s^2 + 2 = 0$

s^1 row → 1 ROZ (~~2nd row~~) ; $\frac{dA \cdot E}{ds} = 4s$

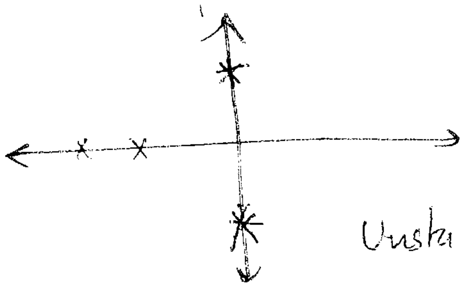
$(AE)_1$

$2s^4 + 4s^2 + 2 = 0$

$s^4 + 2s^2 + 1 = 0$

$(s^2+1)^2 = 0$

$s^2 = \pm j1, \pm j1$



Unstable (Repeated poles on imaginary axis)

Note :-

The number of times, the rows of zeros indicates the number of poles repeated.

whenever in the Routh Tabular form, many times row of zeros occur and all the coefficients in the first column are positive, then the system is unstable because of the poles lies of the imaginary axis which are repeated

Q. Find the number of poles in the right hand side to the given characteristic equation.

ie, $s^4+s^3-s-1=0$

s^4	1	0	-1
s^3	1	-1	
s^2	1	-1	
s^1	0		
s^0	-1		

1 sign change

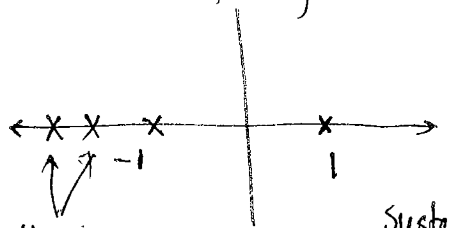
s^2 row → A.E $\Rightarrow s^2-1=0$

$s^2=1$

$s=\pm 1$

s^1 row → 1 ROZ

No imaginary ∵ 2^{nd} order A.E only two roots.

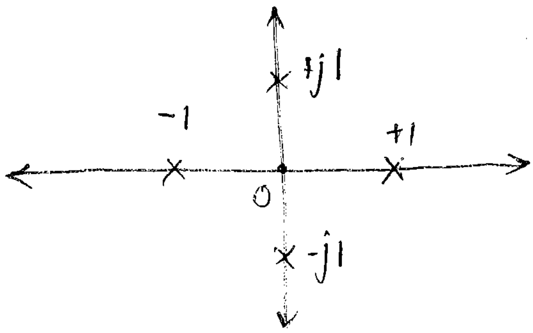


No sign change above A.E.

System Unstable.

Q. Identify the routh tabular form to the given poles locations in the s place.

s^4	
s^3	.
s^2	
s^1	
s^0	



→ 1 ROZ → Symmetrical but not repeated.

→ One sign change below A.E

→ 4^{th} order symmetrical eqn.

→ 1 ~~top~~ time ROZ occurs because the poles are symmetrical but non repeated.

→ One sign change occurs below A.E because there exists a symmetrical pole

$$A\cdot E = (s^2-1)(s^2+1)$$

$$= s^4 - \cancel{s^2} + \cancel{s^2} - 1$$

$$= (s^4-1)$$

$$\begin{array}{c|ccc}
s^4 & 1 & 0 & -1 \\
s^3 & 0 \to 4 & 0 & 0 \\
s^2 & 0 \to \epsilon & -1 & \\
s^1 & \frac{4}{\epsilon} & & \\
s^0 & -1 & &
\end{array}$$

$s^4-1 \to A\cdot E$

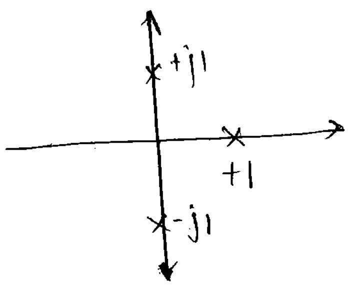
$$\frac{dA\cdot E}{ds} = 4s^3$$

$$\begin{array}{c|ccc}
s^4 & 1 & 0 & -1 \\
s^3 & 0 \to 4 & 0 & 0 \\
s^2 & 0 \to \epsilon & -1 & \\
s^1 & \frac{4}{\epsilon} & & \\
s & -1 & &
\end{array}$$

s^3 row $\to$ 1 RO2

sign change

Q



$A\cdot E \to 2^{nd}$ order.

$A\cdot E \to s^2+1=0$

$C\cdot E \to (s-1)(s^2+1)=0$

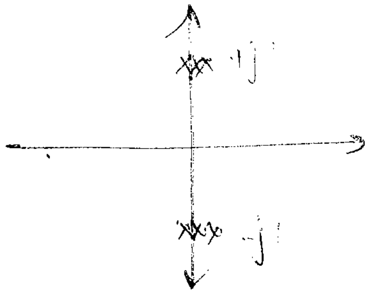
$$s^3-s^2+s-1=0$$

$$\begin{array}{c|cc}
s^3 & 1 & 1 \\
s^2 & -1 & -1 \\
s^1 & 0 & 0 \\
s^0 & -1 &
\end{array}$$

s^1 row $\to$ 1 RO2

1 sign change

Q



$+j1 \quad -j1 \qquad s^2 = -1$

$s^2+1 = 0 \quad s = \pm j$

$$(s^2+1)(s^2+1)(s^2+1)$$

$$\cancel{(s^2+3)^3} \quad s^6 + 3s^4 + 3s^2 + 1 = 0$$

s^6	1	3	3	1	$\rightarrow (A.E)_1$
s^5	$0 \nearrow 6$	$0 \nearrow 12$	$0 \nearrow 6$		$\rightarrow$ 1 ROZ
s^4	1	2	1		$\rightarrow (A.E)_2$
s^3	$0 \nearrow 4$	$0 \nearrow 4$			$\rightarrow$ 2 ROZ
s^2	1	1			$\rightarrow (A.E)_3$
s^1	$0 \nearrow 2$				$\rightarrow$ 3 ROZ
s^0	1				

$\frac{d(A.E)_1}{ds}$

$6s^5 + 12s^3 + 6s = 0$

A.E $\quad s^4 + 2s^2 + 1 = 0$

dA.E

$4s^3 + 4s$.

$s^2+1 = 0$

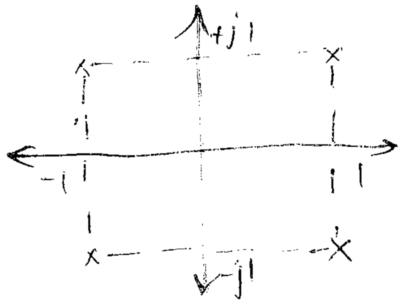
$\frac{d(A.E)_3}{ds} \Rightarrow 2s$

No sign change.

3 ROZ s.

repeated poles.

Q (1)



(1)

$$\cancel{(s+1)^2 (s} \; [(s+1)^2+1] \cdot [(s-1)^2+1] = 0$$

$$(s^2+2s+2)(s^2-2s+2) = 0$$

$$s^4 + 4 = 0$$

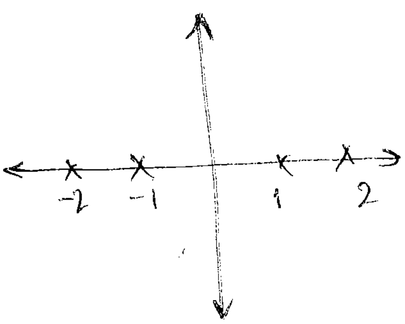
s^4	1	0	4
s^3	$0 \nearrow 4$	0	$\rightarrow$ 1 ROZ
s^2	$0 \nearrow \epsilon$	4	
s^1	$\frac{-16}{\epsilon}$		
s^0	4		

$4s^3$

1 sign change

2 sign change.

(2)



(2) $(s-1)(s+1)(s-2)(s+2)$

$= (s^2-1)(s^2-4)$

$= s^4 - s^2 - 4s^2 + 4$

$= s^4 - 5s^2 + 4$

s^4	1	-5	$+4$ → A.E.
s^3	~~0~~ 4	~~0~~ -10	→ 1 RO2
s^2	$-5/2$	4	
s^1	$\frac{18}{-5}$		
s^0	4		

sign change ($s^2 \to s^1$)

sign change ($s^1 \to s^0$)

$s^4 - 5s^2 + 4 = 0$

$4s^3 - 10s$.

$\frac{-20+10}{4} = \frac{-10}{4} = -5/2$

$\frac{-\frac{5}{2} \times -10 - 4\times 4}{-5/2}$

$\frac{25-16}{-5/2} = \frac{18}{-5}$

Q, Identify the number of poles in the left, right or on imaginary axis of the s plane to the given sample routh tabular form.

s^7	+
s^6	+ → A.E → 6th order.
s^5	0 0 0 → 1RO2
s^4	+
s^3	0 0 0 → 2RO2
s^2	+
s^1	−
s^0	+

sign change ($s^2 \to s^1$)

sign change ($s^1 \to s^0$)

2 repeated poles.

7

2 left ← 2 right

~~4 left~~

~~3 right~~

~~2 imaginary~~

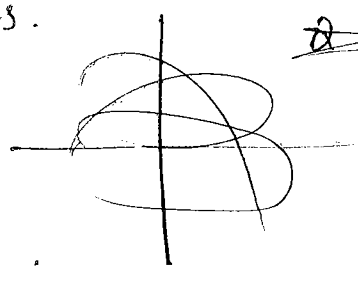
6

2R → 2L

Imagin 2 left.

3 left 2 right

2 imaginary.

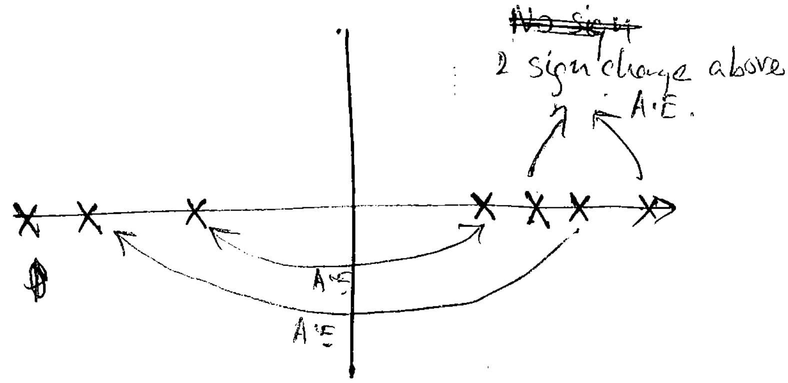


Q,

s^7	+		
s^6	+		
s^5	−		
s^4	+		
s^3	+ 0	0	0
s^2	+		
s^1	−		
s^0	+		

4^{th} order.

$\rightarrow$ IROZ



3 left

3 right.

Q, (i) Find the range of K value for system stability.

(ii) Find the K value to become the system marginal stable. or K value to become the systems undamped.

(iii) Find the natural frequency of oscillations or undamped oscillation to the given characteristic equ.

$$s^3 + 8s^2 + 4s + K = 0$$

K, 32

$K > 0$

stability Internal > External

$32 > K$

$\therefore 0 < K < 32$

s^3	1	4
s^2	8	K
s^1	$\frac{32-K}{8}$	
s^0	K	

(i) For stability

$$\frac{32-K}{8} > 0$$

~~$K > 32$~~ $0 < K < 32$

(ii) $\dfrac{32-K}{8} = 0$

$32 - K = 0$

$\underline{\underline{K = 32}}$

$K_{undamped} = K_{marginal} = 32$

Take only ~~even~~ odd power row equal to zero. Then only A.E contains only even power of s.

Internal = External

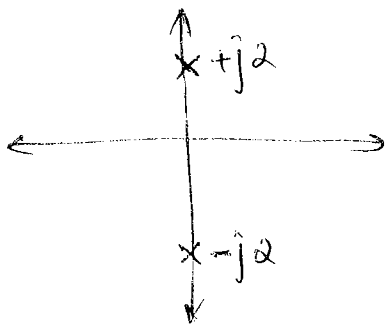
$\underline{32 = K}$

(iii) A.E $\Rightarrow 8s^2 + \cancel{0}K = 0$

$8s^2 + 32 = 0$

$s^2 + 4 = 0$

$s = \pm j2$



<u>Note:</u> For marginal stability, s^0 coefficient not considered. Because if s^0 coefficient made equal to zero, then the row of zero occurs. In this case, it forms the auxillary eqn, it consist the odd power of s terms which is undesirable

Q, $2s^3 + 5s^2 + 10s + (K+5) = 0$

$K + 5 > 0$

$K > -5$

$50 > 2K + 10$

$40 > 2K$

$\underline{K < 20}$

For stability $\rightarrow$ $\underline{\underline{-5 < K < 20}}$

For marginal
Internal = External

$50 = 2(K+5)$

$50 = 2K + 10$

$2K = 40$

$\underline{\underline{K = 20}}$

$K_{marginal} = K_{undamped} = \underline{20}$

f.o.o Even pow of s-term = 0

$5s^2 + 25 = 0$ $s = \pm j\sqrt{5}$ $\omega_n = \sqrt{5}$

$$GH(s) = \frac{K}{s(s+2)(s+4)(s+6)}$$

$$CLTF = \frac{K}{s(s+2)(s+4)(s+6) + K}$$

Characteristic equ $= s(s+2)(s+4)(s+6) + K$

$= (s^2+2s)(s^2+10s+24) + K$

$= s^4 + 10s^3 + 24s^2 + 2s^3 + 20s^2 + 48s + K$

$= s^4 + 12s^3 + 44s^2 + 48s + K = 0$

s^4	1	44	K
s^3	12	48	
s^2	40	K	
s^1	$\frac{1920-12K}{40}$		
s^0	K		

Here in this case, for simplification and easiness, we can divide constant (positive constant) from the entire row.

$$\frac{1920-12K}{40}$$

For stability $K > 0$

$$\frac{1920-12K}{40} > 0$$

$K < 160$

For stability $0 < K < 160$

For marginally stable

$K = 160$

$A.E = 40s^2 + 160 = 0$

$40s^2 = -160$

$s^2 = -4$

$s = \pm j2$

f.o.o $= 2$ rad/s

Q, A unity feedback control system as a forward loop transfer function

$$G(s) = \frac{K}{(s+1)^3(s+4)}$$

(i) Determine the range of K value for closed loop system stability

(ii) K value to become the marginal stable.

(iii) Frequency of oscillations when marginal stable.

C.E $\Rightarrow$ $1+GH = 0$

$$(s+1)^3(s+4) + K = 0$$

$$(s^3 + 3s^2 + 3s + 1)(s+4) + K = 0$$

$$s^4 + 3s^3 + 3s^2 + s + 4s^3 + 12s^2 + 12s + 4 + K = 0$$

$$s^4 + 7s^3 + 15s^2 + 13s + (K+4) = 0$$

s^4	1	15	$(K+4)$
s^3	7	13	
s^2	$\frac{92}{7}$	$K+4$	
s^1	$\dfrac{\frac{1196}{7} - 7(K+4)}{\frac{92}{7}}$		
s^0	$K+4$		

(i) stability

$K+4 > 0$

$K > -4$

$$\frac{1196}{7} - 7(K+4) > 0$$

$$7(K+4) < \frac{1196}{7}$$

$$K < 20.41$$

$-4 < K < 20.41$

(ii) Marginal stable.

$$K = 20.41$$

A.E $\Rightarrow$

$$\frac{92}{7} s^2 + 20.41 = 0$$

$$\frac{92}{7} s^2 = -20.41$$

$$s^2 = \cancel{1.55} \ldots$$

$$s = \pm j\, 1.360$$

(iii) f.o.o = 1.360 rad/s

Q) The loop transfer fn of a closed loop system is

$$GH = \frac{K(s+1)}{s^3 + bs^2 + 3s + 1}$$

oscillates with a frequency of 2 rad/s. Then the values of K and b are ?

CE ?

$$1 + GH = 0$$

Given systems must be marginal stable because it should oscillates with 2 rad/s.

$$\Rightarrow s^3 + bs^2 + 3s + 1 + K(s+1) = 0$$

$$s^3 + bs^2 + (3+K)s + (K+1) = 0$$

s^3	1	$3+K$
s^2	b	$K+1$
s^1		
s^0		

s^3
s^2
s^1

$$b(3+k) = K+1 \qquad b = \frac{K+1}{K+3}$$

$$3b + bK = K+1 \longrightarrow (1)$$

$$bs^2 + (K+1) = 0$$

$$\cancel{\frac{K+1}{K+3} = \frac{K+1}{4}}$$

K

$$bs^2 = -(K+1)$$

$$s^2 = \frac{-(K+1)}{b}$$

$$\sqrt{\frac{K+1}{b}} = 2$$

$$K+1 = 4b$$

$$\frac{K+1}{b} = 4$$

$$\cancel{K = 4}$$

$$b = \frac{K+1}{4}$$

$$\cancel{b = 3}$$

$$\cancel{b = -\tfrac{1}{2}}$$

$$(1) \Rightarrow \cancel{\frac{3(K+1)}{4} + \frac{(K+1)K}{4} = K+1}$$

$$K = 1$$

$$\cancel{\frac{3}{4} + \frac{K}{4} = 0}$$

$$b = 0.5$$

$$\cancel{\frac{K}{4} = \frac{-3}{4}}$$

$$\cancel{K = -3} \qquad \cancel{b = \frac{-3+1}{4} = \frac{-2}{4} = -\tfrac{1}{2}}$$

Q A unity feedback system $G(s) = \dfrac{K(s+2)}{s^3 + Ps^2 + 3s + 2}$ is critically stable, and oscillates with a frequency of 2.5 rad/s. Calculate K & P.

$$s^3 + Ps^2 + (3+K)s + (2K+2) = 0$$

s^3	1	$3+K$
s^2	P	$2K+2$
s^1		
s^0		

$$P(3+K) = 1(2K+2)$$

$$\underline{K = 3.25}$$
$$\underline{P = 1.36}$$

$$P = \frac{2K+2}{K+3}$$

$$Ps^2 + (2K+2) = 0$$

$$s^2 = \frac{-(2K+2)}{P} = 2$$

$$\sqrt{\frac{2K+2}{P}} = 2$$

$$\frac{2K+2}{P} = \frac{25}{4} \qquad \underline{P = 1.36}$$

$$P = \frac{8(K+1)}{25}$$

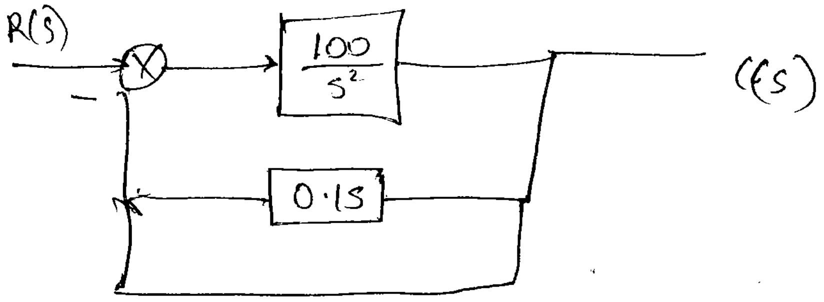
$$\frac{\cancel{25}\cdot \overset{4}{\cancel{8}}\cancel{(K+1)}}{25} = \frac{\cancel{2(K+1)}}{K+3}$$

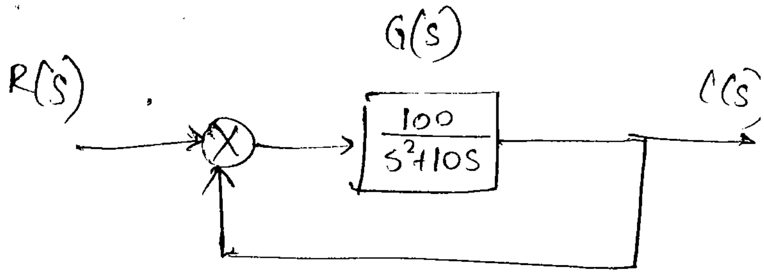
$$4(K+3) = 25$$

$$K+3 = \frac{25}{4}$$

$$K = \frac{13}{4} = 3.25$$

Q Check the stability, to the given block diagram.





$$G(s) = \frac{100}{s^2+10s}$$

~~#~~ CE $\Rightarrow$

$$1+GH = 0$$

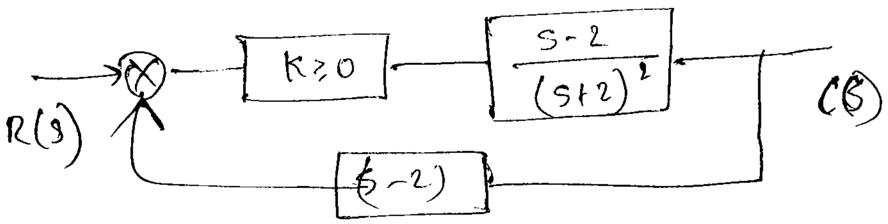
$$s^2+10s+100 = 0$$

$$\begin{array}{c|cc} s^2 & 1 & 100 \\ s^1 & 10 & \\ s^0 & 100 & \end{array}$$

No sign change.
Hence stable.

stable.

Q Find the range of K value.



$$\frac{K(s-2)}{(s+2)^2}$$

$$\frac{C(s)}{R(s)} = \frac{K(s-2)}{(s+2)^2 + K(s-2)^2}$$

$$\frac{C(s)}{R(s)} = \frac{K(s-2)}{(s^2+4s+4) + K(s^2-4s+4)}$$

$$= \frac{K(s-2)}{(K+1)s^2 + (1-K)s + (1+K)4}$$

$1 + GH = 0$

$$(K+1)s^2 + (1-K)s + (1+K)4 = 0$$

s^2	$K+1$	$(1+K)4$
s^1	$(1-K)$	
s^0	$(K+1)4$	

$K+1 > 0$

$K > -1$

$1-K > 0$

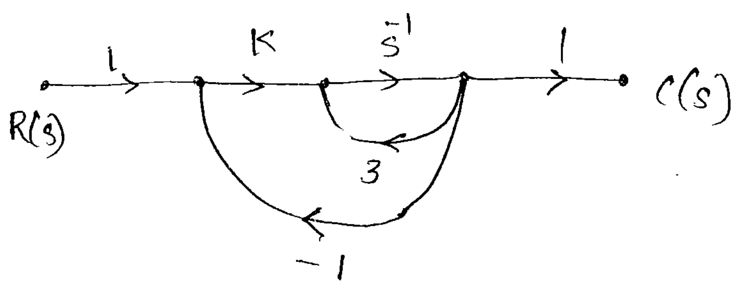
$K < 1$

$-1 < K < 1$

Given $K \geq 0$

Hence $0 \leq K < 1$

Q Find the range of K value,



$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s}}{1 - \frac{3}{s} + \frac{K}{s}}$$

Q. A system,

$$G(s) = \frac{k}{s^3+4s^2+8s}, \quad H(s) = 1$$

For what value of k, the system produce continuous oscillations.

$$1+GH = 0$$

$$s^3+4s^2+8s+K = 0$$

$$K_{marginal} = 32$$

Q. (i) What are the methods for determining the stability of LTI system.

(ii) A third order control system characteristic eqn

$$s^3 + 3408.3s^2 + 1204000s + 1.5\times10^7 K = 0$$

$$K = 273.573$$

(iii) Determine the f.o.o ?

$$3408.3s^2 + \cancel{1.5\times} = -1.5\times10^7 K$$

$$s = \pm j\,1097.269$$

$$\omega_d = 1097.269 \text{ rad/s}$$

(i) There are 5 methods to find the closed loop stability

(i) RH criteria (ii) Root locus (iii) Bode plots (iv) Polar plots (v) Nyquist plots >

$$\frac{K}{S}$$

$$= \frac{K}{s\left(1 + \frac{K-3}{S}\right)}$$

$$= \frac{K}{S + K - 3}$$

S^1	1
S^0	$K-3$

$K - 3 > 0$

$\underline{K > 3}$ for stability.

Q The loop gain of a closed loop system is described by the following expression

$$GH = \frac{K}{s(s+2)(s+4)}$$

The value of K to become the system just stable is

$$s(s+2)(s+4) + K = 0$$

$$s(s^2 + 6s + 8) + K = 0$$

$$s^3 + 6s^2 + 8s + K = 0$$

$6 \times 8 = K \qquad \underline{K = 48}$

Just become stable = Marginal.

(x) A System ~~has~~ has OLTF

$$G(s) = \frac{10}{s(1+0.5s)(1+0.2s)}$$

having a feedback $H(s) = 1+\tau s$.

(i) what should be the value of τ, so that the closed loop system is stable.

$1+GH = 0$

$$s(1+0.5s)(1+0.2s) + 10(1+\tau s) = 0$$

$$s\left(\frac{s^2}{10} + \frac{7}{10}s + 1\right) + 10 + 10\tau s = 0$$

$$\frac{s^3}{10} + \frac{7s^2}{10} + (1+10\tau)s + 10 = 0$$

$$s^3 + 7s^2 + 10(1+10\tau)s + 100 = 0$$

$$7\cancel{0}(1+10\tau) > 10\cancel{0}$$

$$1+10\tau > \frac{10}{7}$$

$$10\tau > \frac{3}{7}$$

$$\tau > \frac{3}{70}$$

$$\underline{\tau > 0.0429}$$

Q (i) Explain the difficulties involved in the application of RH criteria and bring out the limits.

(ii) Find the stability of the control system is given by the characteristic eqn
$(s-1)^2 (s+1)(s+2) = 0$

(i) ~~Difficulty 1 :~~ In the routh stability arises two difficulties

Difficulty 1 : whenever the first element in any row is zero and rest of the elements are non zero, then we cannot continue the routh stability, because, in the next row, ∞ term present.

Remedy : Replace zero by smallest positive constant ϵ and continue the routh stability. Finally replace $\epsilon = 0$ and check the number of sign changes.

Difficulty 2 : when all the elements in the any one row is zero, then routh test is breakdown.

Remedy : In this case, we require to form the auxillary eqn, by using the above the row of zero coefficients and differentiate the auxillary equation and replace the zeros by the coefficients of differential A.E.

LIMITS OF R-H CRITERIA

⇒ The R-H criteria is not applicable for exponential, sine or cose terms.

⇒ R-H criteria is not applicable to finite number of term.

or right or on imaginary axis but ~~not~~ not exact location of the poles. Due to this we cannot identify the ~~number o~~ nature of the system.

RELATIVE STABILITY

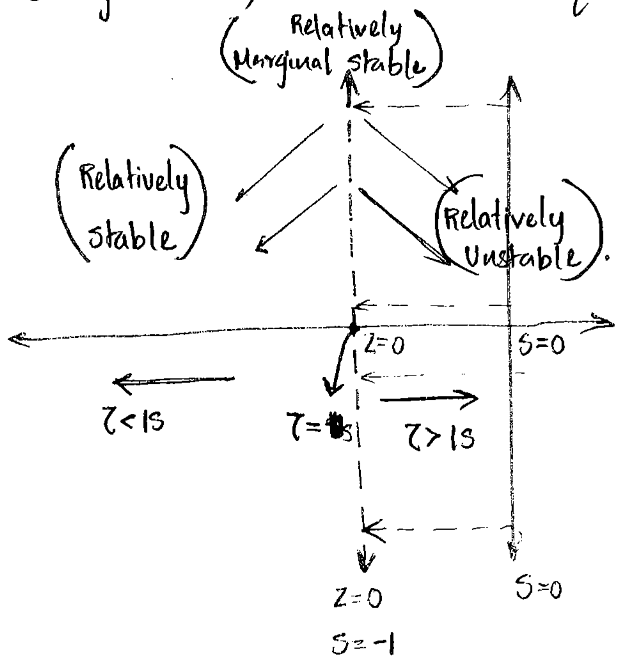
The relative stability concept applied for only closed loop stable system.

Q A system has $G(s) = \dfrac{2}{s(s+1)(s+2)}$, $H(s)=1$. With R-H criteria determine its relative stability about the line $s=-1$.

OR

~~soln.~~

Check whether time constant greater, lesser or equal to 1s.



Soln

$$G(s) = \frac{2}{s(s+1)(s+2)}$$

shift axis location $s=-1$

$$= \frac{2}{(s-1)\,s\,(s+1)}$$

$$= \frac{2}{s(s-1)(s+1)} \quad \text{or} \quad \frac{2}{z(z-1)(z+1)}$$

$CE \Rightarrow z^3 - z + 2$

z^3	1	-1
z^2	$\cancel{0}$ ϵ	2
z^1	$\dfrac{-\epsilon-2}{\epsilon}$	
z^0	2	

2 sign changes.

Relative unstable

2 poles b/w s=0 & -

1 pole LH
s=-1

$$\boxed{s = Z + \text{Shift Axis Location}}$$

$$\boxed{s = z - 1/\tau}$$

<u>Short cut</u> : For such qus as objective, enter eqn to calci and find root and just compare it with the POINT OF RELATIVENESS OR SHIFT AXIS LOCATION.

"The R-H criteria is not applicable for exponential, sine or cosine terms because it gives infinite series. But we can approximate solution to the exponential terms."

Q. Find the range of K value to the given transportation delay system $G(s)H(s) = \dfrac{Ke^{-s\tau}}{s(s+1)}$

$$e^{-s\tau} = 1 - s\tau + \underbrace{\frac{(s\tau)^2}{2!} + \cdots}_{\to 0}$$

Neglect higher order terms.

ie, $G(s)H(s) = \dfrac{K(1-s\tau)}{s(s+1)}$

$CE \Rightarrow 1 + G(s)H(s) = 0$

$$\Rightarrow s^2 + s + K - Ks\tau = 0$$

$$s^2 + (1-K\tau) + K = 0$$

s^2	1
s^1	$1-K\tau$
s^0	K

$K > 0$

$1 - K\tau > 0$

$K\tau < 1 \Rightarrow K < \frac{1}{\tau}$ $\quad\Rightarrow\quad 0 < K < 1/\tau$

ROOT LOCUS

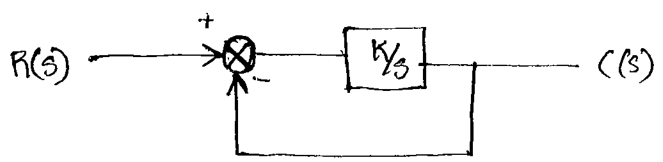
PURPOSE :
1. To find the closed loop system stability.
2. To find the range of K value for system stability.
3. To find the K value to become system marginal stable (undamped)
4. To find the natural frequency of oscillation or undamped oscillati
5. To find the K values for undamped, underdamped, critically damped and overdamped systems.
6. To find the relative stability (i) If the Root Locus branches moving towards the left, then the system is more relatively stable.
   (ii) If the root locus branches moving towards the right, the system is less relatively stable.
   (iii) The best method to find relative stability is Root Locus.
   (iv) The best method to find absolute stability is RH CRITERIA

DEFINITION OF ROOT LOCUS

Root means roots of characteristic equation, which are nothing but closed loop poles. Locus mean path.

Root Locus means closed loop poles path by varying K from $0 < K <$

Q Draw the Root Locus to the following system,



$$CLTF = \frac{K}{S+K}$$

Root locus is nothing but closed loop poles path. The closed loop Poles path given by characteristic eqn $1+G(s) = 0$

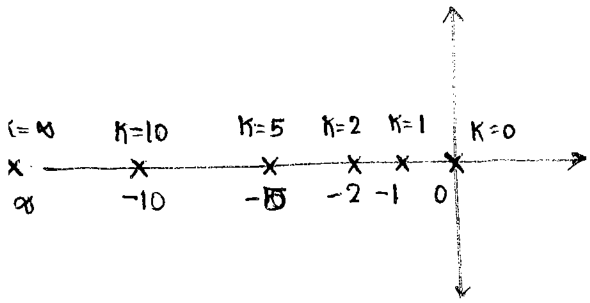
$$1 + G(s) = 0$$

$$1 + K/s = 0$$

$$s + K = 0$$

$$s = -K$$

K value	pole location $= -K$
0	0
1	1
2	-2
5	-5
10	-10
⋮	⋮
∞	$-\infty$



Q $\quad G(s) = \dfrac{K}{s^2}$

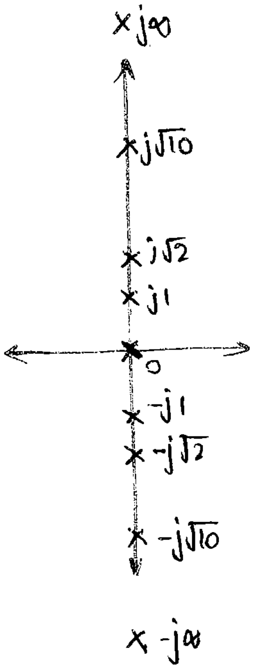
$$1 + G(s) = 0$$

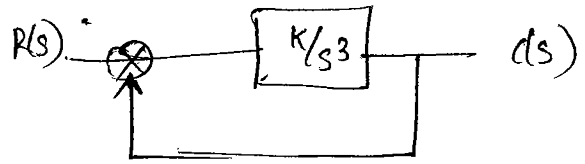
$$s^2 + K = 0$$

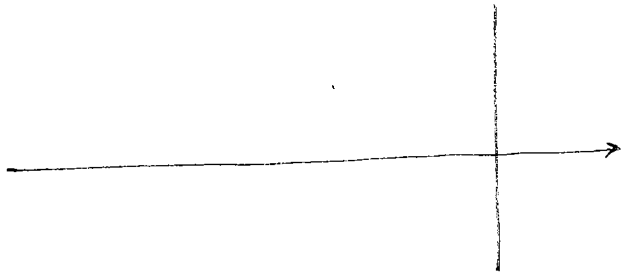
$$s^2 = -K$$

$$s = \pm j\sqrt{K}$$

K value	Pole location $= \pm j\sqrt{K}$
0	0
1	$\pm j1$
2	$\pm j\sqrt{2}$
5	
10	$\pm j\sqrt{10}$







$1 + G(s) = 0$

$s^3 + K = 0$

$s^3 = -K$

$s = (-K)^{1/3} = \sqrt[3]{-K}$

Very difficult. Can't be determined as order increases.

"⇒ As the order increases, finding the roots for the characteristic eqn is very difficult. Hence ~~the~~ we use the open loop transfer function to draw the root locus diagram. But the stability analysis is for closed loop system."

RELATIONSHIP B/W OLTF AND CLTF POLES & ZEROS

Let OLTF $\quad G(s)H(s) = \dfrac{KN(s)}{D(s)} \longrightarrow (1)$

For open Loop Poles $\quad D(s) = 0$

For open Loop Zeros $\quad N(s) = 0$

The closed Loop poles are given by characteristic eqn

$$1 + G(s)H(s) = 0$$

$$1 + K\frac{N(s)}{D(s)} = 0$$

ie, $\underbrace{D(s)}_{\text{openloop poles}} + \underbrace{KN(s)}_{\text{open loop Zeros.}} = 0 \longrightarrow (2)$

→ "The closed Loop poles are nothing but sum of open loop poles and open loop zeros with the function of system gain K."

→ "The closed loop system stability depends on open loop poles and open loop Zeros."

→ But open loop zeros never effect open loop stability and closed loop zeros never effect closed loop stability.

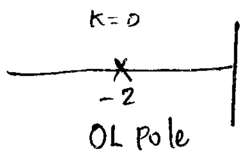
<u>Case (i)</u> $K = 0$

(2) ⇒ ⇒ $K = \dfrac{-D(s)}{N(s)}$

$\therefore K = 0 \Rightarrow D(s) = 0$ which corresponds to open loop poles.

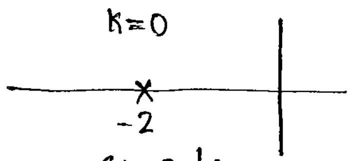
$\boxed{\text{ie, when } K=0 \Rightarrow \text{CLOSED LOOP POLES} = \text{OPEN LOOP POLES}}$

eg: $G(s) = \dfrac{K}{s+2}$, $H(s) = 1$



$CLTF = \dfrac{K}{s+2+K}$

=



<u>Case (ii)</u> $K = \infty$

(2) ⇒ $K = \pm\infty \Leftrightarrow N(s) = 0$ which represents Open loop Zeros.

when $K = +\infty$	⇒	CLOSED LOOP POLES	=	OPEN LOOP ZEROS
when $K = -\infty$	⇒	CLOSED LOOP POLES	=	OPEN LOOP ZEROS

Normally $-K$ values not considered for Root Locus.

→ FOR DIRECT LOCUS (DRL)

$K=0$:→	OPEN LOOP POLE
⇓	⇓
$K=\infty$:→	OPEN LOOP ZERO

× → O Direction of Root Locus.

→ FOR INDIRECT ROOT LOCUS (IRL)

$K=-\infty$:→	OPEN LOOP ZERO
⇓	⇓
$K=0$:→	OPEN LOOP POLE

O → × Direction of Root locus.

→ The direct Root Locus Branch should start at OPEN LOOP POLE ($K=0$) and it should end at OPEN LOOP ZERO ($K=\infty$)

→ The inverse Root Locus Branch should start at OPEN LOOP ZERO ($K=-\infty$) and it should end at OPEN LOOP POLE ($K=0$)

Q. Find where the root locus branches starts and ends.

$$G(s)H(s) = \frac{K(s+1)}{s(s+5)(s+10)}$$

ends OL zeros : $-1, \underbrace{+\infty, +\infty}_{\text{assumed}}$ → $K=\infty$

↑

starts OL POLES : $0, -5, -10$ → $K=0$

~~when K increases from 0 to ∞, pole~~

⇒ In a root Locus Diagram, the number of poles must be equal to number of zeros because the branch start at poles, end at zeros. For each and every pole require a ZERO. If the zeros are less, we assume zeros at infinity (∞). The direction of infinity given by angle of asymptotes.

ANGLE AND MAGNITUDE CONDITION

The Construction Rules of Root Locus are framed by using the angle condition such that the Root Locus Diagram should gives the closed Loop poles path and closed loop system stability.

The closed loop system stability given by characteristic equation.

NEGATIVE FEEDBACK RULES	POSITIVE FEED BACK RULES
DRL / 180° RULES	IRL / CRL / 0° RULES

NEGATIVE FEEDBACK RULES

CE ⇒ $1 + G(s)H(s) = 0$

$$G(s)H(s) = -1 + j0$$

1. ANGLE CONDITION

Angle condition,

$$\angle G(s)H(s) = \angle(-1+j0)$$

= Odd Multiples of $\pm 180^\circ$

$$\boxed{\angle G(s)H(s) = \pm(2q+1)180^\circ}$$

where $q = 0, 1, 2, 3 \ldots$

POSITIVE FEED BACK RULES

CE ⇒ $1 - G(s)H(s) = 0$

$$G(s)H(s) = 1 + j0$$

Angle condition,

$$\angle G(s)H(s) = \angle(1+j0)$$

= Even multiples of $\pm 180^\circ$

$$\boxed{\angle G(s)H(s) = \pm(2q)180^\circ}$$

where $q = 0, 1, 2, 3, \ldots$

	DRL		IRL
In R_4	$2q+1$	⟺	$2q$
In R_3	ODD	⟺	EVEN
In R_6 Case (iii)	Left Most	⟺	Right Most
In R_8	180°	⟺	0°

PURPOSE : The purpose of Angle Condition is to check whether any point lies on Root Locus or Not. ie, All the points on the Root Locus must satisfy the angle Condition.

Q check whether the following points lies on root locus or not, for $G(s)H(s) = \dfrac{K}{s(s+5)(s+10)}$

(i) $s=-3$ (ii) $s=-6$

Angle condition

$$\angle G(s)H(s) = \frac{\angle K}{\angle s \ \angle s+5 \ \angle s+10}$$

(i) $s=-3$

$$= \frac{\angle K}{\angle -3 \ \angle 2 \ \angle 7}$$

$$= \frac{0^\circ}{\pm 180^\circ + 0^\circ + 0^\circ}$$

$\angle G(s)H(s) = 1(\mp 180^\circ) \Rightarrow$ ODD multiples $\pm 1(180^\circ)$

satisfies angle condition

Hence -3 lies on root locus.

(ii) $s=-6$

$$\angle G(s)H(s) = \frac{\angle K}{\angle -6 \ \angle -1 \ \angle +4}$$

$$= \frac{0^\circ}{(\pm 180^\circ)+(\pm 180^\circ)+0}$$

$= \mp 2(180^\circ) \Rightarrow$ EVEN multiple of ± 180

Not satisfies

Hence ~~is~~ -6 not lies on root locus.

2. MAGNITUDE CONDITION (FOR DRL)

$$\boxed{|G(s)\,H(s)| = 1 \quad \text{at any point which is on Root Locus.}}$$

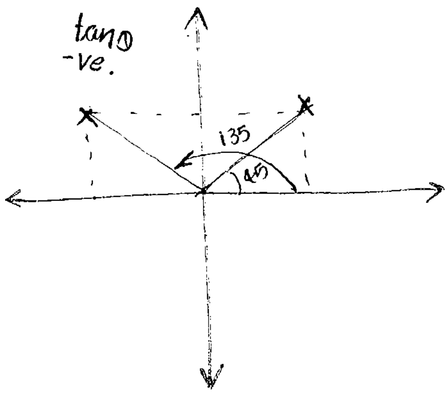
The magnitude condition is valid only when given point is on Root Locus. The given point on root locus is verified by Angle condition. ie, The angle condition must be satisfied to apply magnitude condition.

PURPOSE : The purpose of Magnitude Condition is to find the System Gain at any point which is on the Root Locus.

~~Gt~~

Q. Find the system gain at the point, $s = -5+j5$ to the

$$G(s)\,H(s) = \frac{K}{s(s+10)}$$



$$\angle G(s)\,H(s) = \frac{\angle K}{\angle -5+j5 + \angle 5+j5}$$

$$= \frac{\angle K}{\tan^{-1}-1 + \tan^{-1}1}$$

$$= \frac{\angle K}{-\pi/4 + \pi/4}$$ X wrong

Take angle only w.r.t to origin

$$= \frac{\angle K}{3\pi/4 + \pi/4}$$

$$= 0^\circ - 180^\circ \Longrightarrow \text{odd multiples of } \pm 180^\circ$$

Hence Angle condition satisfies.
Hence can apply Magnitude condition.

$$\left|\frac{K}{s(s+10)}\right|_{at\ s=-5+j5} = 1$$

$$\left|\frac{K}{(-5+j5)(5+j5)}\right| = 1$$

$$\frac{K}{\sqrt{25+25}\ \sqrt{25+25}} = 1$$

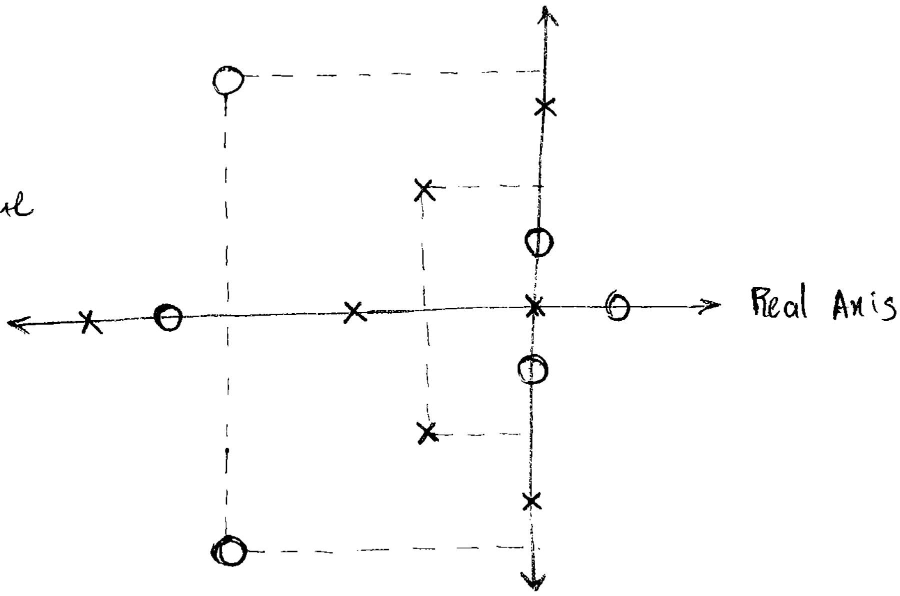
$$\underline{K = 50}$$

CONSTRUCTION RULES OF ROOT LOCUS

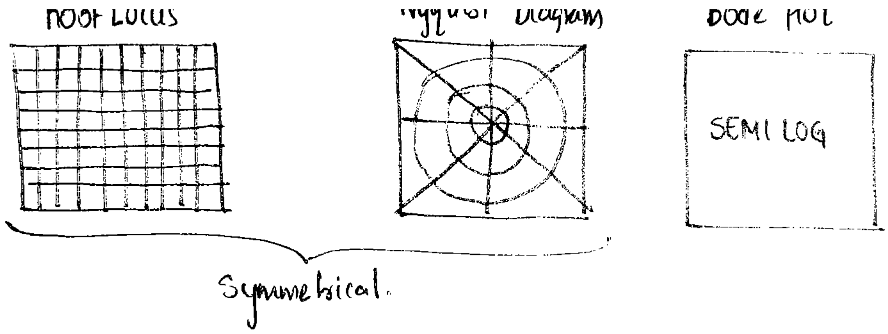
RULE 1 : SYMMETRICAL

→ The Root Locus Diagram is symmetrical about the real axis because the location of the poles and zeros in the splane symmetric about the real axis.

→ (Root Locus, and Nyquist plot is symmetrical, But Bode plot is not symmetrical)



→ The symmetry not only depends on poles and zeros, it depends on the graph sheet on which the diagram is drawn.
The Bode plot is not symmetrical about the real axis because the Bode plot is ~~non linear~~ drawn on semilog sheet which is nonlinear.



RULE - 2

Number of Loci or RL branches depends on number of poles and zeros.

Case (i) Poles > Zeros

$$\boxed{\text{Number of Loci} = \text{Number of Poles}}$$

Poles < zero

$$\boxed{\text{Number of Loci} = \text{Number of zeros}}$$

CONTINUED IN BOOK 2